IJASRM

International Journal of Advanced Scientific Research and Management, Vol. 2 Issue 11, Nov 2017.

www.ijasrm.com

ISSN 2455-6378

A new class of periodic solutions to the Photogravitational Hill problem

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Abstract

In this paper we discuss the nature of the Three Body Problem, which has occupied the center stage of Celestial Mechanics for over 400 years. The Photogravitational Restricted Elliptical Three Body Problem in Hill's approximation is considered. Similar to Hill's method of constructing variational curves for the Lunar theory, we construct a new class of periodic solutions for this problem.

Keywords: *Three Body Problem –Periodic solution-Hill's problem*

1. Introduction

The study of motion of three mass points under their mutual gravitational attraction is called the Three-Body Problem. The nature of the three-body problem is such that even though the three-body problem can be described by a complete set of deterministic equations, yet the behavior of the three-body system maybe chaotic – unpredictable under certain conditions. The three-body problem is solvable but non-integrable.[1]

Closed form solutions seldom exist therefore, to obtain the solutions to the three-body problem, often power-series solutions are used, but they suit the case where either the mass or the value of the variables are small. Numerical solutions are yet another approach to tackle the problem, but they give solutions to special cases. Poincare and Bruns (1887) proved that there does not exist a general closed form solution to the three-body problem. Poincare in his first volume of Les Methodes Nouvelles de la Mecanique Celeste considers the subject of periodic orbits. He proved that for the three-body problem there exist infinitely many distinct periodic solutions. Poincare famous conjecture emphasizes the importance of periodic orbits – it states that if a particular solution of the restricted problem is given, one can always find a periodic solution (with in a period which might be very long) such that the difference between these two solutions is as small as desired or any given length of time. This fact is of immense importance as we can construct innumerable periodic solutions which would determine periodic orbits for artificial satellites.

In this paper we study the Restricted Elliptical Photogravitational Three Body Problem i.e. we investigate the motion of an infinitesimal mass under the influence of two point masses moving in elliptical orbits. Here we also take into account not only the gravitational attraction but also effects due to radiation pressure. The restricted three body problem with radiation pressure has wide applications in motion of artificial satellite under the gravitational field of two major bodies with one body radiating viz. Artificial satellite- Sun-Earth system or

[1] The concept of integrability expressed here refers to the availability of analytic solutions valid for the whole **phase-space**. While solvability refers to availability of particular numeric solutions.

IJASRM
International Journal of Advanced Scientific Research and Management, Vol. 2 Issue 11, Nov 2017.
www.ijasrm.com
ISSN 2455-6378

Artificial satellite- Sun-planet system . We construct a new class of periodic solutions for the generalized Hill's version for the photogravitational restricted elliptical three-body problem.

2. Photogravitational restricted elliptical three-body problem

The photogravitational restricted three-body problem was first formulated by Radzievskii (1950, 1953). The equations of motion of the photogravitational restricted elliptical three-body problem, where the passively gravitating body is subjected not only to gravitational attraction but also a repulsive force due to light pressure from the two actively gravitating bodies, have been obtained by Khasan (1996). Much work has been done on position and stability of librational points of photogravitational restricted elliptical three-body problem by Ammar M.K. (2008), Avdhesh Kumar & B.Ishwar (2014), Singh. J. & Umar (2014) and Usha, Narayana, Ishwar (2013). In this paper we construct a new class of periodic solutions in Hill's approximation.

Let □□□and □□ denote respectively the positions of actively gravitating bodies M_1 and M_2 with respect to their center of mass masses and let masses of these bodies be denoted by m_1 and m_2 respectively. Let $m_3 (\cong 0)$ denote the mass of the infinitesimal point mass of the third body M_3. The primary masses follow Keplerian elliptical orbits about their common centre of mass. Taking masses $m_1 + m_2 = 1$, distance between the actively gravitating masses i.e. □□□+□□ =1 and choosing units such that the Universal Gravitational constant, G to be unity, the equation of motion in the rotating frame of reference- rotating about the z-axis with mean velocity n are given by:

$$\left.\begin{aligned} \ddot{\xi} - 2n\dot{\eta} &= \frac{\partial\Omega}{\partial\xi} \\ \ddot{\eta} + 2n\dot{\xi} &= \frac{\partial\Omega}{\partial\eta} \\ \ddot{\varsigma} &= \frac{\partial\Omega}{\partial\varsigma} \end{aligned}\right\} \quad (1)$$

where (□□□□□□□□□denote the position of the infinetisemal□mass.

$$\Omega = \frac{n^2}{2}\left(\xi^2 + \eta^2\right) + \frac{m_1 q_1}{\rho_1} + \frac{m_2 q_2}{\rho_2} \qquad (2)$$

Here ρ_1 and ρ_2 denote the distance of the passively gravitating body from m_1 and m_2 respectively q_1 and q_2 are constants which denote the coefficients of mass reduction of m_1 and m_2 respectively and $p = \sum_{j=1}^{\infty} p_k \lambda^k$ denotes the mean motion of actively gravitating masses.

2.1 Hill's version

Hill (1902) in his seminal works on Lunar Theory assumed one of the actively gravitating bodies, namely the Sun to be infinitely far from the other two bodies and assumed it's mass to be infinitely large.

$$\text{i.e } \lim_{\substack{|M_1M_2| \to \infty \\ m_2 \to \infty}} \frac{m_1 + m_2}{\left|M_1 M_2\right|^3} = n^2 \qquad (3)$$

The equation of motion of the Photogravitational restricted elliptical three-body problem in Hill's approximation assumes the forms:

$$\left.\begin{aligned} &\frac{d^2\xi}{d\tau^2} - 2m\frac{d\eta}{d\tau} + \frac{q_1 k\xi}{\rho_1^3} + (q_2 - 1)m^2\xi = 3m^2 q_2 \xi \\ &\frac{d^2\eta}{d\tau^2} + 2m\frac{d\xi}{d\tau} + q_1 k\eta + (q_2 - 1)m^2\eta = 0 \\ &\frac{d^2\zeta}{d\tau^2} + \frac{q_1 k\zeta}{\rho_1^3} + q_2 m^2\zeta = 0 \end{aligned}\right\} \quad (4)$$

where $\tau = n_1(t - t_0)$. Here τ and n_1 denote two arbitrary constants, and

$$m = \frac{n}{n_1}, \qquad k = \frac{\mu}{n_1^2} \qquad (5)$$

Similar to Hill's method of constructing variational curves for the Lunar theory we construct periodic solutions for this problem. For the case $\varsigma = 0$, in the equations (4) we artificially define a new parameter λ :

$$\left.\begin{aligned} &\frac{d^2\xi}{d\tau^2} - 2m\frac{d\eta}{d\tau} + [\frac{q_1 k}{\rho_1^3} + (q_2 - 1)m^2]\xi - \frac{3}{2}q_2 m^2\xi = \frac{3}{2}q_2\lambda\xi \\ &\frac{d^2\eta}{d\tau^2} + 2m\frac{d\xi}{d\tau} + [\frac{q_1 k}{\rho_1^3} + (q_2 - 1)m^2]\eta - \frac{3}{2}q_2 m^2\eta = \frac{3}{2}q_2\lambda\eta \end{aligned}\right\} (6)$$

At the final stage we substitute $\lambda = m^2$.

For $\lambda = 0$ the equations (6) have the solutions:

$$\xi = a\cos\tau \quad , \eta = a\sin\tau \tag{7}$$

where a denotes a constant defined by the equality:

$$\frac{q_1 k}{a^3} = 1 + 2m + (\frac{q_2}{2} + 1) + m^2 \tag{8}$$

We make the following transformations:

$$u = x + iy \quad , \quad v = x - iy \tag{9}$$

and

$$u = a(1-p)e^{i\tau}, v = a(1-q)e^{-i\tau} \tag{10}$$

We obtain the equations:

$$\left.\begin{aligned} &\frac{d^2 p}{d\tau^2} + (1+m)i\frac{dp}{d\tau} + l(1-p) - l(1-p)^{-1/2}(1-s)^{-3/2} = \frac{3}{2}q_2\lambda(s-1)e^{-2i\tau} \\ &\frac{d^2 s}{d\tau^2} - (1+m)i\frac{ds}{d\tau} + l(1-s) - l(1-p)^{-3/2}(1-s)^{-1/2} = \frac{3}{2}q_2\lambda(p-1)e^{2i\tau} \end{aligned}\right\} \mathbf{(11)}$$

where

$$l = 1 + 2m + (\frac{q_2}{2} + 1)m^2 \tag{12}$$

For small λ we look for periodic solutions with period 2π . We look for p and q in the following series form:

$$p = \sum_{j=1}^{\infty} p_k\lambda^k \quad , s = \sum_{j=1}^{\infty} s_k\lambda^k . \tag{13}$$

Where we take p_k and s_k in the form:

$$\left.\begin{aligned} p_j &= \sum_{k=0}^{j} a_{j,j-2k}\lambda^j e^{2(j-2k)i\tau} \\ s_j &= \sum_{k=0}^{j} a_{j,j-2k}\lambda^j e^{-2(j-2k)i\tau} \end{aligned}\right\} (14)$$

Substituting expressions (13) in (14) we get p and s as:

$$\left.\begin{aligned} p &= \sum_{j=1}^{\infty}\sum_{k=0}^{j} a_{j,j-2k}\lambda^j e^{2(j-2k)i\tau} \\ &, \\ s &= \sum_{j=1}^{\infty}\sum_{k=0}^{j} a_{j,j-2k}\lambda^j e^{-2(j-2k)i\tau} \end{aligned}\right\} (15)$$

Substituting expressions (15) in equations (11) we obtain the coefficients in series in (15) to be:

$$\left.\begin{aligned} a_{1,1} &= \frac{-9}{16}q_2\frac{[2 + 4m + (q_2 + 2)m^2]}{[6 - 4m + (3q_2 - 2)m^2]} \\ a_{1,-1} &= \frac{3}{16}q_2\frac{[38 + 28m + 3(q_2 + 2)m^2]}{[6 - 4m + (3q_2 - 2)m^2]} \\ a_{r,\sigma} &= \frac{\frac{3}{2}lA_{r,-\sigma} - [4\sigma^2 - 4(1+m)\sigma + \frac{3}{2}l]A_{r,\sigma}}{2\sigma^2[2(4\sigma^2 - 1) - 4m + (3q_2 - 2)m^2]} \text{,if } \sigma \neq 0 \\ a_{r,0} &= \frac{-1}{31}A_{r,0} \quad \text{if } \sigma = 0 \end{aligned}\right\} (16)$$

where $A_{r,\pm\sigma}$ $A_{r,\pm\sigma}$ are polynomials with positive coefficients with respect to the constants $a_{j,\pm\sigma}$ $a_{j,\pm\sigma}$, for $j < r$ $.j < r$. Convergence of the series has been established.

Hence equations (15) with coefficients given by (16) give us a new class of class of periodic solutions for the Photogravitational restricted elliptical three body problem in Hill's approximation.

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