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Homomorphic and Isomorphic images of some soft structures over a semigroup

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Abstract

This paper deals with some basic notions of sub semigroups, soft semigroups, ideals in semi groups, soft left and soft right ideals over a semigroup and by utilising these structures some interesting results on their homomorphic and isomorphic images are established.

Keywords: *Soft semi groups, Soft left ideal, Soft right ideal, Homomorphic image and Isomorphic image.*

1. Introduction

The difficulties of uncertain data in many fields such as engineering, economics, Environmental Science, Sociology, Medical Science, Computer Science etc are dealt by a large number of researchers. The problem of imperfect knowledge is tackled by many mathematicians, logicians. This has become a crucial issue for computer scientists in particular those working in the area of artificial intelligence. We have many approaches such as Fuzzy set theory and Rough set theory to the problem of how to understand and manipulate imperfect knowledge. Theories of Fuzzy sets and Rough sets are powerful mathematical tools to check uncertainties of various problems. Soft set theory is still a better approach to deal the problems of uncertainty.

The concept of soft sets was first introduced by D.Molodsov[5] in the year 1999 as a completely new mathematical tool for solving many typical problems having uncertainties. He proposed new approach of modelling uncertainty in soft sets using the role of parameters by showing several applications in the fields of economics, engineering and medicine. Gradually this theory became good source of research for many mathematicians of recent years because of its wide range of applicability. Maji et.al [3] gave first practical application to the problems of decision making where he showed that this is based on the knowledge of reduction in rough set theory. Aktas and Cagman [1] proposed the concept of soft groups first by using soft set theory and as a result many soft algebraic structures have been investigated K.Moinuddin and D.Madhusudhana Rao [4] studied soft groups and soft subgroups by assigning a group structure to the universal set and established some interesting results.

The soft set theory became a very good source of research for many mathematicians and computer scientists of recent years. The development in the fields of soft set theory and its applications has been taking place in a rapid pace.

In this paper we introduce some basic notions of sub semigroups, soft semigroups, ideals in semi groups, soft left and right ideals and by utilising these structures some interesting results on their homomorphic and isomorphic images are established.

2. Preliminaries of Soft Sets and Soft Groups

In this section some basic notions in soft set theory are presented. Let U be an initial universe and E_U be the set of all possible parameters under consideration with respect to U. The power set of U (i.e., the set of all subsets of U) is denoted by $P(U)$ and A is a subset of E. Usually parameters are characteristics or properties of objects in U.

Definition 2.1: A pair (F, A) is called a *soft set* over U, if $A \subset E$ and $F : A \to P(U)$ and we write F_A for (F, A).

Example 1:

Let $U = \{ f_1, f_2, ... f_6 \}$ be the set of flats under consideration and E={Set of Parameters}
={Expensive, cheap, Beautiful, with good specifications, with bad specifications}

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Suppose p_1 : expensive; p_2 : cheap; p_3 : Beautiful; p_4 : good specifications; p_5 : bad specifications and let

$F(p_1)=\{f_2, f_4\}$, $F(p_2)=\{f_1, f_3\}$,

$F(p_3)=\{f_3, f_4, f_5\}$, $F(p_4)=\{f_3, f_5\}$,

$F(p_5)=\{f_1\}$

To define a soft set means to point out expensive flat, cheap flat, and so on. The soft set (F,E) describes the attractiveness of the flat which some person '*P*' is going to buy. The soft set (F,E) is a parametrized family $\{F(p_i)/i=1,2,3,4,5\}$ of subsets of the set U and it gives us a collection of nature of objects.

Consider the mapping which is " $flats(\circ)$"

where dot is some $p_i \in E$

$\therefore F(p_1)$ means "flats(expensive)" $=\{f_2, f_4\}$

and so on, which means that a soft set

$$(F,E)=\left\{\begin{array}{l} Expensive\ \ flats=\{f_2, f_4\}, \\ cheap\,flats=\{f_1, f_3\}, \\ Beautiful\ \ flats=\{f_3, f_4, f_5\}, \\ Good\ specification\ flats=\{f_3, f_5\}, \\ Bad\ specification\ flats=\{f_1\} \end{array}\right\}$$

Example 2:

Let $U=\{metro\,cities\,of\ \ India\}$

$=\{C_1,C_2,....C_5\}$; C_1 : Delhi; C_2 : Mumbai; C_3 : Kolkatta; C_4 : Chennai; C_5 : Hyderabad and $E=\{e_1,e_2,e_3,e_4,e_5\}$ where the parameters

e_1 : High literacy rate; e_2 : Densely populated city; e_3 : High employability; e_4 : low employability and e_5 : poverty.

Let $A=\{e_1,e_2,e_3\}$ we define $F:A\to P(U)$ by $F(e)=C_1$ for $e\in A$ then F_A is a soft set and we describe F_A as

$\therefore F(e_1)=F(e_2)=F(e_3)=C_1$ which means city Delhi has high literacy rate which is densely populated with high employability.

Definition 2.2: Let F_A and G_B be soft sets over a common universe set U and $A,B\subset E$. Then we say that

(a) F_A is a *soft subset* of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if (i) $A\subseteq B$ and

(ii) $F(e)\subseteq G(e)\ \ \forall\, e\in A$.

(b) F_A equals G_B, denoted by $F_A \tilde{=} G_B$, if $F_A \tilde{\subseteq} G_B$ and $G_B \tilde{\subseteq} F_A$.

Definition 2.3: A soft set F_A over U is called a *null soft set*, denoted by Φ, if for $e\in A$, $F(e)=\phi$.

Definition 2.4: A soft set F_A over U is called an *absolute soft set*, denoted by $\tilde{A}$, if for $e\in A$, $F(e)=U$.

Definition 2.5: The *union* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C=A\cup B$, and for all $e\in C$

$$H(e)=\begin{cases} F(e) & if\ \ e\in A-B \\ G(e) & if\ \ e\in B-A \\ F(e)\cup G(e) & if\ \ e\in A\cap B \end{cases}$$

We write $F_A \cup G_B \tilde{=} H_C$.

Definition 2.6: The *intersection* of two soft sets F_A and G_B over a common universe U is the soft set H_C, where $C=A\cap B$, and for all $e\in C$, $H(e)=F(e)\cap G(e)$. We write $F_A \cap G_B \tilde{=} H_C$.

Definition 2.7: For a soft set F_A over U, the relative *complement* of F_A is denoted by F_A^c and is defined by $F_A^c \tilde{=} F_A^1$, where $F^1:A\to P(U)$ is a mapping given by

$F^1(e)=U-F(e)$ for all $e\in A$.

Definition 2.8: Suppose a binary operation denoted by o, is defined for all subsets of U. Let F_A and G_B be two soft sets over U. Then the operation o for the soft sets is defined in the following way :

$F_A \, o \, G_B = (H, A \times B)$ Where

$$H(\alpha, \beta) = F(\alpha) o G(\beta), \ \alpha \in A \text{ and } \beta \in B.$$

Replacing the universal set structure U with the groups G and G' and with $P(G)$ and $P(G')$ the power sets of G and G' respectively, E being the set of parameters we define the following:-

Definition 2.9: We say that a soft set F_E is a *soft subgroup* over G if $F(e)$ is a subgroup of G for every $e \in E$.

Example: Let $G = \{1, -1, i, -i\}$ and $H = \{1, -1\}$ Then G forms an abelian group with respect to multiplication and H is a subgroup of G if $F : E \to P(G)$ is defined by $F(e) = H$ for every $e \in E$ then F_E is a soft subgroup over G. Here E any set of parameters.

Definition 2.10: We say that a soft set F_E is a *soft normal subgroup* over G if $F(e)$ is a normal subgroup of G for every $e \in E$.

Definition 2.11: Let $S(G)$ be the collection of all soft subgroups over G. That is $S(G)$ is the collection of all soft sets F_E, where $F : E \to P(G)$ is defined by $F(e)$ is a subgroup of G for every $e \in E$. We say that $S(G)$ is a *soft group* over G.

Definition 2.12: We say that a soft set F_E is a *soft cyclic subgroup* over G if $F(e)$ is a cyclic subgroup of G for every $e \in E$.

Definition 2.13: A mapping $\varphi : G \to G'$ is said to be a *homomorphism* of G into G' if

$\varphi(xy) = \varphi(x)\varphi(y)$ for all x and y in G.

Definition 2.14: A homomorphism $\varphi : G \to G'$ is said to be an *isomorphism* of G into G' if φ is one-one.

Definition 2.15: A homomorphism $\varphi : G \to G$ is said to be an *automorphism* of G onto itself if φ is a bijection.

Definition 2.16: Let $F : E \to P(G)$ be a soft set and $\varphi : G \to G'$, a mapping of G into G'. Then we define a soft set $\varphi_F : E \to P(G')$ as follows.

$$\varphi_F(e) = \varphi(F(e)) = \{\varphi(x) : x \in F(e)\}$$

If $\varphi : G \to G'$ is a homomorphism of G into G' then we call the soft set $\varphi_F : E \to P(G')$ to be the *homomorphic image* of the soft set F_E.

Proposition: If $\varphi : G \to G'$ is a homomorphism of G into G' and $F : E \to P(G)$ is a soft subgroup over G then $\varphi_F : E \to P(G')$ is soft subgroup over G'.

Proposition: If $\varphi : G \to G'$ is a homomorphism of G into G' and $F : E \to P(G)$ is a soft normal subgroup over G then $\varphi_F : E \to P(G')$ is soft normal subgroup over G'.

Proposition: If $\varphi : G \to G'$ is a homomorphism of G into G' and $F : E \to P(G)$ is a soft

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cyclic subgroup over G then $\varphi_F : E \to P(G')$ is soft cyclic subgroup over G'.

3. Soft Semi groups, Soft Ideals over a semi group and their Homomorphic and Isomorphic images

In this section a few interesting results on the homomorphic images and the isomorphic images of soft sub semigroups and soft ideals over a semi group are investigated. Throughout this section, S and S' denote two semi groups. Let $P(S)$ and $P(S')$ be the power sets of S and S' respectively. E be the set of all possible parameters under consideration with respect to the universes S and S' and $A \subset E$.

Definition 3.1: If S is any semi group of a group G then non-empty subset P of S is called a sub semigroup of S if $P^2 \subseteq P$ that is $ab \in P \,\forall\, a, b \in P$.

Definition 3.2: **A** sub semi group '*P*' of a semi group S is called a *left ideal* of S if $SP \subseteq P$ i.e $sp \in P \;\forall s \in S, p \in P$.

Definition 3.3: **A** sub semi group '*P*' of a semi group S is called a right ideal of S if $PS \subseteq P$ i.e $ps \in P \;\forall s \in S, p \in P$.

'*P*' is said to be an ideal of a semi group S if it is both left and right ideal of *S*.

Definition 3.4: A soft set *(F,A)* over a semi group S is called a soft semigroup if $(F, A) \neq \Phi$ and $(F, A) \circ (F, A) \tilde{\subseteq} (F, A)$.

The Soft set *(F,A)* over a semi group S is called a *soft semi group* over S iff $\forall\; a \in A, F(a)$ is a sub semigroup of *S*.

Definition 3.5: A soft set $(F, A) \neq \Phi$ over a semi group S is called a *soft left ideal* over S if $\tilde{A}_S \circ (F, A) \tilde{\subseteq} (F, A)$.

A soft set $(F, A) \neq \Phi$ over a semi group S is also said to be a *soft left ideal* over S if *F(a)* is a left ideal of $S \;\forall\; a \in A$.

Definition 3.6: A soft set $(F, A) \neq \Phi$ over a semi group S is called a *soft right ideal* over S if $(F, A) \circ \tilde{A}_S \tilde{\subseteq} (F, A)$.

A soft set $(F, A) \neq \Phi$ over a semi group S is also said to be a *soft right ideal* over S if *F(a)* is a right ideal of $S \;\forall\; a \in A$.

Definition 3.7: A soft set $(F, A) \neq \Phi$ over a semi group S is called a *soft ideal* over S if *F(a)* is an ideal of $S \;\forall\; a \in A$.

Definition 3.8: A mapping $\varphi : \mathrm{S} \to S'$ is said to be *homomorphism* of S into S' if $\varphi(xy) = \varphi(x)\varphi(\mathrm{y})$ for all x and y in *S*.

Definition 3.9: A homomorphism $\varphi : \mathrm{S} \to S'$ is said to be an *isomorphism* of S into S' if φ is one-one.

Definition 3.10: A homomorphism $\varphi : \mathrm{S} \to S$ is said to be an *automorphism* of S onto itself if φ is a bijection.

Definition 3.11: Let $F : \mathrm{A} \to P(S)$ be a soft set and $\varphi : \mathrm{S} \to S'$ a mapping of S into S'. Then we define a soft set $\varphi_F : \mathrm{A} \to P(S')$ as follows.

$$\varphi_F(\mathrm{a}) = \varphi(F(\mathrm{a})) = \{\varphi(x) : x \in F(\mathrm{a})\}$$

If $\varphi : \mathrm{S} \to S'$ is a homomorphism of S into S' then we call the soft set

$\varphi_F : A \to P(S')$ to be the *homomorphic image* of the soft set F_A.

Similarly if $\varphi : S \to S'$ is an isomorphism of S into S' then $\varphi_F : A \to P(S')$ is said to be the *isomorphic image* of the soft set F_A.

Proposition: -

If $\varphi : S \to S'$ is a homomorphism of S into S' and $F : A \to P(S)$ is a soft semigroup over S then $\varphi_F : A \to P(S')$ is a soft semi group over S'.

Proof:-Given $\varphi : S \to S'$ is a homomorphism of S into S' and $F : A \to P(S)$ is a soft semigroup over *S*.

we have to prove $\varphi_F : A \to P(S')$ is a soft semigroup over S'

we have $\varphi_F(a) = \varphi(F(a)) = \{\varphi(x) : x \in F(a)\}$

since F_A is a soft semi group over *S*, *F(a)* is a

sub semigroup of S $\forall\, a \in A$. $\therefore$ for each $a \in A$, $\varphi(F(a)) = \{\varphi(x) : x \in F(a)\}$ is the homomorphic image of sub semigroup *F(a)* of *S*.

$\therefore$ $\varphi(F(a))$ is also a sub semigroup of S' $\forall\, a \in A$.

$\therefore$ $\varphi_F(a)$ $\forall\, a \in A$ is soft semigroup over S'.

Hence $\varphi_F : A \to P(S')$ is a soft semi group over S'.

Proposition: - If $\varphi : S \to S'$ is an isomorphism of S into S' and $F : A \to P(S)$ is a soft left ideal over S then $\varphi_F : A \to P(S')$ is a soft left ideal over S'.

Proposition: -

If $\varphi : S \to S'$ is an isomorphism of S into S' and $F : A \to P(S)$ is a soft right ideal over S then $\varphi_F : A \to P(S')$ is a soft right ideal over S'.

Proposition: -

If $\varphi : S \to S'$ is an isomorphism of S into S' and $F : A \to P(S)$ is a soft ideal over S then $\varphi_F : A \to P(S')$ is a soft ideal over S'.

Proposition: - Automorphic images of soft ideals over a semigroup S are soft ideals over *S*.

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References

[1]Aktas. H., and Cagman. N., *Soft sets and Soft groups*, Inf. Sci., Vol-177, pp.2726-2735(2007).

[2]Herstein I.N., *Topics in Algebra*,. 2nd edition, Wiley&Sons, New York, 1975.

[3]Maji. P. K., Biswas. R., and Roy. A. R., *Soft set theory* , Comp. Math. Appl., Vol-45, pp.3029-3037(2008).

[4]Moinuddin.K,Madhusudana Rao.D., *A novel approach:soft groups*,International Journal of Algebra,Vol.9,No.2,pp79-83(2015)

[5]Molodtsov. D., *Soft Set Theory- First Results*, Comp. Math. Appl., Vol-37, pp. 19-31(1999).

[6]Muhammad Irfan Ali., *Some Contributions to Soft Semigroups and related Structures*, Ph.D Thesis,Quaid-i-Azam University(2009).

[7] Srinivaskumar. V, Moinuddin.K., *On homomor phic images of Soft Subgroups,* Global Journal of Pure and Applied Mathematics,Vol.11,No.3,pp1223-229(2015).