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# Sensitivity and stability of super-efficiency in data envelopment analysis models

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**Abstract**

Data Envelopment Analysis (DEA) a significant measure tool of efficiency which has been applied for various fields. By applying Standard DEA models can classify the Decision Making Unit (DMUs) into efficiency and inefficiency, and efficient production frontier could be determined but the basic DEA failed to distinguish the difference degree of efficiency when more indicators inputs or outputs were introduced into this model and multiple DMUs were efficient simultaneously. To distinguish efficient units further we employed Super Efficiency (SE) model but the analysis would be infeasible sometimes and we extend our analysis to locate the position of the unit under evaluation when infeasibility occur. Here it is identified the endpoint position of the extreme efficient units. This paper developed analytical methods for studying the sensitivity of DEA results to variations in the data. Necessary and sufficient conditions for preserving a DMU's efficiency classification are developed when various data changes are applied to all DMUs. In our study we have been taken the district Central Cooperative Banks in India to investigate empirically and the results based on empirical investigations are highlighted.

**Keywords: Data Envelopment Analysis, Decision Making Units, Super-Efficiency.**

## 1. Introduction:

Data Envelopment Analysis (DEA) is a methodology based upon an interesting application of linear programming. It was originally developed for performance measurement. It has been successfully employed for assessing the relative performance of a set of firms that uses a variety of identical inputs to produce a variety of identical outputs generally known as Decision Making Units (DMUs). DMU can include manufacturing units, departments of big organizations such as universities, schools, bank branches, hospitals, power plants, police stations, tax offices, prisons, defence bases, a set of forms or even practicing individuals such as medical practitioners, chartered accountants etc.., Farrel (1957) emphasis on the importance of measuring more than one input when facing a problem of efficiency and introduced "efficient production function", as a function to be fitted from empirical data. Charnes, Cooper and Rhodes (CCR) in 1978 is the first and fundamental DEA model, built on the notion of efficiency as defined in the classical engineering ratio with multiple inputs and multiple outputs. The obtained efficiency is never absolute as it is always measured relative to the field. The CCR model admits Constant Returns to Scale (CRS). Banker, Charnes and Cooper (BCC) (1984) model measures Technical efficiency with the convexity constraint, which admits variable returns to scale. Seiford and Thrall (1990) found that mathematical programming procedure used by DEA for efficient frontier estimation is comparatively robust. A large number of papers have extended and applied the DEA methodology (Coelli, 1996). Bhattacharyya et al. (1997) examined the productive efficiency of 70 Indian commercial banks during early stages (1986-1991) prior to liberalization. Charnes, Haag, Jaska and semple ( 1992 ) use a super efficiency model to study the sensitivity of the efficiency classifications. Zhu (1996b) and Seiford and Zhu ( 1998d) to determine the efficiency stability region. Andersen and Petersen ( 1993) propose using the CRS super-efficiency model in ranking the efficient DMUs. Thrall (1996) identifying extreme efficient DMUs. Seiford and Zhu (1999c) study the infeasibility of various super-efficiency models. Seiford and Zhu (1999) further analyzed the causes of infeasibility problem. Lovell and Rouse (2003) proposed a new super-efficiency model, which generated the same scores as conventional model for all units having feasible solution. Sathye (2003) measured the productive efficiency of banks in India using DEA. Ray S. C, "The Directional Distance Function and Measurement of Super-Efficiency(2008). Subbarayan and Prakash ( 2009 ) study on ranking the efficiency performance by DEA. M. Khodabakhshi, M., "Asgharian, and Greg N. Gregoriou. (2010) An Input-oriented Super-efficiency Measure in Stochastic Data Envelopment Analysis: Evaluating Chief Executive Officers of Us Public Banks and Thrifts", Expert Systems with Applications. Dwivedi and Charyulu (2011) seek to determine the impact of various market and regulatory initiatives on efficiency improvements of Indian banks. Prakash, Jayarani and Rajesh ( 2013 ) Performance evaluation of public sector banks in India using DEA approach. GUO Bing, WANG Xia, WANG Yanhong (2013)- A Modified Super-efficiency DEA Model Based on Piecewise Returns to Scale. The study shows that on comparing both CRS and VRS model, it is identified that the number of efficient banks are more in VRS than CRS. This paper organized as follows: Section 2 discusses DEA methodology; Section 3 describes the extension of DEA models for finding Super-Efficiency scores and performing Sensitivity Analysis. Section4 deals with empirical investigations. Finally conclusions are given in Section 5.

## 2. DEA METHODOLOGY

Consider $N$ DMUs and each DMU consumes $q$ inputs and produces $p$ outputs. Let the input and output vectors of $DMU_n$ $(n = 1,2,\dots,N)$ be $X_i = (x_{1n}, x_{2n}, \dots, x_{qn})'$ , $Y_j = (y_{1n}, y_{2n}, \dots, y_{pn})'$ respectively and $X_i > 0, Y_j > 0$. The production possibility set $T$ under CRS assumption and possibility postulates is defined as

$$T = \{(X,Y) \mid X \geq \sum_{n=1}^{N} \lambda_n X_n , \ Y \leq \sum_{n=1}^{N} \lambda_n Y_n , \lambda_n \geq 0, \ n = 1,2,\dots,N\}$$

Under $T$ the CCR model in respect of output oriented is stated as follows

**Max $\varphi_m$**
**such that**

$$\sum_{n=1}^{N} y_{jn}\lambda_n \geq \varphi_m y_{jm} \quad ; \ j=1,2,\dots\dots p$$

$$\sum_{n=1}^{N} x_{in}\lambda_n \leq x_{im}; \ i=1,2,\dots\dots q$$

$$\lambda_n \geq 0 \ ; \ n = 1,2,\dots\dots N$$

**$\varphi_m$ unrestricted**

Similarly the production possibility set under convexity constraint and possibility postulates is defined as

$$T = \{(X,Y) \mid X \geq \sum_{n=1}^{N} \lambda_n X_n , \ Y \leq \sum_{n=1}^{N} \lambda_n Y_n , \sum_{n=1}^{N} \lambda_n = 1, \lambda_n \geq 0, \ n = 1,2,\dots,N\}$$

Under $T$ the BCC model in respect of output oriented is stated as follows

$$\mathbf{Max} \ \varphi_m$$
**s.t**
$$Y\lambda \geq \varphi_m Y_m$$
$$X\lambda \leq X_m$$
$$\sum_{n=1}^{N} \lambda_n = 1$$
$$\lambda_n \geq 0;$$
**$\varphi_m$ Unrestricted**

Based on the above two models it is possible to calculate the efficiency scores of each DMU , construct the reference set formed by peers to each inefficient DMU and rank the DMUs based on the peer count summary . It may be noted that the tie occurs among the efficient DMUs. To break up the tie the Super-efficiency DEA model suggested by Andersen and Petersen is adopted.

## 3. Extensions of DEA - Super Efficiency model and Sensitivity Analysis

On the basis of the DEA models described above when a DMU under evaluation is not included in the reference set the resulting DEA models are called Super-efficiency DEA models. Thus the output-oriented super-efficiency DEA model is as follows

$$Max \ \varphi^{SE}$$

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$$s.t \quad \sum_{\substack{n=1\\n\neq o}}^{N} y_{jn}\lambda_n \geq \varphi^{SE} y_{jo} \quad ,j = 1,2,\dots\dots\dots p;$$

$$\sum_{\substack{n=1\\n\neq o}}^{N} x_{in}\lambda_n \leq x_{io}\,, \qquad i = \quad 1,2,\dots\dots.q;$$

$$\sum_{n=1}^{N} \lambda_n = 1$$

$$\lambda_n \geq 0 \quad ,n = 1,2,\dots\dots\dots.N$$

The above output-oriented super-efficiency DEA model excludes the DMU under evaluation from the reference set so the efficient DMUs may have efficiency scores less than or equal to one. The efficiency scores classify the DMUs into four different classes $C_1, C_2, C_3$ *and* $C_4$ as follows.

(i) $C_1$ contains extreme efficient DMUs ( $\varphi^{SE}$ <1 )

(ii) $C_2$ contains efficient DMUs but not an extreme ( $\varphi^{SE}$ = 1 with zero slacks )

(iii) $C_3$ contains weakly efficient DMUs ( $\varphi^{SE}$ = 1 with non-zero slacks )

(iv) $C_4$ contains inefficient DMUs ($\varphi^{SE}$ >1)

The procedure of excluding the evaluated DMU from the reference set in the SE model may leads to the problem of infeasibility. A necessary condition states that infeasibility occurs in the case of extreme efficient DMU. Also it may be noted that extreme efficient either has feasible solution with SE score strictly lesser than unity or it has an infeasible solution.

Zhu(2012) stated that for a extreme efficient DMU under evaluation the output-oriented VRS SE model is infeasible if and only if the standard output-oriented VRS model is efficient. The following theorem proposed by Zhu (2012) identifies the infeasibility in output-oriented super-efficiency DEA model.

## Necessary and Sufficient condition for infeasibility

### Theorem:-

The output-oriented VRS super-efficiency model is infeasible if and only if $h^*$ where $h^* > 1$ is the optimal value to following model

$$h^* = \min \text{h}$$

Subject to

$$\sum_{\substack{n=1\\n\neq 0}}^{N} x_n\lambda_n \leq hx_o$$

$$\sum_{\substack{n=1\\n\neq 0}}^{N} \lambda_n = 1$$

$$\lambda_n \geq 0 \quad n \neq 0$$

This model obtains the value of $h$ may be greater than one or less than one. $h > 1$ exhibits infeasibility and $h < 1$ implies not infeasible. Further sensitive issue of $C_2$ and $C_3$ may be ignored because the data variation in inputs and outputs will swift $C_2$ to $C_4$ and $C_3$ remains same. So sensitivity issue may focus only on $C_1$ that is the class of extreme efficient DMUs.

## SENSITIVITY ANALYSIS

One important issue in DEA which has been studied by many DEA researchers is the efficiency sensitivity to perturbations in the data. In this paper the author focuses on the DEA sensitivity analysis methods based on super-efficiency DEA models that are developed by Zhu( 1996b;2001a) and Seiford and Zhu(1998d;f). Since an increase of any output or a decrease of any input cannot worsen the efficiency of $DMU_0$. In this study the output oriented is considered. So the worst case scenario that is the proportional decreases of outputs of the form is taken for sensitivity analysis

$$\hat{y}_{i0} = \alpha_r y_{r0}\,; \qquad 0 < \alpha_r \leq 1, \; r = 1,2,\dots.,s$$

where $y_{r0}$ $(r = 1,2,\dots s)$ are the outputs for a specific extreme efficient $DMU_m$ among $N$ DMUs. Zhu(1996b) provides the following to compute a stability region in which $DMU_0$ remains efficient.

$$\max \boldsymbol{\alpha_k^m} \qquad for\ each\ k = 1,2,\dots..p$$

$$\text{s.t} \sum_{\substack{n=1\\n\neq m}}^{N} \lambda_n y_{kn} \geq \alpha_k^m y_{jm}$$

$$\sum_{\substack{n=1\\n\neq m}}^{N} \lambda_n y_{kn} \geq y_{jm}\,, \qquad j \neq k$$

$$\sum_{\substack{n=1 \\ n\neq m}}^{N} \lambda_n x_{in} \geq x_{im} \quad i = 1,2, \dots, q$$

$$\sum_{\substack{n=1 \\ n\neq m}}^{N} \lambda_n = 1$$

$$\lambda_n, \alpha_k^m \geq 0$$

## 4. Empirical investigation

In this paper secondary data were collected from " District Central Cooperative Banks in India (2012-2013) directory published in the year 2013-2014. We considered 19 states which contribute to National economy. Here each state is considered as DMU. Each DMU involves with five inputs namely Number of DCC banks, Paid up capital (in lakhs), Total reserves (in lakhs), working capital (in lakhs), Number of employees were transformed to generate three outputs namely total deposits (in lakhs), investment (in lakhs), total loans issued(in lakhs).

By applying BCC output Oriented model the efficiency scores for all the DMUs included in this set, peers ( Reference set ) for each inefficient DMU and its weights are listed in the following table

**Table 1 Efficiency Scores and Peers**

| DMU | States | ϕ | Peer Weights | Peer counts | Rank |
|---|---|---|---|---|---|
| 1 | Andhra Pradesh | 1 | | 1 | 2 |
| 2 | Bihar | 1 | | | |
| 3 | Chhattisgarh | 1.00215 | λ6=0.5454; λ7=0.1292;λ10=0.0032;λ13=0.32213 | | |
| 4 | Gujarat | 1 | | | |
| 5 | Haryana | 1 | | | |
| 6 | Himachal Pradesh | 1 | | 2 | 1 |
| 7 | Jammu and Kashmir | 1 | | 1 | 2 |
| 8 | Jharkhand | 1 | | | |
| 9 | Karnataka | 1.2102 | λ1=0.0191; λ6=0.189;λ10=0.3538;λ15=0.4375 | | |
| 10 | Kerala | 1 | | 2 | 1 |
| 11 | Madhya Pradesh | 1 | | | |
| 12 | Maharashtra | 1 | | | |
| 13 | Orissa | 1 | | 1 | 2 |
| 14 | Punjab | 1 | | | |
| 15 | Rajasthan | 1 | | 1 | 2 |
| 16 | Tamil Nadu | 1 | | | |
| 17 | Uttar Pradesh | 1 | | | |
| 18 | Uttar Khand | 1 | | | |
| 19 | West Bengal | 1 | | | |

It is observed from the above 17 out of 19 DMUs are efficient and the remaining DMUs are relatively

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inefficient. Peers form the reference set and linear combination of them provides input and output targets for the inefficient DMU to improve its efficiency. Ranking of DMUs has been carried out based on peer count summary and it is found that the tie occurs among the DMUs. Therefore to break up the tie the Super-Efficiency model is applied.

## Super-Efficiency Model

The Super efficiency output oriented VRS model obtained the extreme efficient scores for all the DMUs and were listed below

Table 2. Super-Efficiency

| DMU | $\varphi^{SE}$ | Status |
|---|---|---|
| 1 | 0.9792 | Extreme Efficient |
| 2 | 0.5566 | Extreme Efficient |
| 3 | 1.0021 | Inefficient |
| 4 | 0.8109 | Extreme Efficient |
| 5 | 0.9721 | Extreme Efficient |
| 6 | Infeasible | Extreme Efficient |
| 7 | Infeasible | Extreme Efficient |
| 8 | Infeasible | Extreme Efficient |
| 9 | 1.2102 | Inefficient |
| 10 | 0.4769 | Extreme Efficient |
| 11 | 0.9934 | Extreme Efficient |
| 12 | 0.3664 | Extreme Efficient |
| 13 | 0.4929 | Extreme Efficient |
| 14 | 0.9881 | Extreme Efficient |
| 15 | 0.7601 | Extreme Efficient |
| 16 | 0.6209 | Extreme Efficient |
| 17 | 0.9119 | Extreme Efficient |
| 18 | 0.7776 | Extreme Efficient |
| 19 | 0.7018 | Extreme Efficient |

From the above table it is found that out of 19 DMUs 17 were found to be extreme efficient falls under $C_1$ classification and 2 are inefficient DMUs are comes under $C_4$ classification. It is to be noted that three are infeasible among extreme efficient DMUs. Further, the necessary and sufficient condition for infeasibility is analyzed and results are highlighted in the following table.

**Table 3 Necessary Condition for infeasibility**

| DMU | Input1 | Input2 | Input3 | Input4 | Input5 | $h^*$ |
|---|---|---|---|---|---|---|
| 1 | 22 | 105279 | 150670 | 1463030 | 4373 | 0.209197 |
| 2 | 23 | 15874 | 5948 | 195765 | 1064 | 0.805817 |
| 3 | 7 | 20601 | 47791 | 542351 | 2183 | 0.475002 |
| 4 | 18 | 41750 | 315068 | 2111231 | 7229 | 0.156633 |
| 5 | 19 | 37235 | 54010 | 1110548 | 3087 | 0.276298 |
| 6 | 2 | 501 | 69835 | 69806 | 1424 | **2.076127** |
| 7 | 3 | 827 | 8401 | 149945 | 1068 | **2.964041** |
| 8 | 8 | 3324 | 4793 | 91147 | 590 | **1.807973** |
| 9 | 21 | 46702 | 95322 | 1614126 | 4330 | 0.213772 |
| 10 | 14 | 23590 | 134715 | 2676351 | 5662 | 0.200639 |
| 11 | 38 | 70442 | 189214 | 1786564 | 6804 | 0.12981 |
| 12 | 31 | 218774 | 731685 | 6993396 | 24775 | 0.0731 |
| 13 | 17 | 59464 | 25398 | 1128922 | 2257 | 0.348977 |
| 14 | 20 | 16574 | 107592 | 1672875 | 3726 | 0.240298 |
| 15 | 29 | 38477 | 72091 | 1430619 | 2318 | 0.266148 |
| 16 | 23 | 126099 | 276088 | 2751437 | 4243 | 0.210314 |
| 17 | 50 | 67863 | 166934 | 1720833 | 7371 | 0.111497 |
| 18 | 10 | 4093 | 48065 | 525223 | 1115 | 0.654177 |
| 19 | 17 | 13855 | 32909 | 7679555 | 1992 | 0.374544 |

The optimal value $(h^* \geq 1)$ of objective function indicates infeasibility. So it is observed the infeasibility occurs in DMUs 6,7 and 8 respectively. Thus the above test ensures the infeasibility occurs in same DMUs 6,7 and 8 respectively.

## Ranking of DMUs

We rank the DMUs excluding the infeasible units based on SE scores is given below:
Table 4

| DMU | States | SE score $\varphi^{SE}$ | Rank |
|---|---|---|---|
| 1 | Andhra Pradesh | 0.9792 | 3 |
| 2 | Bihar | 0.5566 | 11 |
| 4 | Gujarat | 0.8109 | 6 |
| 5 | Haryana | 0.9721 | 4 |
| 10 | Kerala | 0.4769 | 13 |
| 11 | Madhya Pradesh | 0.9934 | 1 |
| 12 | Maharashtra | 0.3664 | 14 |

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| | | | |
|---|---|---|---|
| 13 | Orissa | 0.4929 | 12 |
| 14 | Punjab | 0.9881 | 2 |
| 15 | Rajasthan | 0.7601 | 8 |
| 16 | Tamil Nadu | 0.6209 | 10 |
| 17 | Uttar Pradesh | 0.9119 | 5 |
| 18 | Uttar Khand | 0.7776 | 7 |
| 19 | West Bengal | 0.7018 | 9 |

It is found that DMU 11 (Madhya Pradesh ) stood rank 1, DMU 14 (Punjab) gets rank 2 etc. ,

Sensitivity analysis is essential for studying DEA. Besides sensitivity analysis is applied in the worst case scenario. The VRS $k^{th}$ output specific model is applied and the results are presented in the following table 5.

### Table 5 Sensitivity analysis

| DMU | $\varphi^{SE}$ | $\alpha^m_{k=1}$ | $\alpha^m_{k=2}$ | $\alpha^m_{k=3}$ |
|---|---|---|---|---|
| 1 | 0.9792 | Infeasible | Infeasible | 0.979151 |
| 2 | 0.5566 | Infeasible | Infeasible | Infeasible |
| 4 | 0.8109 | Infeasible | 0.8109 | Infeasible |
| 5 | 0.9721 | Infeasible | Infeasible | 0.9721 |
| 6 | 0.0602 | Infeasible | Infeasible | Infeasible |
| 7 | 0.5012 | Infeasible | Infeasible | Infeasible |
| 8 | 0.7198 | Infeasible | Infeasible | Infeasible |
| 10 | 0.4769 | Infeasible | Infeasible | Infeasible |
| 11 | 0.9934 | Infeasible | 0.9874 | 0.98632 |
| 12 | 0.3664 | Infeasible | Infeasible | Infeasible |
| 13 | 0.4929 | Infeasible | Infeasible | Infeasible |
| 14 | 0.9881 | Infeasible | Infeasible | 0.988123 |
| 15 | 0.7600 | Infeasible | Infeasible | Infeasible |
| 16 | 0.6209 | Infeasible | Infeasible | Infeasible |
| 17 | 0.9119 | Infeasible | 0.9119 | Infeasible |
| 18 | 0.7776 | Infeasible | Infeasible | Infeasible |
| 19 | 0.7018 | Infeasible | 0.6102 | Infeasible |

The above table reflects the super-efficiency scores of specific output which we are interested. It is observed that the VRS $k^{th}$ output specific model reflects some of the results are infeasible and feasible. With these efficiency scores obtained in the table 5 a stability region for each DMU could be formed in the following table 6.

### Table 6:- Stability Region

| DMU | $\varphi^{SE}$ | $\alpha^m_{k=1}$ | $\alpha^m_{k=2}$ | $\alpha^m_{k=3}$ |
|---|---|---|---|---|
| 1 | 0.9792 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | 0.979151< $v \leq 1$ |
| 2 | 0.5566 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 4 | 0.8109 | $\alpha_k \leq 1$ | 0.8109< $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 5 | 0.9721 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | 0.9721< $\alpha_k \leq 1$ |
| 6 | 0.0602 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 7 | 0.5012 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 8 | 0.7198 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 10 | 0.4769 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 11 | 0.9934 | $\alpha_k \leq 1$ | 0.9874< $\alpha_k \leq 1$ | 0.98632< $\alpha_k \leq 1$ |
| 12 | 0.3664 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 13 | 0.4929 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 14 | 0.9881 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | 0.988123< $\alpha_k \leq 1$ |
| 15 | 0.7600 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 16 | 0.6209 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 17 | 0.9119 | $\alpha_k \leq 1$ | 0.9119< $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 18 | 0.7776 | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |
| 19 | 0.7018 | $\alpha_k \leq 1$ | 0.6102< $\alpha_k \leq 1$ | $\alpha_k \leq 1$ |

The VRS $k^{th}$ output specific model helps to maintains extreme efficient irrespective of the data variation made within the stability region. A stability region corresponding to VRS Super-Efficiency standard model reflects efficient and stability region corresponding to VRS $k^{th}$ output specific model results infeasibility. The sufficient condition for preserving efficiency permits the choice of $\alpha_k$ must be lies between $\alpha^m_k < \alpha_k \leq 1$ in case of feasible. In case of infeasibility the choice of $\alpha_k$ must be $\alpha_k \leq 1$. It is to be noted that in case of infeasibility the value of $\alpha_k$ has no limitations. Also the necessary conditions for preserving inefficiency is the choice of $\alpha_k$ must satisfy $\alpha_k < \alpha^m_k$. The data can be changed according to the limits of the stability region and different DEA models can be applied to calculate the efficiency scores of Decision making units. Thus the infeasible DMU are maintains its same classification and it is called as infeasible and stable.

## 5. Conclusions

Finally it is concluded that the standard BCC model indicates there are 17 DMUs were efficient. When the efficient DMUs are ranked tie of rank occurs among the efficient DMUs. To break up the tie of ranks the super-efficiency model is applied and it exhibits 14 DMUs are extreme efficient and 3 DMUs are infeasible. The infeasibility test also witnesses the same DMUs 6,7 and 8 are infeasible. Then DMUs that are infeasible are excluded and remaining efficient DMUs are ranked. Sensitivity analysis has performed to determine a stability region for each extreme efficient DMU. Stability region obtains a range for each extreme efficient DMU and this range indicates the admissible amount of data variation to preserve the same efficiency classification.

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