IJASRM

International Journal of Advanced Scientific Research and Management, Volume 3 Issue 2, Feb 2018.

www.ijasrm.com

ISSN 2455-6378

Fluid flow and Heat transfer of a viscous fluid of finite depth over a fixed thermally insulated bottom

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Abstract

A steady flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable and thermally insulated bottom is studied in this paper. Velocity, Flow rate, Temperature and Heat transfer rate are obtained through exact solutions from the momentum and energy equations. The effect of various parameters on different fields have been analyzed through the graphs.

Keywords: *Velocity, Flow rate, Temperature, Nusselt Number*

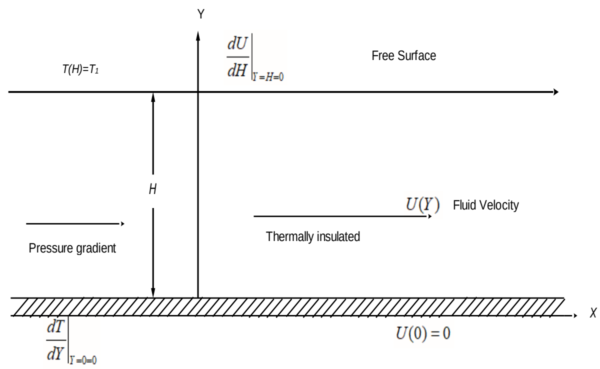
1. Introduction

The study of flow problems of fluids and associated heat transfer are of widespread interest in almost all the fields of engineering as well as in astrophysics, biology, biomedicine and scores of other disciplines. In the year 2006 Rajesh Yadav [2] studied convective heat transfer through a porous medium in channels and pipes. Sharma,Veena Kumari, Mishra [3] examined thermo solute convection flow in a porous medium. In the year 2017 a heat transfer problem of viscous liquid in porous media was studied by K.Moinuddin [1]. This paper deals with the flow of a viscous liquid of viscosity μ and of finite depth through a porous medium with some permeability coefficient over a fixed thermally insulated impermeable bottom .

The flow is generated by a constant horizontal pressure gradient parallel to the fixed bottom. The momentum equation considered is the generalized darcy's law proposed by Yama Moto and Iwamura[4] which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force. Velocity and temperature fields are obtained through exact solutions of the basic equations of momentum and energy. Employing these , the flow rate and the heat transfer rate on the free surface have been obtained and their variations are illustrated graphically.

2. Mathematical Formulation

Steady forced convective flow of a viscous liquid through a porous medium of viscosity coefficient μ and of finite depth H over a fixed horizontal impermeable bottom is considered in this problem. The flow is generated by a constant pressure gradient parallel to the plate and the bottom plate is thermally insulated whereas the free surface kept at temperature T_1 . With reference to a rectangular Cartesian coordinate system with the origin O on the bottom, the X-axis in the flow direction and the Y-axis vertically upwards the bottom is represented as Y=0 and the free surface as Y=H. Let the flow be characterized by a velocity *U =(U(Y), 0, 0)*. This choice of velocity evidently satisfies the continuity equation $divU = 0$.



Flow Configuration

Let the convective flow be characterised by the velocity field *U = (U(Y), 0, 0)* and the temperature *T(Y)*. The choice of the velocity satisfies the continuity equation

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$$divU = 0 \qquad (1)$$

The momentum equation

$$-\frac{\partial P}{\partial X} + \mu \frac{d^2U}{dY^2} - \frac{\mu U}{k^*} = 0 \qquad \text{-- (2)}$$

and the energy equation

$$\rho c U \frac{dT}{dX} = K \frac{d^2T}{dY^2} + \mu \left(\frac{dU}{dY} \right)^2 \qquad \text{-- (3)}$$

In the above equations ρ is the fluid density, 'c' is the specific heat, K the thermal conductivity of the fluid and P the fluid pressure.

Boundary conditions:

The bottom is fixed $\therefore$ $U(0) = 0$ -- (4a)

The free surface is shear stress free

$$\therefore \mu \frac{dU}{dY} = 0 \text{ on } Y=H \qquad \text{-- (4b)}$$

The bottom is thermally insulated

$$\therefore \frac{dT}{dY}\Big|(Y=0) = 0 \qquad \text{-- (5a)}$$

The free surface is exposed to the atmosphere

$\therefore T(H) = T_1$ = temperature of the atmosphere. - (5b)

In terms of the non-dimensional variables defined hereunder:

X=ax ; Y=ay ; H=ah ; $U = \frac{\mu u}{\rho a}$; $P = \frac{\mu^2 p}{\rho a^2}$;

$k^* = \frac{a^2}{\alpha^2}$; $-\frac{\partial P}{\partial X} = \frac{\mu^2}{\rho a^3} c_1, \left(c_1 = \frac{-\partial p}{\partial x} \right)$;

$T = T_0 + (T_1 - T_0)\theta$ $\quad \frac{\partial T}{\partial X} = \frac{(T_1 - T_0)c_2}{a}$

where $c_2 = \frac{\partial \theta}{\partial x}$ $\quad P_r$ (Prandtl number)$= \frac{\mu c}{K}$;

$$E = \frac{\mu^3}{\rho^2 a^2 K (T_1 - T_0)} \qquad \text{-- (6)}$$

(where a is some standard length and T_0 the temperature at the bottom) the basic field equations can be rewritten as :

Momentum equation:

$$\frac{d^2u}{dy^2} - \alpha^2 u = -c_1 \qquad \text{- -(7)}$$

and the Energy equation

$$\frac{d^2\theta}{dy^2} = P_r c_2 u - E\left(\frac{du}{dy} \right)^2 \qquad \text{-- (8)}$$

together with the boundary conditions:

for velocity

$$u(0)=0 \text{ and } \frac{du}{dy}\Big|_{y=h} = 0 \qquad \text{--(9)}$$

and for temperature

$$\frac{d\theta}{dy}\Big|_{y=0} = 0 \quad \text{and} \quad \theta(h) = 1 \qquad \text{--(10)}$$

The solution of these equations together with the related boundary conditions yield

the velocity distribution:

$$u(y) = \frac{c_1}{\alpha^2}\left[1 - \frac{\cosh \alpha (h-y)}{\cosh(\alpha h)} \right] \qquad \text{--(11)}$$

$$\theta(y) = 1 + \frac{P_r c_1 c_2}{\alpha^2} \left\{ \begin{matrix} \frac{(y^2 - h^2)}{2} + (h-y)\frac{\tanh \alpha h}{\alpha} + \\ \frac{1}{\alpha^2 \cosh \alpha h}(1 - \cosh \alpha (h-y)) \end{matrix} \right\} + \frac{Ec_1^2}{2\alpha^2}\left\{ \frac{(y^2 - h^2)}{2\cosh^2 \alpha h} + \frac{(h-y)\tan \alpha h}{\alpha} + \frac{(1 - \cosh 2\alpha (h-y))}{4\alpha^2 \cosh^2(\alpha h)} \right\} \qquad \text{--(12)}$$

The flow rate:

$$q = \int_0^h u(y)dy = \frac{c_1}{\alpha^2}\left(h - \frac{\tanh(\alpha h)}{\alpha} \right) \qquad \text{--(13)}$$

The rate of Heat transfer (Nusselt number) on the free surface :

$$\frac{d\theta}{dy}\Big|_{y=h} = \frac{P_r c_1 c_2}{\alpha^3}(h\alpha - \tanh \alpha h) - \frac{Ec_1^2}{4\alpha^3 \cosh^2 \alpha h}(\sinh 2\alpha h - 2\alpha h) \qquad \text{--(14)}$$

3. Results and Discussion

It is noticed that the velocity of the fluid decreases with the increase in the values of the porosity parameter α (Fig.1).

From Fig.2 it is evident that the velocity profiles are negligibly small for smaller values of the pressure gradient c_1 and increases with the increasing larger values c_1.

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For larger values of α it has been observed that the thickness of the boundary layer increases with the increasing values of the pressure gradient c_1. (Fig.3)

For the smaller values of the porosity parameter the flow rate increases with the increasing values of *h*.(Fig.4)

It is noticed that the flow rate increases with the increasing values of the pressure gradient c_1.(Fig.5)

From Fig.6 when α =0.1 it is evident that the flow rate is very low in the lower part of the channel and increases with the increasing pressure gradient c_1.

For large α the flow rate increases with the increasing values of pressure gradient.(Fig.7)

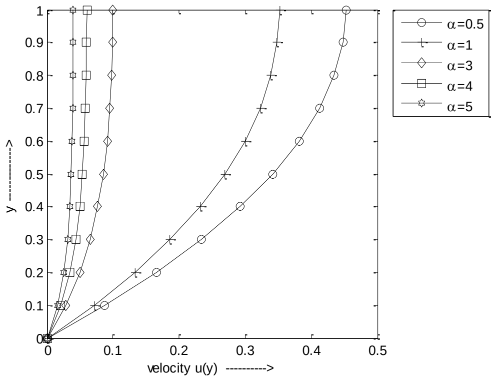

Fig.1 velocity profile for c1=1 h=1

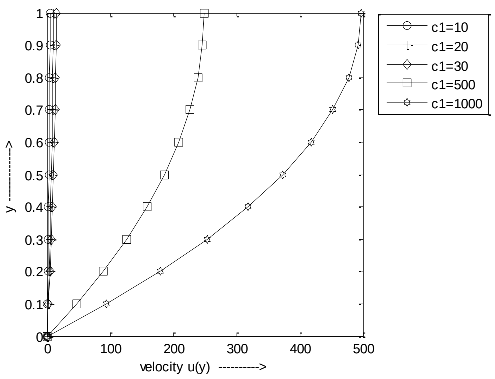

Fig.2 velocity profile for α=0.1 h=1

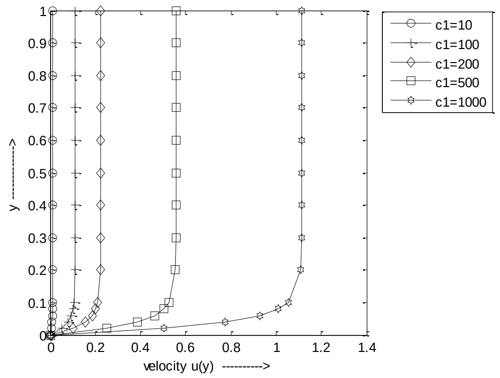

Fig.3 velocity profile for α=30 h=1

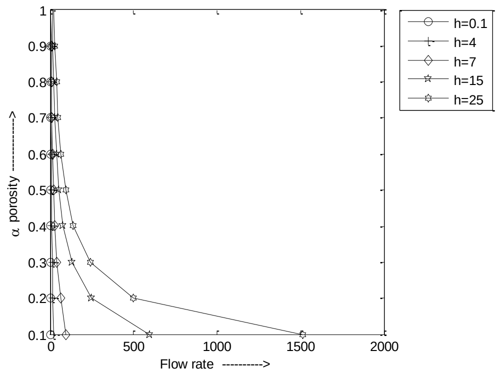

Fig.4 Flow rate for c1=1

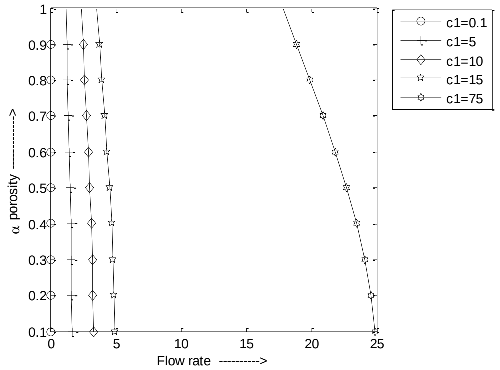

Fig.5 Flow rate for h=1

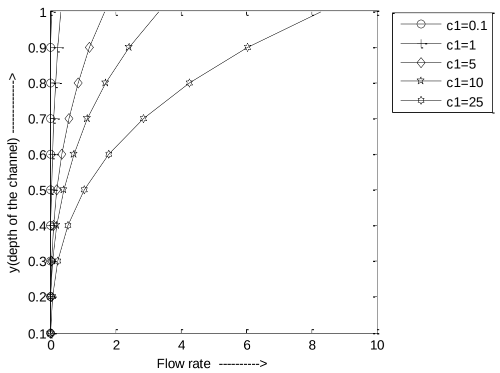

Fig.6 Flow rate for h=1 α=0.1

ISSN 2455-6378

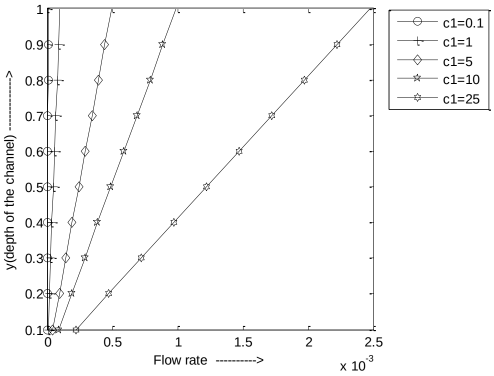

Fig.7 Flow rate for h=1 α=100

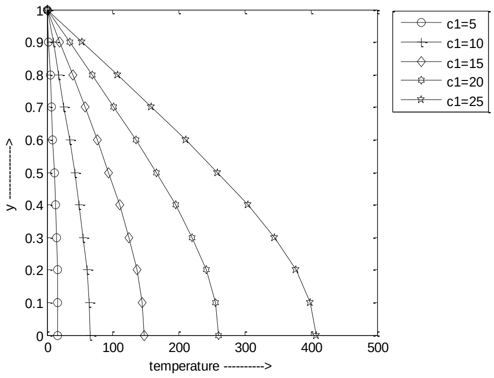

Fig.8 Temperature distribution for p=1 ,h=1,E=5,α=1,c2=1

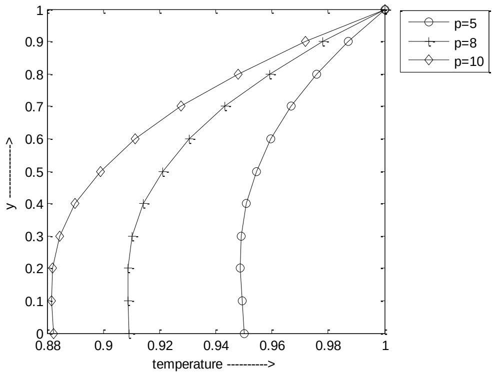

Fig.9 Temperature distribution for ,h=1,E=5,α=1,c1=1,c2=1

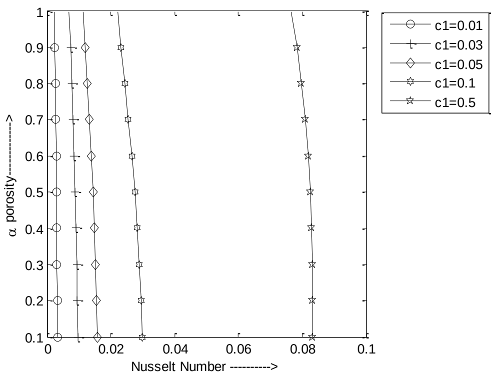

Fig.10 Nusselt Number on free surface for h=1,E=1,p=1,c2=1

It is noticed that with the increasing values of the pressure gradient the temperature profile increases (Fig.8).

From Fig.9 it is clear that with the increase in the prandtl number the temperature of the flow region decreases.

Fig.10 clearly shows the increase in the heat transfer rate (Nusselt number) with the increasing values of the pressure gradient c_1.

4. Conclusions

In this study it has been noticed that the velocity of the fluid decreases with the increase in the values of the porosity parameter and also we have seen that with the increasing values of the pressure gradient the temperature profile increases. Further the heat transfer rate increases with the increasing values of the pressure gradient.

Acknowledgments

I sincerely thank my colleague Dr.Syed Waseem Raja, Faculty of Mathematics, MANUU, for his support and encouragement in the preparation of this manuscript.

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