IJASRM International Journal of Advanced Scientific Research and Management, Volume 3 Issue 2, Feb 2018.
www.ijasrm.com
ISSN 2455-6378

A study of Nuclear Binding Energy of Magic Number Nuclei and Energy Splitting considering Independent particle shell model

Dipesh Chanda

Department of Physics, Siliguri College,
Siliguri,West Bengal, India. Pin 734001

Abstract

It is well established about the higher nuclear binding energies of atoms having magic number nuclei compared to their nearest neighbouring nuclei and as a whole. In the present case there will be a comparative study about the nuclear binding energies of highly stable nucleus and the nuclei having one more or less nucleons using nuclear liquid drop model and independent particle shell model simultaneously. In addition to that, there would be a try to explain the reason of higher nuclear binding energies of magic number nuclei by considering additional $s-l$ interactions of nucleons. This $s-l$ interaction are considered to be much stronger and opposite in sign to the $s-l$ coupling of electrons in the atom. The $s-l$ interaction is very strong coupling of spin and orbital motion of the nucleons in the nucleus of the magic number nuclei with negative sign is required to confirm energy splitting.

Keywords: Magic number nuclei, stable nucleus, independent particle model, $s-l$ coupling

1. Introduction

From the study of atoms, it is found that when number of electrons in an atom are 2, 10, 18, 36, 54, 86 show high chemical stability, ionization potentials are also high and do not interact with other atoms. These are called atomic magic numbers and chemical stability of these atoms can be explained on the basis of electronic closed shells and sub-shells according Pauli's exclusion principle. He(2), Ne(10), Ar(18), Kr(36), Xe(54), are the examples of atomic magic numbers. Electrons are filled in shells and sub-shells according to the configuration 1s-2s-2p-3s-3p-4s-3d-4p-5s-4d-5p-6s.

Similarly nuclear binding energies are high compared to their adjacent nuclei when proton number and or neutron numbers are 2, 4, 8, 20, 50, 82, 126. The nuclei having the number of protons and or neutrons are called magic nuclei. These nuclei are not only highly stable but show addition some characteristics, which are greater relative abundances in nature, greater number of stable isotopes, the separation energy of one proton or one neutron very large, probability of capturing a neutron is much lower, the energy of first excited states of nuclei is high, the alpha disintegration energy is small etc. These unexpected behaviors can be explained by introducing a nuclear model, experimental results also reflects these..

To explain the properties of the nucleus, various nuclear models are proposed. Among these, liquid drop model is one of the most suitable one to calculate the exact value of nuclear binding energy. In the model the atomic nucleus is assumed to be a small water droplet composed of molecules. The properties like shape, density independent of its volume, short range force, etc. of a nucleus are analogous to the microscopic depiction of a water drop. According the model, when binding energy of the nucleus are calculated, different factors like volume energy/effect, surface energy/effect, coulomb energy/effect, asymmetric energy/effect and pairing energy/effect, of a nucleus are assumed to be the only factors effected the actual binding energy of the nucleus, as a result weizsacker assumed a formula that is called semi empirical mass formula. Out of the five factors volume effect increases the binding energy , whereas the next three factors decreases the binging energy and the last factors increases the value for even- even N and Z, decreases when both N and Z are odd.

From the study of quadrupol moments of the ground state of magic number nuclei, shows zero value i.e. these indicates their spherical shape having closed structure.

The single particle nuclear shell model proposed from the basic as was found in atoms, where electrons moves independently in a central orbit round the nucleus due to the central Coulomb force created by the nucleus. In the model it assumed that the nucleons are moving independently in a combined force of central and non-central force field. For central field, it is explained as a short-range potential. Various forms of potential had used for the calculation of nuclear energy level, like square well potential and harmonic oscillator well etc. Three dimensional Schrodinger equation for harmonic oscillator can be solved using spherical polar coordinates. Hence, achieved eigen value and eigen vectors. In the eigen vectors two parts, one is radial function and other spherical harmonics. For the non central part, MM.Mayer and H.Jensen separately assumed the existence of nuclear $s-l$ interaction (nuclear spin-orbit coupling interaction). Nuclear spin-orbit coupling interaction is like that the atomic cases but much stronger and opposite in sign to the $s-l$ interaction of electrons. From this special property of coupling, energy splitting of shells are created in nuclear magic numbers, which are also verified experimentally.

2. Theory and Calculation

Atomic nucleus is considered as a liquid drop so binding energies of the highly stable nucleus i.e. magic nuclei and their adjacent nuclei can be estimated using semi empirical mass formula. The formula used to calculate binding energy is,

$$B.E = avA - bsA^{2/3} - c_c Z(Z-1)/A^{1/3} - d_a (A-2Z)^2 / A \pm \delta / A^{3/4}$$ --------- (1),

where the successive terms are volume energy/effect, surface energy, coulomb energy, asymmetry energy and pairing energy. A and Z are the mass no. and atomic no. of the nucleus and a_v, b_s, c_c, d_a, and ð are constants, their values are 15.8, 17.8, 0.71, 23.7 and 34 respectively. When A and Z both are even ð is positive, it is zero, when any one is odd and negative when both are odd. Finally binding energy and binding energy per nucleon for magic nuclei and their neighbours can be estimated.

Calculation of nuclear binding energy from the mass defect:

Nuclear binding energy =

$$931\text{x}[Zm_p + (A-Z)m_n - M]\ \text{MeV}$$ ------------- (2)

Where A=mass number and M= isotopic mass of the nucleus. m_p, m_n , mass of protons and neutrons, are used standard values as m_p = 1.007825u, m_n =1.008665u

Using the above formula nuclear binding energy of magic nuclei as well as for their neighbours can be calculated.

Calculation of Separation energy of neutron S_n and proton S_p :

Separation energy of a neutron or a proton can be treated as the amount of energy required to get free one neutron or a proton from the nucleus. i.e. minimum energy required for the separation of one neutron or one proton from a nucleus. These can be estimated by the equations

$$S_n = [m_n + M(Z, A-1) - M(Z, A)] \times 931\ \text{MeV}.$$

And

$$S_p = [m_p + M(Z-1, A-1) - M(Z, A) \times 931\ \text{MeV}$$

-------------- (3) where, m_n and m_p are the masses of neutron and proton, $M(Z, A-1), M(Z-1, A-1)$ and $M(X, A)$ are the masses of nuclei of one neutron short, one proton short and the mass of original nuclei.

Estimation of energy of a shell and splitting energy:

For the explanation of higher stability of magic nuclei, MM.Mayer and H.Jensen separately assumed the existence of nuclear $s-l$ interaction (nuclear spin-orbit). The interaction is stronger and opposite sign compared to spin orbit coupling interactions in atoms. The strong interaction of spin and orbital motion of the nucleons in the nucleus in the nucleus having opposite sign, is the result of energy splitting in agreement with the experimental nuclear magic numbers. Spin orbit interaction term is considered to be added to the central potential which is non central and may written in the form

$V_{ls} = f(r)(\vec{s}.\vec{l})$ ----- (4), where $f(r)$ is the potential function, $\vec{s}$ and $\vec{l}$ are spin and orbital angular momentum vectors, both combine to form total angular momentum $\vec{j}$.

After simplification of cosine law

$$(\vec{s}.\vec{l}) = \frac{j(j+1) - l(l+1) - s(s+1)}{2}$$ and

$$j_{max} = l + \frac{1}{2} \quad \text{and} \quad j_{min} = l - \frac{1}{2}$$

i.e. $(\vec{s}.\vec{l})_{j_{\max}} = \frac{l}{2}$ and $(\vec{s}.\vec{l})_{j\min} = -\frac{(l+1)}{2}$ ----(5)

Ultimately using equation (5), the spin orbit energy splitting of two levels

$\Delta V_{sl} = \frac{1}{2} f(r)(2l+1)$ ----- (6)

The equation (6) confirms that the state with $j = l + \frac{1}{2}$ lies below the state $j = l - \frac{1}{2}$.

Hence the nuclear orbit in each shells splits and according energy states these are arrange as $1s_{1/2}$ $1p_{3/2}$,$1p_{1/2}$,$1d5_{/2,}$$1d_{3/2}$,$2s_{1/2}$,$1f_{7/2}$, $1f_5/2$,$2p3_/2$,$2p1_{/2}$,$1g_{9/2}$,$1g7_{/2}$ and so on.

Solving the potential function $f(r)$ (in the form of square well), it is found spin orbit energy splitting i.e. energy separation between the two J states increases with the increase of l - values and it is approximately proportional to

$(2l+1)/A^{1/3}$ ---- (7)

The energy of last neutron is estimated when one splitting shell is completed i.e. the energy at the closed shell $1p_{1/2}$, $1d_{3/2}$, $1f_{7/2}$ etc., these are calculated with the help of mass defects of two isotopes of an atom, one isotope is chosen, whose neutron is sufficient enough to complete a shell, other one having one more or one less neutron than the earlier one. For an example to calculate energy of $1d_{3/2}$ neutron shell

B.E=[M(39)+M_n+M(40)]x931 MeV. and for $1f_{7/2}$,
B.E=[M(40)+M_n+M(41)]x931 MeV--- 8

is used. Appling this methods, B.E.of [$1p_{1/2}$,$1d_{5/2}$$1d_{3/2}$, $1f_{7/}2$, $1f7_{/2}$, $2p_{3/}2$, $2p1_{/2}$, $1g_{9/}2$, $1g7_{/2}$,$1d_{5/}2$, $2d3_{/2}$, $1h_{11/2}$] of neutron shell are estimated out and from this the energy difference between two nearest complete shells are determined.

3. Results

At first considered eight magic nuclei, starting from $_{2}He^{4}$ to $_{82}Pb^{208}$ and using semi empirical mass formula different energy/ factors of the total binding energy are calculated separately of each nucleus, i.e. volume energy (E_v), surface energy (E_s), Coulomb energy (E_c), asymmetry energy (E_{as}), and paring energy(E_p). Out of all these factors only first term increase the total energy, though for magic nuclei, even even nucleons pairing energy adds with the binding energy, but all middle terms of the mass formula diminishes the binding energy. These energy terms are calculated, using different parts of equation (1) and the standard values of the constants used are already shown earlier. After that, ratio E_s to E_v, E_c to Ev, and $E_{as}$to E_v are estimated out. Than total binding are calculated and binding energy per nucleon are estimated. Next once again binding energy of each magic number nuclei are calculated using mass defect, For this calculation equation (2) is utilized. Ultimately binding energy per nucleon is determined.

For the estimation of separation energies of neutron and proton equation (3) are used. After that some nuclei are chosen which have one more nucleon compared to magic nuclei. Following above mention process and by using equations (1), (2) and (3), the different ratios of energy terms, binding energy, binding energy per nucleon, (two different ways) and separation energies of neutron and proton are estimated. For all the calculations the required data, like mass of proton, mass of neutron and atomic masses etc. are used from the book of Nuclear Physics by S.N.Ghoshal, reprint 2004, S.Chand & company Ltd.

The calculated values of volume energy (E_v), surface energy (E_s), Coulomb energy (E_c), asymmetry energy (E_a), and pairing energy(E_p) of magic nuclei and the nuclei one more nucleon, from two different ways are shown in the table. In addition to these the varies ratios of E_s/E_v, E_c/E_v, and E_a/E_v, i.e how surface energy, coulomb energy and asymmetry energy varies with volume energy are calculated, these are also shown in the table no.1. In the table no. 2, total binding energy of the above mention nuclei including the magic nuclei are calculated using two different methods i.e. semi empirical mass formula and mass defect, after that binding energy per nucleon are calculated for each nuclei, all these calculated data are shown in the table no.2. Using equation (3) neutron separation energy as well as proton separation energy are calculated for magic nuclei and one more nucleons with respect to magic nuclei, all the calculated results are shown in table no. 2. Lastly using mass difference of different nuclei, the energies of different shell which are well established from theory as well as experimental point view are calculated. Like the sub-shells [$1p_{1/2}$,$1d_{5/2}$$1d_{3/2}$, $1f_{7/}2$, $1f7_{/2}$, $2p_{3/}2$, $2p1_{/2}$, $1g_{9/}2$, $1g7_{/2}$,$1d_{5/}2$, $2d3_{/2}$, $1h_{11/2}$]. These are estimated only due to estimate the energies of one complete splitting shell. From the difference of two successive energy values, it may be compared between two nearest splitting shells of either neutron or proton. Due to the spin–orbit interaction the each shell splits into two, and the splitting (difference of energy levels of the splitting) also can be calculated (not exact but some proportionate value) for the both neutron shell and proton shell, using mass defects. The various calculated values of energy of each complete sub-shells and their differences i.e. proportionate splitting energy are shown in the table no. 3.

Fig 1.1 and Fig 1.2 show the variation of the ratios (magnitude) of surface energy to volume energy (E_s/E_v), coulomb energy to volume energy (E_c/E_v) and asymmetry energy to volume energy (E_a/E_v) energy with mass number of nucleus for magic nuclei and magic nuclei with nuclei having one more nucleon compared to magic nuclei (all energy are negative except volume energy). The points Blue(top most line), Red(middle line) and Green(bottom line) indicate the ratios (E_s/E_v), (E_c/E_v) and (E_a/E_v) .

Fig 2.1 and Fig 2.2 stand the variation of total volume energy (E_v), surface energy (E_s), coulomb and energy (E_c) with mass number of nucleus for magic nuclei and magic nuclei plus nuclei having one more nucleon compared to magic nuclei. The points Blue (top line), Red (square points) and Green(triangle points) indicate the volume energy Violet (star symbol) points stand the resultant of the above mentioned three energies.

Fig 3.1 and Fig 3.2 represent the variation of volume energy (E_v), surface energy (E_s) and coulomb energy (E_c) per nucleon with mass number and the variation of the resultant of the three energies of nucleus for magic nuclei and magic nuclei plus nuclei having one more nucleon compared to magic nuclei. Blue(top most line), Red(square symbol) and Green(triangle symbol) points represent the volume energy (E_v), surface energy (E_s) and coulomb energy (E_c) per nucleon and Violet(star symbol) points indicates the sum of the three energy values per nucleon.

$E_a/E_{v)}$, pairing energy(E_a) and pairing energy per nucleon not plotted with the others due to smallness of numerical values of the former with respect to others.

TableNo.1

Nuclei.	Volume energy (E_V) in MeV.	Surface energy (E_S) in MeV	Coulomb energy (E_C) in MeV	Asymmetry energy (E_a) in MeV	Value of(magnitude) E_s/E_v	Value of (magnitude) E_c/E_v	Value of (magnitude) E_a/E_v
$_{2}He^{4}$	63.008	44.779	0.890	0	0.71	0.01412	0
$_{8}O^{16}$	252.032	112.780	15.790	0	0.44	0.06265	0
$_{20}Ca^{40}$	630.080	207.620	78.980	0	0.3295	0.0777	0
$_{28}Ni^{58}$	913.616	265.913	138.850	1.633	0.2910	0.1519	0.00178
$_{38}Sr^{88}$	1386.176	351.010	224.760	38.756	0.2532	0.1621	0.02795
$_{50}Sn^{120}$	1890.240	431.550	353.229	78.948	0.2283	0.1868	0.04176
$_{58}Ce^{140}$	2205.280	478.210	452.790	97.445	0.2168	0.2053	0.04418
$_{82}Pb^{208}$	3276.416	622.480	797.330	220.448	0.1899	0.2434	0.06728
$_{8}O^{17}$	267.78	117.429	15.470	1.3932	0.4385	0.0577	0.0052
$_{37}Sr^{87}$	1370.42	348.350	225.620	46.008	0.2500	0.1646	0.0335
$_{50}Sn^{119}$	1874.78	429.153	354.215	71.849	0.2289	0.1889	0.0383
$_{50}Sn^{121}$	1905.99	433.943	352.250	86.322	0.2276	0.1848	0.0428
$_{59}Pr^{141}$	2221.03	480.480	467.570	88.850	0.2163	0.2105	0.0400
$_{83}Bi^{209}$	3292.17	624.470	815.72	209.530	0.1890	0.2477	0.0636

IJASRM International Journal of Advanced Scientific Research and Management, Vol. Issue , 2018.
www.ijasrm.com
ISSN 2455-6378

Table No.2

Nuclei.	B.E in M eV. [using mass formula]	B.E/A in M eV.	B.E. in MeV. [using mass defects]	B.E/A in MeV.	Neutron separation Energy in MeV.	Proton separation energy in MeV.
$_{2}He^{4}$	29.16	7.29	28.28	7.07	20.56	19.77
$_{8}O^{16}$	127.64	7.97	127.55	7.97	12.90	12.42
$_{20}Ca^{40}$	345.58	8.64	341.87	8.55	11.59	8.62
$_{28}Ni^{58}$	508.81	8.77	506.197	8.73	8.12	7.64
$_{38}Sr^{88}$	772.80	8.82	768.05	8.73	11.10	10.62
$_{50}Sn^{120}$	1027.13	8.56	1019.99	8.50	9.09	8.86
$_{58}Ce^{140}$	1177.65	8.41	1172.03	8.37	8.92	8.43
$_{82}Pb^{208}$	1636.75	7.87	1635.56	7.86	7.36	6.88
$_{8}O^{17}$	133.48	7.85	131.69	7.74	4.14	3.36
$_{37}Sr^{87}$	762.31	8.76	753.55	8.66	5.50	4.72
$_{38}Sr^{87}$	763.50	8.77	756.94	8.70	8.42	7.64
$_{50}Sn^{119}$	1019.26	8.56	1010.00	8.49	6.48	5.70
$_{50}Sn^{121}$	1034.75	8.55	1025.77	8.47	6.56	5.56
$_{59}Pr^{141}$	1184.11	8.39	1177.25	8.34	6.00	5.23
$_{83}Bi^{209}$	1642.43	7.85	1639.36	7.84	4.50	3.80

Table No.3

Difference between the splitting shells (for neutron)	Energy in MeV.	Splitting	Splitting energy proportional to $(2l+1)A^{-1/3}$ in MeV.				
			value of l	value of A for proton shell	value of A for neutron shell	Energy in Mev. Proton shell	Energy in MeV Neutron shell
[1] $1p_{1/2}$-$1d_{5/2}$	8.76	1p	1	16	16	1.19	1.19
[2] $1d_{3/2}$-$1f_{7/2}$	7.27	1d	2	36	36	1.51	1.51
[3] $1f_{7/2}$-$2p_{3/2}$	3.88	1f	3	74	62	1.66	1.77
[4] $2p_{1/2}$-$1g_{9/2}$	2.54	1g	4	136	106	1.75	1.91
[5] $1g_{7/2}$-$2d_{5/2}$	2.92	1h	5	238	156	1.78	2.04
[6] $2d_{3/2}$-$1h_{11/2}$	2.28						

International Journal of Advanced Scientific Research and Management, Vol. Issue , 2018.
www.ijasrm.com
ISSN 2455-6378

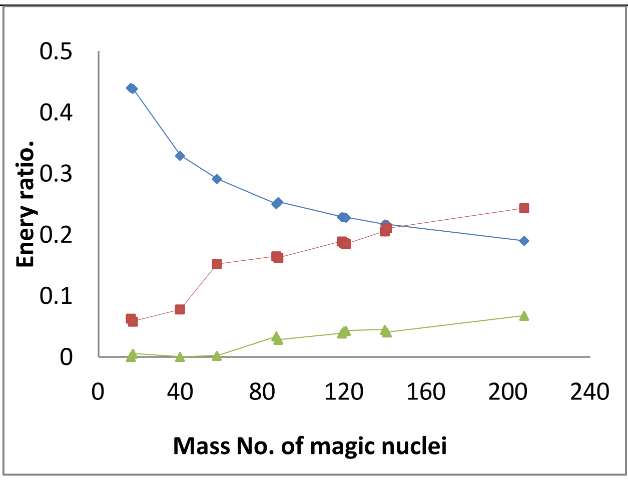

Fig 1.1

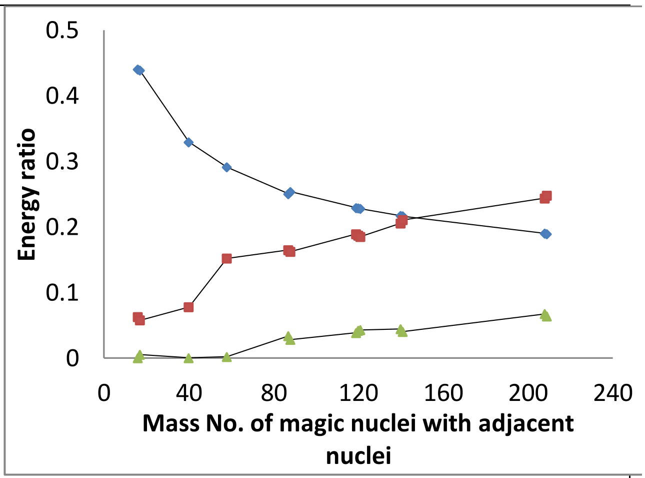

Fig 1.2

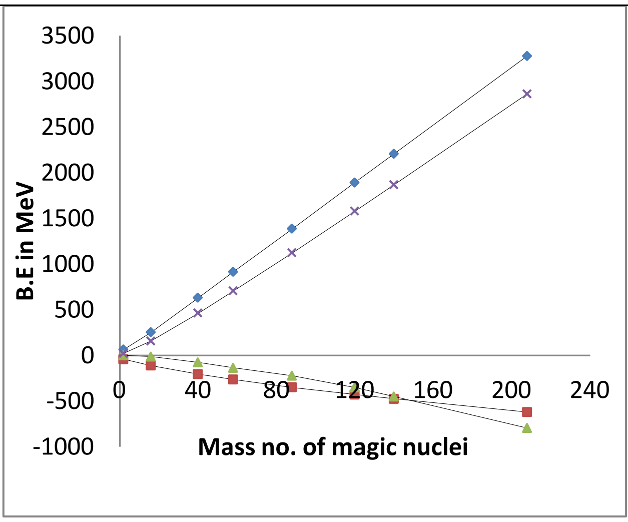

Fig 2.1

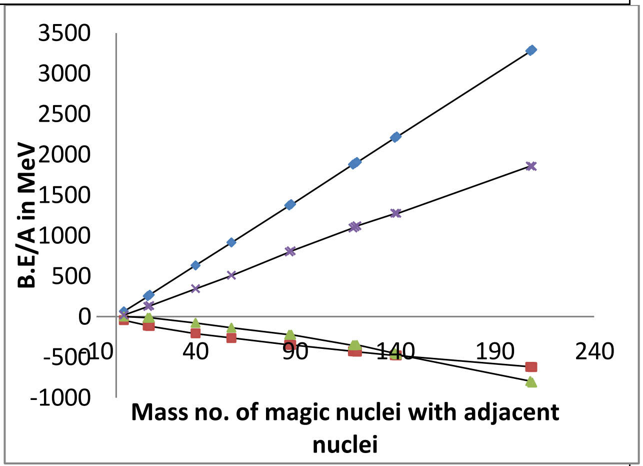

Fig 2.2

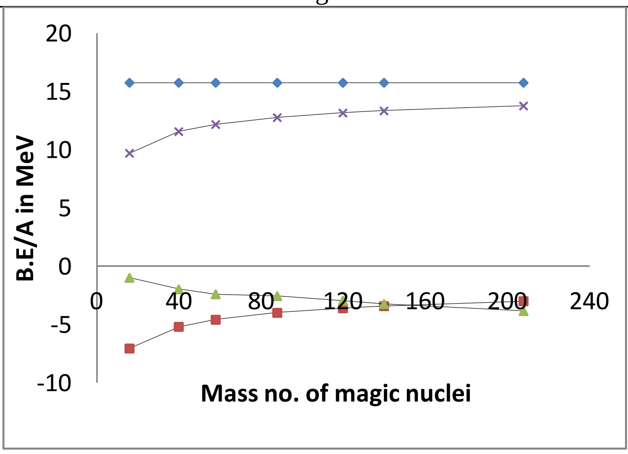

Fig 3.1

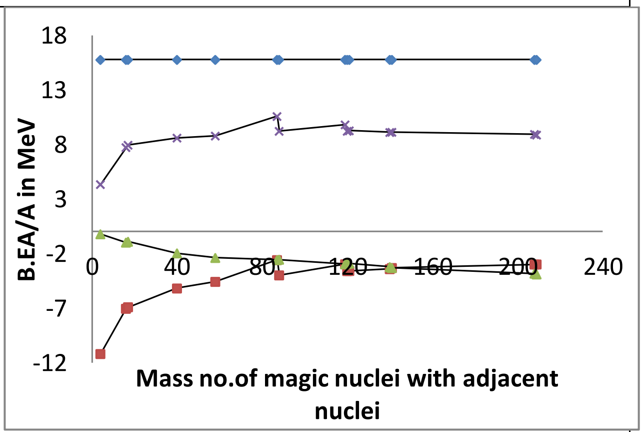

Fig 3.2

IJASRM
International Journal of Advanced Scientific Research and Management, Vol. Issue , 2018.
www.ijasrm.com
ISSN 2455-6378

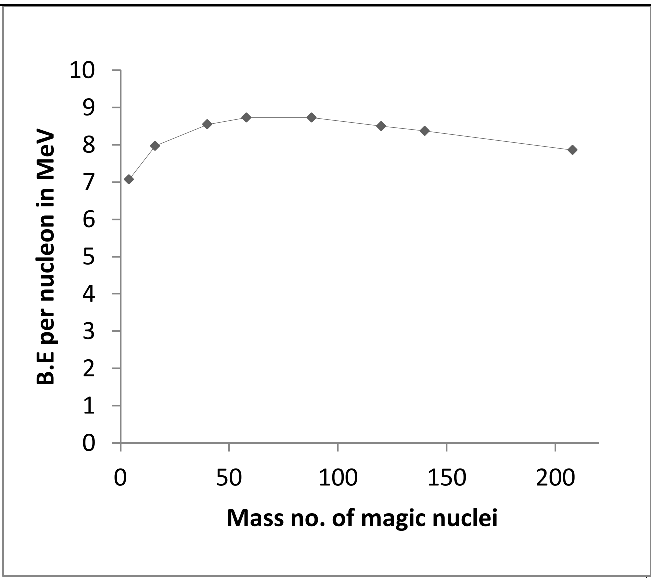

Fig 4.1

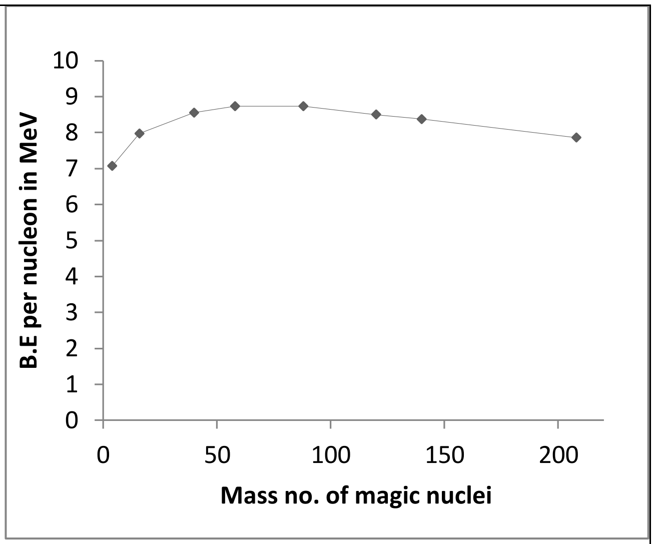

Fig 4.2

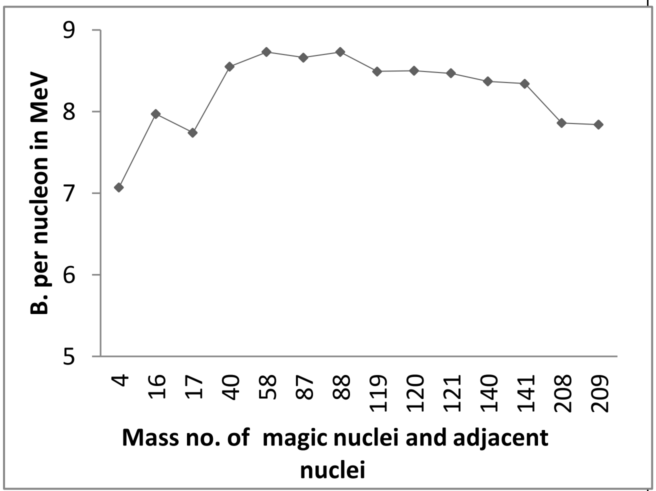

Fig 5.1

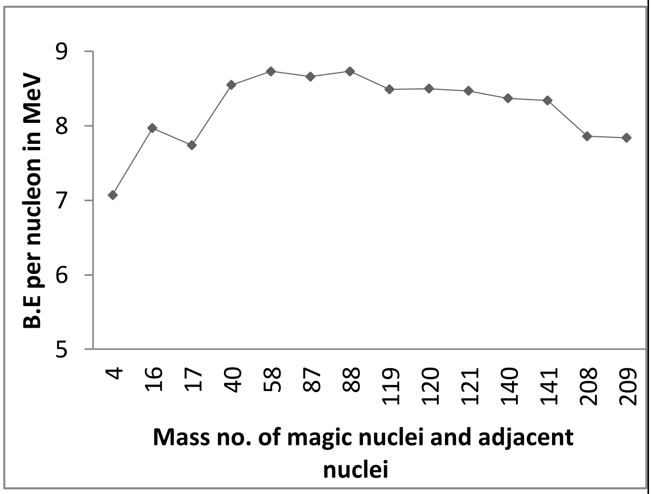

Fig 5.2

Fig 4.1 and Fig 4.2 indicate the variation of binding energy per nucleon with mass number of magic nuclei using semi-empirical mass formula and mass defects, respectively.

Fig 5.1 and Fig 5.2 indicate the variation of binding energy per nucleon with mass number of magic nuclei with other nuclei having one more nucleons compared to magic nuclei, using semi-empirical mass formula and mass defects respectively.

Fig 6.1 represents the variation of neutron separation energy with mass number of magic nuclei. Whereas Fig 6.2 represents the variation of neutron separation energy with mass number of magic nuclei with adjacent other nuclei having one more nucleon compared to magic nuclei.
Fig 7.1 represents the variation of proton separation energy with mass number of magic nuclei. And Fig 7.2 represents the variation of proton separation energy with mass number of magic nuclei with adjacent other nuclei having one more nucleon compared to magic nuclei.

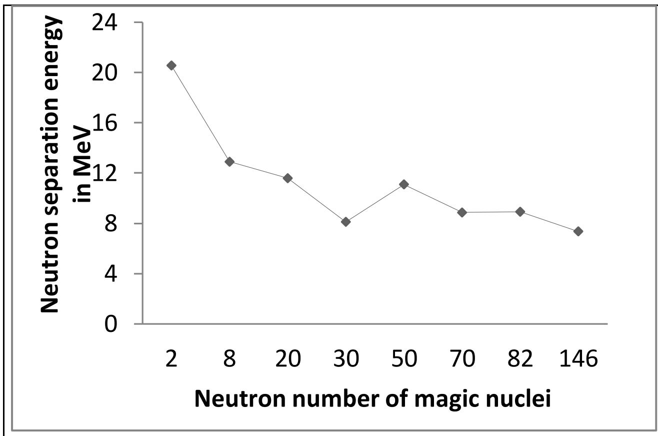

Fig 6.1

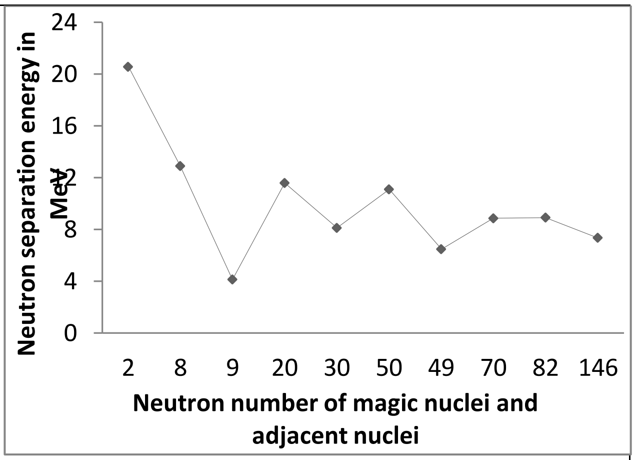

Fig 6.2

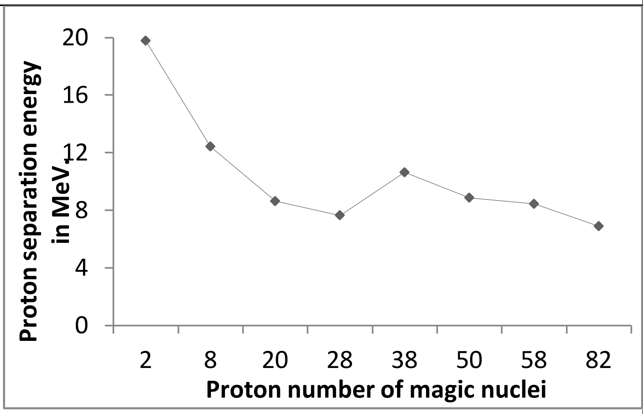

Fig 7.1

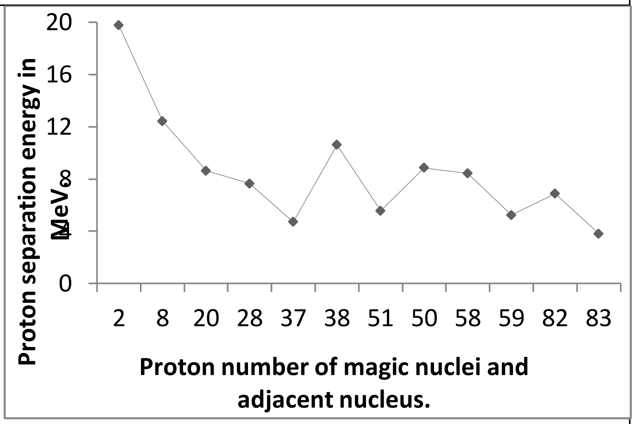

Fig 7.2

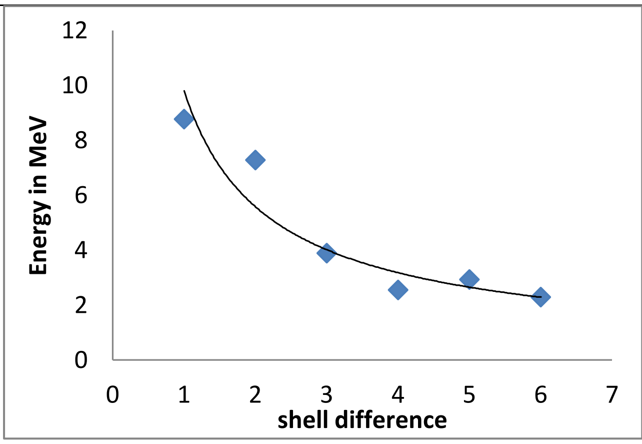

Fig 8.1

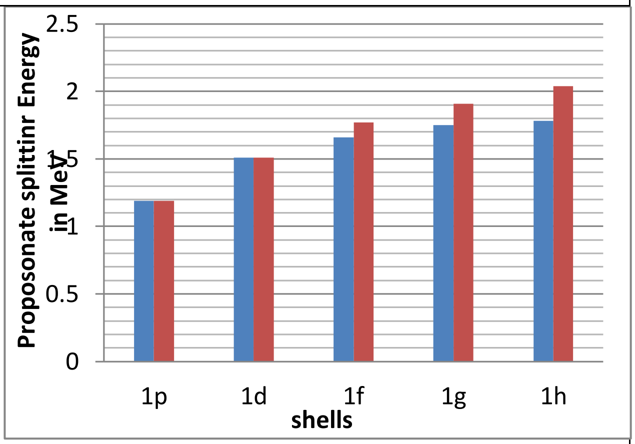

Fig 8.2

Fig 8.1 indicates the variation of energy difference of one complete splitting shell to the next complete splitting shell with the difference of shell(for neutron shell), Where the difference of shell from the list of the chart indicates as [1], [2], [3], [4], [5] and [6] respectively.

Fig 8.2 represents the proportionate energy of proton and neutron for different sub-shell start from lowest value of *l* (i.e. 1p, 1d, 1f, 1g and 1h). In the bar representation blue and red colour bar indicates the proportionate energy of proton and neutron successively.

4 Discussions and Conclusion

It is observed from the Fig 1.1 and from Fig 1.2, that the ratio of surface energy to volume energy (E_s/E_v) decreases with mass number of the both, magic nuclei and other nuclei having one more nucleon. But the other ratios i.e. (E_c/E_v) and (E_a/E_v), [magnitude] increases with mass number. Through all the three variations are symmetric. From the Fig 2.1 and Fig 2.2 symmetrical variations of different energy factors are also noticed. As the nature of volume energy is opposite of the other two, so as expected, the resultant value lies within the variation lines. It is well established from mass formula that nuclear binding energy is directly proportional to volume energy factors, but surface energy, coulomb energy and asymmetric energy all are negative, hence their contributions are reverse to that of earlier one. Calculated results once again verify these. So from the Fig 3.1 and Fig 3.2, it is noted that the variation of volume energy per nucleon changes with mass number for magic nuclei and magic nuclei with adjacent nuclei shows the almost same result. Other two variation as these are negative, so increases with mass number. The resultant (i.e. sum of these three energies) first increases with mass number and then almost constant and then decreases slightly for both magic nuclei and nuclei adjacent to magic nuclei. But from the Figs it is true that the binding energy of the magic nuclei is always slightly greater than that of nuclei adjacent to magic nuclei (though due to scaling all the points are not clearly visualized in the variation curve). From the Fig 4.1 and Fig 4.2, it is found that binding energy per nucleon for magic number nuclei are higher for both semi-empirical mass formula and mass defects considerations, compared to average binding energy per nucleon. For magic nuclei these are higher than 8MeV.per nucleon. From the curve it is also observed that the highest value of it is for ${}_{38}Sr^{8}$, and these values are 8.82MeV. and 8.73MeV from two calculations. From the study of the Fig 5.1 and Fig 5.2, binding energies per nucleon for the nuclei having one less or more nucleon compared to the magic nuclei are less with respect to the values of magic nuclei. It is further observed that this value is 7.97MeV. for ${}_{8}O^{16}$ and it is 7.85MeV. for ${}_{8}O^{17}$ (nucleus having one more neutron compared to magic nucleus ${}_{8}O^{16}$).Whereas it is 8.82MeV.for ${}_{38}Sr^{88}$ and 8.77MeV. for ${}_{38}Sr^{87}$ (one neutron less than ${}_{38}Sr^{88}$, magic nucleus). Fig. 6.1, reflects the neutron separation energy decreases with neutron number of magic nuclei. But these values are greater than the values of average binding energy for each case as well as for average binding energy of the all nuclei. .Neutron separation energy of ${}_{28}Ni^{58}$ is surprisingly found to be less compared to other magic nuclei, other than this, the variation of neutron separation energy with neutron number is almost symmetrically decreases with neutron number. In the is also noted that value S_n for ${}_{38}Sr^{38}$ is comparatively higher than that of other magic nuclei. From the observation of the Fig 6.2, it is concluded that neutron separation energy for adjacent nuclei to magic number nuclei is comparatively less than that of magic nuclei. S_n values for the adjacent nuclei even less than their average binding energy. It is the least for the nucleus ${}_{9}O^{17}$ in the series of considered nuclei. In the Fig 7.1 it is shown that proton separation energy S_p decreases with proton number for magic nuclei. The variation is almost exponential fall; the erratic behavior (value) of proton separation energy is observed ${}_{38}Sr^{88}$. Proton separation energies for each of the members of the magic nuclei are higher than their average binding energy and less with respect to their individual neutron separation energy. The study of Fig 7.2 gives the information that the values of proton separation energy of the nearest nuclei of magic nuclei are less than that of the magic nuclei. The proton separation energy of magic number nuclei and a nucleus one less or more proton differ abruptly and the later always less compared to the magic nuclei. Proton separation energies of the very adjacent nuclei less than that of the magic nuclei, and these values are also less than their individual average nuclear binding energies. Here S_p value is the least for the adjacent nuclei, ${}_{8}O^{17}$.It is noticed that energy decreases with shell differences i.e. energy separation for two successive l (lower value of J to higher value of J) diminishes from lower to higher values. It also confirms when moves from beneath to top of the standard energy chart, distances come closer compared to the reverse direction. From the Fig 8.2, it is also predicted the for two J values, the proportionate energy splitting or energy separation increases with the increase of the value l .

References:

[1] Mayer. M. G, On closed Shells in Nuclei phy. phy.Rev.78, 16 (1950)

[2] Daniel. S. K. Total Binding ---- Experiments.Phys.Rev.Lett.28, (182), 1972.

[3] .Harrey .J.A: phy.Rev.81,53(1951).

[4] Nordling.C. Siegbahn.K. Phys Rev.105. 1676(1957)

[5] Nuclear Physics, Ghosal.N.S, reprint 2004,S. Chand & co.ltd.

[6] Nuclear physics: experimental and theoretical. .Hans. S.H. 2nd Ed.2010.New Age----(p) ltd.

[7] Avalid.M.M Study of Nuclear shell Model,... open Science journal of Modern physics 2(3): 19-22, 2015.
[8] Shpol'skii, Atomnain. E.V. Fizika,vol.(1),5th Ed.Moscow,1963.
[9] Maria. G. Mayer, On closed Shells in Nuclei phy.Rev.74, 235(1950)
[10] Nuclear physics,Patel.S.B,reprint,1996.New Age International—(P) ltd.
[11] Atomic and Nuclear Physics, Sharma.S, 1st Ed.2008. RearsonEducation.
[12] Mottenson.B.R, Volatin.J.G Phys. Rev.Lett.5, 511, 1960.
[13] Utsuno.Y, Phys .Rev. C 60, 054315. 1999.
[14] Ede. R.J,,Eemery.V.J,. Procedings of the Royal Society.1958
[15] Nuclear Physics,Prakash.S,2nd Ed..2011.Pragati Prakashan.
[16] Concepts of Modern Physics. Beiser.A,5th. Ed.1999,Tata McGraw Hill.
[17]Wang.Z,Zhang.X and Wang.X , phys.A,356,225 (1996)
[18] Ozawa.A, Kobayashi.T, Suzuki.T, Yoshida.K, and Taninata.I, phys. Rev.Lett.84, 5493, 12 June 2000.
[19] Caurier.G, Pinedo.M, Nowacki.F,Poves.A, and Zuker.A.P,The shell model as a unified view of nuclear structu Rev.Mod.phys.77,427.June2005.
[20] Chanda.D, A Review ---of Atom.IJIR, Vol-2 Issue-11, 2016
[21] Nuclear physics , Kaplan. I, (2nd edition) Oxford & IBH Publishing Co.pvt..Ltd.1962.