Hyperbolic Tangent Fluid Flow Through A Porous Medium in an Inclined Channel with Peristalsis

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Abstract

In the present analysis, we investigated the peristaltic transport of a hyperbolic tangent fluid through a porous medium in a two-dimensional symmetric inclined channel under the assumptions of low Reynolds number and long wavelength. The expression for the velocity and axial pressure gradient are obtained by employing regular perturbation technique. It is observed that the pumping is less for hyperbolic tangent fluid than that of Newtonian fluid. The effects of various pertinent parameters on the time-averaged flow rate are discussed with the help of graphs.

Keywords: *Hyperbolic tangent fluid, Symmetric inclined channel, Porous medium*

1. Introduction

Peristaltic transport is a form of fluid transport generated by a progressive wave of area contraction or expansion along a length of a distensible tube containing fluid. Peristaltic transport widely occurs in many biological systems for example, food swallowing through the esophagus, intra-urine fluid motion, circulation of blood in small blood vessels and the flows of many other glandular ducts. Several theoretical and experimental studies have been undertaken to understand peristalsis through abrupt changes in geometry and realistic assumptions.

Peristaltic transport of Newtonian fluids has been studied by Fung and Yih (1968), Shapiro et al. (1969) and Subba Reddy et al. (2005) under different conditions. Moreover, it is well known that most physiological fluids including blood behave as non-Newtonian fluids. So, the study of peristaltic transport of non-Newtonian fluids may help to get better understanding of the working biological systems. Fluids in which viscosity depends on the shear stress or on the flow rate are of considerable practical importance and is considered as special class of fluids. For most of the non-Newtonian fluids, viscosity is usually a nonlinear decreasing function of the generalized shear rate. This is known as shear-thinning behavior. Such fluid is a hyperbolic tangent fluid (Ai and Vafai (2005)). The peristaltic flow of a hyperbolic tangent fluid in an asymmetric channel was discussed by Nadeem and Akram (2009). Nadeem and Akram (2011) have analyzed the effect of magnetic field on the peristaltic motion of a hyperbolic tangent fluid in a vertical asymmetric channel with heat transfer. The peristaltic transport of a Tangent hyperbolic fluid in an endoscope numerically was investigated by Nadeem and Akbar (2011). Akbar et al. (2012) have studied the effects of slip and heat transfer on the peristaltic transport of a hyperbolic tangent fluid in an inclined asymmetric channel.

A porous medium is the matter which contains a number of small holes distributed throughout the matter. Flows through a porous medium occur in filtration of fluids. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium were investigated by Hayat et al. (2007). El-Dabe et al. (2010) have discussed the effect of magnetic field on the peristaltic motion of a Carreau fluid through a porous medium with heat transfer. Navaneeswar Reddy et al. (2012) have analyzed the peristaltic flow of a Prandtl fluid through a porous medium in a channel. Peristaltic pumping of a fourth grade fluid through a porous medium in an asymmetric channel was investigated by Jyothi et al. (2012). Subba Reddy and Nadhamuni Reddy (2014) have studied the peristaltic flow of a non-Newtonian fluid through a porous medium in a tube with variable viscosity using Adomian decomposition method.

In view of these, we investigated the peristaltic transport of a hyperbolic tangent fluid through a porous medium in an inclined channel under the assumptions of low Reynolds number and long wavelength. The expression for the velocity and axial pressure gradient are obtained by employing perturbation technique. The effects of various pertinent parameters on the time-averaged flow rate are discussed with the help of graphs.

2. MATHEMATICAL FORMULATION

We consider the peristaltic motion of a hyperbolic tangent fluid through a porous medium in a two-dimensional symmetric channel of width . The flow is generated by sinusoidal wave trains propagating with constant speed along the channel walls. The channel walls are inclined at an angle to the horizontal. Fig. 1 shows the schematic diagram of the channel. The wall deformation is given by

$$Y = \pm H(X, t) = \pm a \pm b \sin \frac{2\pi}{\lambda}(X - ct) \qquad (1)$$

where b is the amplitude of the wave, λ - the wave length and X and Y - the rectangular co-ordinates with X measured along the axis of the channel and Y perpendicular to X . Let (U, V) be the velocity components in fixed frame of reference (X, Y) .
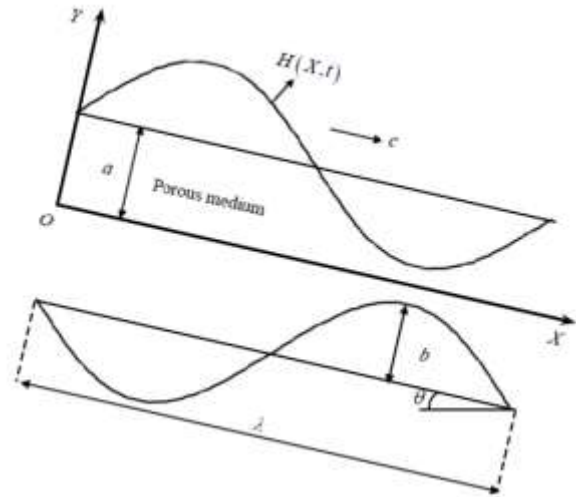

Fig. 1 The physical model

The flow is unsteady in the laboratory frame . However, in a co-ordinate system moving with the propagation velocity c (wave frame (x, y)), the boundary shape is stationary. The transformation from fixed frame to wave frame is given by

$$x = X - ct, y = Y, u = U - c, v = V \qquad (2)$$

where (u, v) and (U, V) are velocity components in the wave and laboratory frames respectively. The constitutive equation for a Hyperbolic Tangent fluid is

$$\tau = -\left[\eta_\infty + (\eta_0 + \eta_\infty)\tanh\left(\Gamma\dot\gamma\right)^n\right]\dot\gamma \qquad (3)$$

where τ is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_o is the zero shear rate viscosity, Γ is the time constant, n is the power-law index and $\dot\gamma$ is defined as

$$\dot\gamma = \sqrt{\frac{1}{2}\sum_i\sum_j \dot\gamma_{ij}\dot\gamma_{ji}} = \sqrt{\frac{1}{2}\pi} \qquad (4)$$

where π is the second invariant stress tensor. We consider in the constitutive equation (3) the case for which $\eta_\infty = 0$ and $\Gamma\dot\gamma < 1$, so the Eq. (3) can be written as

$$\tau = -\eta_0 \left(\Gamma \dot{\gamma} \right)^n \dot{\gamma} = -\eta_0 \left(1 + \Gamma \dot{\gamma} - 1 \right)^n \dot{\gamma} = -\eta_0 \left(1 + n \left[\Gamma \dot{\gamma} - 1 \right] \right) \dot{\gamma} \tag{5}$$

The above model reduces to Newtonian for $\Gamma = 0$ and $n = 0$.

The equations governing the flow in the wave frame of reference are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \left(\frac{\eta_0}{k} \right)(u + c) + \rho g \sin\theta \tag{7}$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} - \frac{\eta_0}{k} v - \rho g \cos\theta \tag{8}$$

where ρ is the density σ is the electrical conductivity, B_0 is constant transverse magnetic field and k is the permeability of the porous medium. The corresponding dimensional boundary conditions are

$$u = -c \qquad \text{at} \qquad y = H \tag{9}$$

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at} \qquad y = 0 \tag{10}$$

Introducing the non-dimensional variables defined by

$$\overline{x} = \frac{x}{\lambda}, \quad \overline{y} = \frac{y}{a}, \quad \overline{u} = \frac{u}{c}, \quad \overline{v} = \frac{v}{c\delta}, \quad \delta = \frac{a}{\lambda}, \quad \overline{p} = \frac{pa^2}{\eta_0 c \lambda}, \phi = \frac{b}{a}$$

$$h = \frac{H}{a}, \quad \overline{t} = \frac{ct}{\lambda}, \quad \overline{\tau}_{xx} = \frac{\lambda}{\eta_0 c}\tau_{xx}, \quad \overline{\tau}_{xy} = \frac{a}{\eta_0 c}\tau_{xy}, \quad \overline{\tau}_{yy} = \frac{\lambda}{\eta_0 c}\tau_{yy},$$

$$\text{Re} = \frac{\rho ac}{\eta_0}, \quad We = \frac{\Gamma c}{a}, \quad \overline{\dot{\gamma}} = \frac{\dot{\gamma} a}{c}, \quad \overline{q} = \frac{q}{ac} \tag{11}$$

into the Equations (6) - (8), reduce to (after dropping the bars)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{12}$$

$$\text{Re}\,\delta \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} - \delta^2 \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} - \frac{1}{Da}(u + 1) + \frac{\text{Re}}{Fr}\sin\theta \tag{13}$$

$$\text{Re}\,\delta^3 \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} - \delta^2 \frac{\partial \tau_{xy}}{\partial y} - \delta \frac{\partial \tau_{yy}}{\partial y} - \frac{\delta^2}{Da}v - \delta \frac{\text{Re}}{Fr}\cos\theta \tag{14}$$

where $\qquad \tau_{xx} = -2 \left[1 + n \left(We\dot{\gamma} - 1 \right) \right] \frac{\partial u}{\partial x}$,

$$\tau_{xy} = -\left[1 + n \left(We\dot{\gamma} - 1 \right) \right] \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right)$$

$$\tau_{yy} = -2\delta \left[1 + n \left(We\dot{\gamma} - 1 \right) \right] \frac{\partial v}{\partial y}$$

$$\dot{\gamma} = \left[2\delta^2 \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right)^2 + 2\delta^2 \left(\frac{\partial v}{\partial y} \right)^2 \right]^{\frac{1}{2}}$$

$Fr = \dfrac{c^2}{ag}$ is the Froude number, and $Da = \dfrac{k}{a^2}$ is the Darcy number.

Under lubrication approach, neglecting the terms of order δ and Re, the Equations (13) and (14) become

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left\{ \left[1 + n \left(We \frac{\partial u}{\partial y} - 1 \right) \right] \frac{\partial u}{\partial y} \right\} - \frac{1}{Da}(u + 1) + \frac{\text{Re}}{Fr}\sin\theta \tag{15}$$

$$\frac{\partial p}{\partial y} = 0 \tag{16}$$

From Eq. (15) and (16), we get

$$\frac{dp}{dx} = (1 - n)\frac{\partial^2 u}{\partial y^2} + nWe\frac{\partial}{\partial y}\left[\left(\frac{\partial u}{\partial y} \right)^2 \right] - \frac{1}{Da}(u + 1) + \frac{\text{Re}}{Fr}\sin\theta \tag{17}$$

The corresponding non-dimensional slip boundary conditions in the wave frame are given by

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$$u = -1 \text{ at } \qquad y = h = 1 + \phi \sin 2\pi x \qquad (18)$$

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at} \qquad y = 0 \qquad (19)$$

The volume flow rate q in a wave frame of reference is given by

$$q = \int_0^h u\, dy . \qquad (20)$$

The instantaneous flow $Q(X, t)$ in the laboratory frame is

$$Q(X, t) = \int_0^h U\, dY = \int_0^h (u + 1)\, dy = q + h \qquad (21)$$

The time averaged volume flow rate $\overline{Q}$ over one period $T\left(= \dfrac{\lambda}{c} \right)$ of the peristaltic wave is given by

$$\overline{Q} = \frac{1}{T} \int_0^T Q\, dt = q + 1 \qquad (22)$$

3. SOLUTION

Since Eq. (17) is a non-linear differential equation, it is not possible to obtain closed form solution. Therefore we employ regular perturbation to find the solution.

For perturbation solution, we expand u, $\dfrac{dp}{dx}$ and q as follows

$$u = u_0 + We\, u_1 + O\left(We^2\right) \qquad (23)$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + We\, \frac{dp_1}{dx} + O\left(We^2\right) \qquad (24)$$

$$q = q_0 + We\, q_1 + O\left(We^2\right) \qquad (25)$$

Substituting these equations into the Eqs. (17) - (19), we obtain

3.1. System of order We^0

$$(1-n)\frac{\partial^2 u_0}{\partial y^2} - \frac{1}{Da} u_0 = \frac{dp_0}{dx} - \frac{Re}{Fr}\sin\theta + \frac{1}{Da} \qquad (26)$$

and the respective boundary conditions are

$$u_0 = -1 \qquad \text{at} \qquad y = h \qquad (27)$$

$$\frac{\partial u_0}{\partial y} = 0 \qquad \text{at} \qquad y = 0 \qquad (28)$$

3.2. System of order We^1

$$(1-n)\frac{\partial^2 u_1}{\partial y^2} - \frac{1}{Da} u_1 = \frac{dp_1}{dx} - \frac{\partial}{\partial y}\left[\left(\frac{\partial u_o}{\partial y} \right)^2 \right] \qquad (29)$$

and the respective boundary conditions are

$$u_1 = 0 \quad \text{at } y = h \qquad (30)$$

$$\frac{\partial u_1}{\partial y} = 0 \text{ at } y = 0 \qquad (31)$$

3.3 Solution for system of order We^0

Solving Eq. (26) using the boundary conditions (27) and (28), we obtain

$$u_0 = \frac{1}{\Omega^2(1-n)}\left(\frac{dp_0}{dx} - \frac{Re}{Fr}\sin\theta \right)\left[\frac{\cosh\Omega y}{\cosh\Omega h} - 1 \right] - 1 \qquad (32)$$

where $\Omega = 1/\sqrt{Da(1-n)}$

The volume flow rate q_0 is given by

$$q_0 = \frac{1}{\Omega^3(1-n)}\left(\frac{dp_0}{dx} - \frac{Re}{Fr}\sin\theta \right)\left[\frac{\sinh\Omega h - \Omega h\cosh\Omega h}{\cosh\Omega h} \right] - h \qquad (33)$$

From Eq. (33), we have

$$\frac{dp_0}{dx} = \frac{(q_0 + h)\Omega^2(1-n)\cosh\alpha_1 h}{[\sinh\Omega h - \Omega h\cosh\Omega h]} + \frac{Re}{Fr}\sin\theta \qquad (34)$$

3.4 Solution for system of order We^1

Substituting Eq. (32) in the Eq. (29) and solving the Eq. (29), using the boundary conditions (30) and (31), we obtain

$$u_1 = \frac{1}{\Omega^2(1-n)}\frac{dp_1}{dx}\left[\frac{\cosh\Omega y}{\cosh\Omega h} - 1\right] \qquad (35)$$

$$+ \frac{n}{3}\frac{\left(\frac{dp_0}{dx} - \frac{Re}{Fr}\sin\theta\right)^2}{[\Omega(1-n)\cosh\Omega h]^3}\left[\begin{array}{l}(\sinh 2\Omega h - 2\sinh\Omega h)\cosh\Omega y \\ + (2\sinh\Omega y - \sinh 2\Omega y)\cosh\Omega h\end{array}\right]$$

The volume flow rate q_1 is given by

$$q_1 = \frac{1}{\Omega^3(1-n)}\frac{dp_1}{dx}\left[\frac{\sinh\Omega h - \Omega h\cosh\Omega h}{\cosh\Omega h}\right] + \frac{Kn\left(\frac{dp_0}{dx} - \frac{Re}{Fr}\sin\theta\right)^2}{6\Omega^2(1-n)^3\cosh^3\Omega h} \qquad (36)$$

where $K = 4 - \cosh\Omega h + 2\sinh 2\Omega h\sinh\Omega h - \cosh\Omega h\cosh 2\Omega h$.

From Eq. (36) and (34), we have

$$\frac{dp_1}{dx} = \frac{q_1\Omega^3(1-n)\cosh\Omega h}{[\sinh\Omega h - \Omega h\cosh\Omega h]} - \frac{Kn}{6\Omega(1-n)^2}\frac{\left(\frac{dp_0}{dx} - \frac{Re}{Fr}\sin\theta\right)^2}{\cosh^2\Omega h(\sinh\Omega h - \Omega h\cosh\Omega h)} \qquad (37)$$

Substituting Equations (34) and (37) into the Eq. (24) and using the relation

$$\frac{dp_0}{dx} = \frac{dp}{dx} - We\frac{dp_1}{dx} \quad \text{and neglecting terms}$$

greater than $O(We)$, we get

$$\frac{dp}{dx} = \frac{(q+h)\Omega^3(1-n)\cosh\Omega h}{[\sinh\Omega h - \Omega h\cosh\Omega h]} - \frac{WeKn\Omega^5}{6}\frac{(q+h)^2}{(\sinh\Omega h - \Omega h\cosh\Omega h)^3} + \frac{Re}{Fr}\sin\theta \qquad (38)$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx}dx \qquad (39)$$

Note that, as $\theta \to 0$, $Da \to \infty$, $We \to 0$ and $n \to 0$ our results coincide with the results of Shapiro et al. (1969).

4. RESULTS AND DISCUSSIONS

The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of We with $\phi = 0.5$, $n = 0.5$, $Fr = 0.2$, $Re = 1$, $\theta = \frac{\pi}{4}$ and $Da = 0.1$ is illustrated in Fig. 2. It is noted that, the time-averaged volume flow rate $\overline{Q}$ increases with increasing We in pumping $(\Delta p > 0)$, free-pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ regions.

Fig. 3 shows the variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of n with $\phi = 0.5$, $Fr = 0.2$, $Re = 1$, $\theta = \frac{\pi}{4}$, $We = 0.01$ and $Da = 0.1$. It is observed that, the time-averaged volume flow rate $\overline{Q}$ decreases with an increase in n in both the pumping and free pumping regions, while it increases with increasing n in co-pumping region for chosen $\Delta p (< 0)$.

The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of

Da with $\phi = 0.5$, $Fr = 0.2$, $\mathrm{Re} = 1$, $\theta = \dfrac{\pi}{4}$, $n = 0.5$ and $We = 0.01$ is shown in Fig. 4. It is noted that, the time-averaged volume flow rate $\overline{Q}$ decreases with increasing Da in the pumping region, while it increases with increasing Da in both the free-pumping and co-pumping regions.

Fig. 5 shows the variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of Re with $\phi = 0.5$, $Da = 0.1$, $n = 0.5$, $Fr = 0.2$, $\theta = \dfrac{\pi}{4}$ and $We = 0.01$. It is found that, the time-averaged volume flow rate $\overline{Q}$ increases with increasing Re in all the three regions.

The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of Fr with $\phi = 0.5$, $Da = 0.1$, $n = 0.5$, $\mathrm{Re} = 1$, $\theta = \dfrac{\pi}{4}$ and $We = 0.02$ is shown in Fig. 6. It is observed that, the time-averaged volume flow rate $\overline{Q}$ decreases with increasing Fr in all the three regions.

Fig. 7 depicts the variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of θ with $\phi = 0.5$, $Da = 0.1$, $n = 0.5$, $Fr = 0.2$, $\theta = \dfrac{\pi}{4}$ and $We = 0.02$. It is noted that, the time-averaged volume flow rate $\overline{Q}$ increases with increasing θ in all the three regions.

The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of ϕ with $\mathrm{Re} = 1$, $Da = 0.1$, $n = 0.5$, $Fr = 0.2$, $\theta = \dfrac{\pi}{4}$ and $We = 0.02$ is depicted in Fig. 8. It is found that, the time-averaged volume flow rate $\overline{Q}$ increases with an increase in ϕ in both the pumping and free-pumping regions, while it decreases with increasing ϕ in the co-pumping region for chosen $\Delta p \left(< 0 \right)$.

5. CONCLUSIONS

In this paper, we studied the peristaltic flow of a hyperbolic tangent fluid through a porous medium in a planar channel under the assumptions of low Reynolds number and long wavelength. The expression for the velocity and axial pressure gradient are obtained by employing perturbation technique. It is observed that, in the pumping region the time-averaged flow rate $\overline{Q}$ increases with increasing We, Re, θ and ϕ, while it decreases with increasing n, Da and Fr. Moreover, it is observed that the pumping is less for hyperbolic tangent fluid than that of Newtonian fluid.
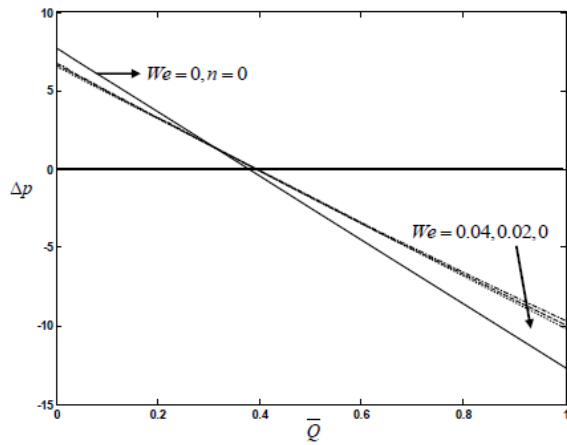
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Fig. 2 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of We with $\phi = 0.5$, $n = 0.5$, $Fr = 0.2$, $Re = 1$, $\theta = \dfrac{\pi}{4}$ and $Da = 0.1$.
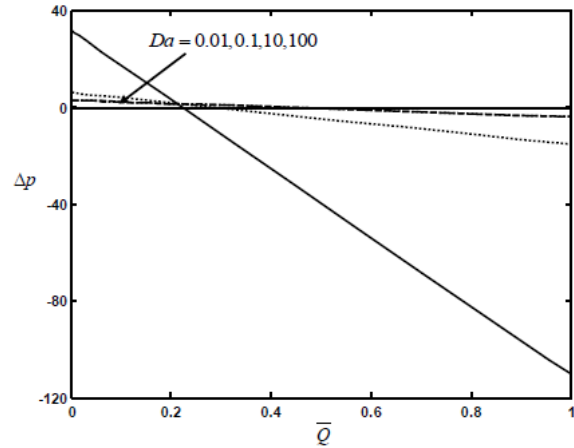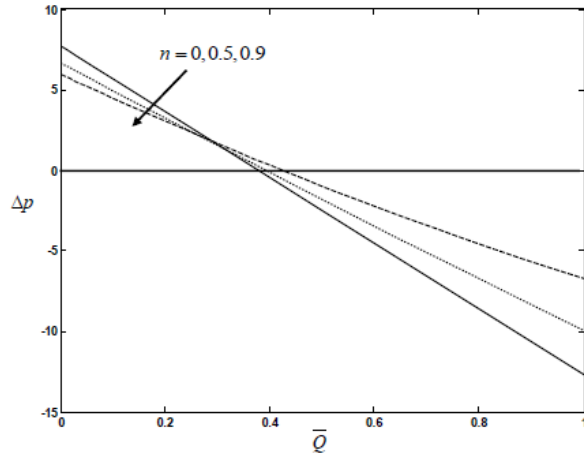

Fig. 4 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of Da with $\phi = 0.5$, $n = 0.5$, $Fr = 0.2$, $Re = 1$, $\theta = \dfrac{\pi}{4}$ and $We = 0.02$.

Fig. 3 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of n with $\phi = 0.5$, $Fr = 0.2$, $Re = 1$, $\theta = \dfrac{\pi}{4}$, $We = 0.02$ and $Da = 0.1$.
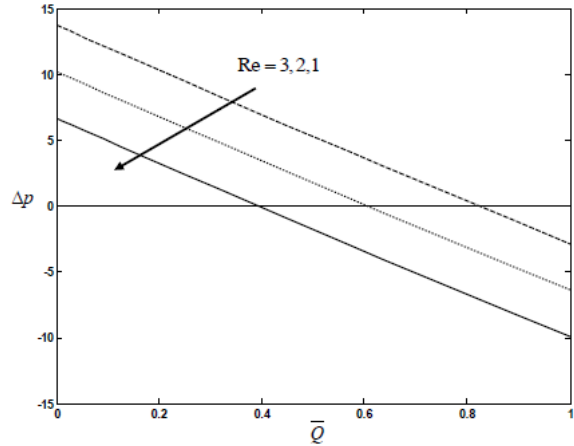

Fig. 5 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of Re with $\phi = 0.6, Da = 0.1$, $n = 0.5$, $Fr = 0.2$, $\theta = \dfrac{\pi}{4}$ and $We = 0.01$.
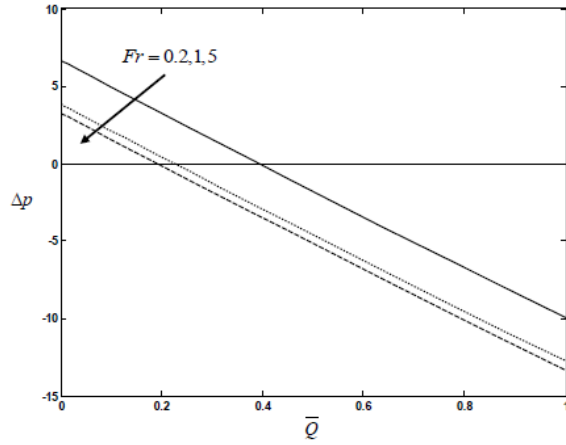
Fig. 6 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of Fr with $\phi = 0.6, Da = 0.1$, $n = 0.5$, $Re = 1$, $\theta = \dfrac{\pi}{4}$ and $We = 0.02$.
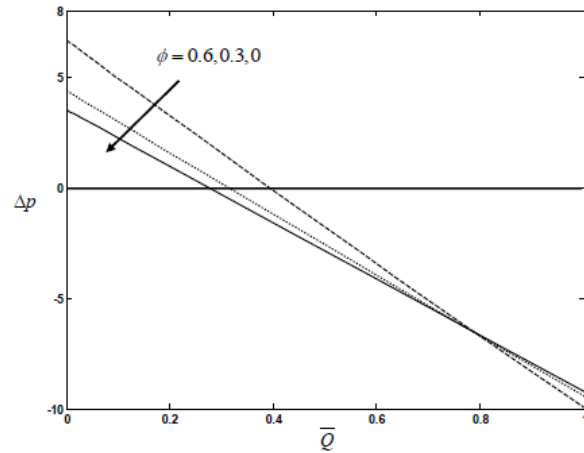
Fig. 8 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of ϕ with $We = 0.02$, $n = 0.5$, $Fr = 0.2$, $Re = 1$, $\theta = \dfrac{\pi}{4}$ and $Da = 0.1$.
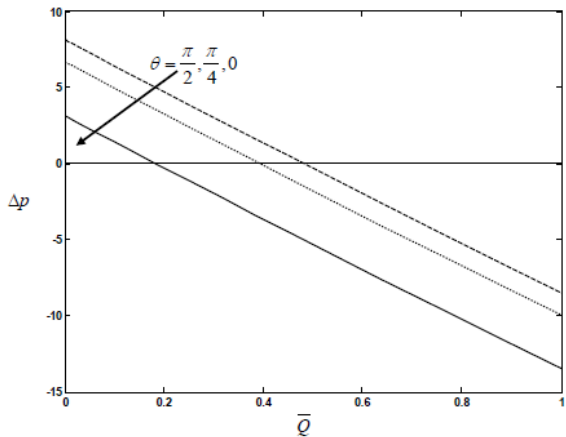
Fig. 7 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of θ with $\phi = 0.6, Da = 0.1$, $n = 0.5$, $Fr = 0.2$, $\theta = \dfrac{\pi}{4}$ and $We = 0.02$.

References

[1] Ai, L. and Vafai, K., An investigation of Stokes' second problem for non-Newtonian fluids. *Numerical Heat Transfer, Part A,* 47, 955–980 ,(2005).

[2] Akbar, N.S., Hayat, T., Nadeem, S. and Obaidat, S. , Peristaltic flow of a Tangent hyperbolic fluid in an inclined asymmetric channel with slip and heat transfer. *Progress in Computational Fluid Dynamics*, *an International Journal* 12(5), 363-374 ,(2012).

[3] El-Dabe, N. T.M., Fouad, A. and Hussein, M.M. , Magnetohydrodynamic flow and heat transfer for a peristaltic motion of Carreau fluid through a porous medium. *Punjab University Journal of Mathematics* 42, 1-16,(2010).

[4] Fung, Y.C. and Yih, C.S.,Peristaltic transport, *Trans. ASME J. Appl. Mech.*, 35, 669-675,(1968).

[5] Hayat, T., Ali, N. and Asghar, S, Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. *Phys. Lett. A,* 363, 397- 403, (2007).

[6] Jyothi, S., Subba Reddy, M. V. and Ramakrishna Prasad, A., Peristaltic Pumping Of a Fourth Grade Fluid Through a Porous Medium in An Asymmetric Channel. *International Journal of Mathematical Archive*-3(1), Page: 330-341,(2012).

[7] Nadeem, S. and Akram, S., Peristaltic transport of a hyperbolic tangent fluid model in an asymmetric channel. *Z. Naturforsch.,* 64a, 559 – 567,(2009).

[8] Nadeem, S. and Akram, S., Magnetohydrodynamic peristaltic flow of a hyperbolic tangent fluid in a vertical asymmetric channel with heat transfer. *Acta Mech. Sin.,* 27(2), 237–250,(2011).

[9] Nadeem, S. and Akbar, S. , Numerical analysis of peristaltic transport of a Tangent hyperbolic fluid in an endoscope. *Journal of Aerospace Engineering* 24(3), 309-317,((2011).

[10] Navaneeswara Reddy, S., Viswanatha Reddy, G. and Subba Reddy, M. V., Peristaltic flow of a Prandtl fluid through a porous medium in a channel. *Int. J. Mathematical Archive,* 3(11), 4072 – 4080, (2012).

[11] Shapiro, A.H., Jaffrin, M.Y and Weinberg, S.L. , Peristaltic pumping with long wavelengths at low Reynolds number. *J. Fluid Mech.* 37, 799-825,(1969).

[12] Subba Reddy, M. V., Mishra, M., Sreenadh, S. and Ramachandra Rao, A., Influence of lateral walls on peristaltic pumping in a rectangular duct. *Trans. ASME. Journal of Fluids Engineering* 127, 824 – 827,(2005).

[13] Subba Reddy, M. V. and Nadhamuni Reddy, C. ,Peristaltic flow of a non-Newtonian fluid through a porous medium in a tube with variable viscosity using Adomian decomposition method. *International Review of Applied Engineering Research* 4(1), 137-146, (2014).