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Blood Flow Through a Capillary With Variable Wall Permeability

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Abstract

The present investigation describes the mathematical model of the blood flow through the capillary with variable wall permeability. Considering blood as homogenous Newtonian fluid, it is assumed that the permeability of the fluid across the capillary wall obeys Starling's law. Hence the present study gives an analytical expression for velocity, pressure of the flow. The results are discussed graphically with the effects of various parameters.

Keywords: *permeability, capillary, Starling's law*

1. Introduction

Blood flow through capillary has always been an area of great interest of the researchers with various aspects. Mackey R.I [1] [2] have studied the various pressure flow pattern with reabsorbing wall and focused on the governing principles of hydrodynamic in renal tubule. Appellate [3] and Palatt.P.J [4] have done the mathematical analysis of capillary tissue fluid exchange while Marshal.E.A et al. [5] have found the transmural seepage at the wall obeys Darcy's law, which can be related to starling's hypothesis. Murata [6] have studied in detail the effect of linearly varying permeability on capillary tissue exchange. While the cause of exponentially decreasing the bulk flow and its effects on pressure drops in the renal tubules are found exactly in agreement with the established results by Radhakrishnamacharaya.G et.al [7]. The simultaneous variable permeability with non uniform cross section with their application to the renal tubules has been discussed by Chaturani et al. [8]. Sandeep .K et.al [9] have found that the constant wall permeability and its effects on the pressure drop are compliance with the Germans solution. Mussy .Y et.al [10] have investigated that the radial , axial velocity and pressure distribution over an array of permeable channels reduces the cylindrical coordinates of the Navier-Stokes equation into a fourth order non linear differential equation. Pozrikidis. C [11] have obtained the high accuracy solution to the problem obeying starling law of normal fluid velocity by formulating an integral representation to the stokes flow. While the transverse velocity is no longer necessarily small when compared with the axial velocity, which relaxes that the permeability of the wall is small relative to the non-dimensional parameter was proposed by Herschlag .G et.al [16]. Bernals. B [14] have extended the study of Yaun of non-uniform leakage model allowing a new ODE(ordinary differential equation) which extend to small or moderate transverse Reynolds number Rt (based on transpiration velocity) and also obtained the analytical solution with increasing accuracy to the problem WNL(weak non-linear case).Krishna. J.S.V.R et al. [13] have simultaneously discussed the dual nature of rigidity and permeability of the wall while assuming the permeability as axial function and obeys starling's law and studied the effect of low Reynolds number and permeability on flow characteristics. Tesfahun Berhane [20] has studied the flow of a Newtonian fluid in a non-uniform wavy and permeable tube. Muthu.P et.al [12] considered the flow in a asymmetry channel by considering the fluid absorption through permeable walls. Muthu. P and Varun Kumar [17],[18] have seen the effect of constriction, re-absorption coefficient of walls on the entrance length of channel and tube at different cross-section to discuss the velocity profile, streamlines and pressure drop by considering the Newtonian fluid through a channel and tube with permeable wall. Waseem Raja. S et.al [19] studied the flow through renal tubule of asymmetry channel of varying cross-section by assuming the effect of fluid absorption thorough permeable walls. Further Waseem Raja. S et.al [15] have studied the effect of slip on radial, axial velocities and pressure drop at different cross-section of permeable walls of convergent, divergent channel by considering the Newtonian fluid in permeable boundaries. In this present work, the objective of the study is to understand the effect of linearly varying permeability on blood flow in a capillary.

2. Mathematical Formulation

In the present study we consider the steady incompressible, homogenous, Newtonian fluid flow of a constant viscosity. The axisymmetry with the wall permeability varying along the axis of the tube, we use the cylindrical coordinates as (r, θ, z) in which z-axis denotes the axis of the tube. Where r and θ are radial and angular coordinates.

The governing equations are

$$\frac{\partial p}{\partial r} = \eta\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad (1)$$

$$\frac{\partial p}{\partial r} = \eta\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right) \qquad (2)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial v}{\partial z} = 0 \qquad (3)$$

Where $\boldsymbol{u}$ and $\boldsymbol{v}$ are radial and axial components of velocities, respectively and $\boldsymbol{P}$ is the hydrostatic pressure inside the tube and $\boldsymbol{\eta}$ is the viscosity of the blood.

The corresponding boundary conditions are

$$\frac{\partial v}{\partial r} = 0 \text{ at } r=0 \qquad (4)$$

$$u = 0 \text{ at } r=0 \qquad (5)$$

$$\frac{\partial v}{\partial r} = 0 \text{ at } r=R \qquad (6)$$

$$U = \beta(z)[p-\alpha] \text{ at } r = R \qquad (7)$$

$$\bar{P} = p_a \text{ at } r=0 \qquad (8)$$

$$\bar{P} = p_v \text{ at } z=L \qquad (9)$$

The approximate non dimensional parameters are

$$P = \frac{p-p_v}{\Delta P}, \quad U = \frac{u}{\beta_0 \Delta P}, \quad V = \frac{v\eta}{R\Delta p}, \quad y = \frac{r}{R}, \quad x = \frac{z}{L}$$

Where $\Delta P = p_a - p_v$

Here p_a and p_v are the arterial and venous pressures respectively, where R and L are the radius and length of the tube respectively and $\bar{P}$ is the average pressure over the cross section of the tube. The condition obtained in equation (7) is the consequence of starling's law. Where β is the hydraulic wall permeability which is given by

$$\beta(z) = \beta_0 - \beta_1\left(\frac{z}{L}\right)^2$$

where β_0 and β_1 are the constants, α is the net opposing pressure assumed to be constant.

The basic governing equations assume the form

$$\frac{\partial P}{\partial y} = \varepsilon\left(\frac{\partial^2 U}{\partial y^2} + \frac{1}{y}\frac{\partial U}{\partial y} - \frac{U}{y^2} + \delta^2\frac{\partial^2 U}{\partial x^2}\right) \quad (10)$$

$$\delta\frac{\partial P}{\partial y} = \frac{\partial^2 V}{\partial y^2} + \frac{1}{y}\frac{\partial V}{\partial y} + \delta^2\frac{\partial^2 V}{\partial x^2} \quad (11)$$

$$\frac{\varepsilon}{y}\frac{\partial}{\partial y}(Uy) + \delta\frac{\partial V}{\partial x} = 0 \quad (12)$$

Where $\varepsilon = \frac{\beta_0 \eta}{R}$, $\delta = \frac{R}{L}$

The boundary conditions in non dimensional forms are

U=0 at y=0 (13)

$\frac{dv}{dy}$ =0 at y=0 (14)

$\frac{dv}{dy}$ =0 at y=1 (15)

U =(1-β x)(P + c) at y=1 (16)

$\overline{P} = 0$ at x=0 (17)

$\overline{P} = -\frac{1}{2}$ at x=l (18)

Where β= $\frac{\beta_1}{\beta_0}$, c= $\frac{\Delta\alpha}{\Delta p}$,in which $\Delta\alpha = p_a - \alpha$ and $\overline{P}(= \int_0^1 pydy)$ is the average pressure.

3. Method of Successive Approximation

The governing equation with the initial terms is highly non-linear. Thus in this paper we adopt the successive approximation method to solve the linear equations subject to the same boundary conditions. Thus we get the solution of the non-linear system.

4. Solution By the Method of Successive Approximation

We employed successive approximation method to solve the above equations. On dropping the terms of o(ε) and $o(\delta)^2$ in the equations (10),(11) and(12) we obtain the following approximations.

First Approximation

$$\frac{\partial P_1}{\partial y} = 0 \quad (19)$$

$$\delta\frac{\partial P_1}{\partial x} = \frac{\partial^2 V_1}{\partial y^2} + \frac{1}{y}\frac{\partial V_1}{\partial y} \quad (20)$$

$$\frac{\varepsilon}{y}\frac{\partial(U_1 y)}{\partial y} + \delta\frac{\partial V_1}{\partial X} = 0 \quad (21)$$

Second Approximation

$$\frac{\partial P_2}{\partial y} = \varepsilon\left(\frac{\partial^2 U_1}{\partial y^2} + \frac{1}{y}\frac{\partial U_1}{\partial y} - \frac{U_1}{y^2}\right) \quad (22)$$

$$\delta\frac{\partial P_2}{\partial x} = \frac{\partial^2 V_2}{\partial y^2} + \frac{1}{y}\frac{\partial V_2}{\partial y} + \delta^2\frac{\partial^2 V_1}{\partial x^2} \quad (23)$$

$$\frac{\varepsilon}{y}\frac{\partial(U_2 y)}{\partial y} + \delta\frac{\partial V_2}{\partial x} = 0 \quad (24)$$

On solving equations (19), (20)and (21)subject to boundary conditions (13)and (14) we get

$$V_1 = \frac{\delta}{4}\frac{dP_1}{dx}(y^2 - 1) \quad (25)$$

$$U_1 = \frac{1}{\mu^2}\frac{d^2P_1}{dx^2}(2y - y^3) \quad (26)$$

From (16) and (17), we have

$$\frac{1}{\mu^2}\frac{d^2P_1}{dx^2} = (1 + \beta x)(P_1 + c) \quad (27)$$

Where $\mu^2 = \frac{16\varepsilon}{\delta^2}$

Let $1 + \beta x = X, P_1 + c = Y$

Then eqn. (21) becomes

$$\frac{d^2Y}{dx^2} - \lambda^2 XY = 0 \quad (28)$$

Where $\lambda^2 = \frac{\mu^2}{\beta^2}$

Differential equation (28) can be solved by series solutions

Let Y= $a_0 + a_1X + a_2X^2$ ... (29)

(i.e.) Y= $\Sigma a_i X^i$

$$\frac{d^2Y}{dx^2} = \sum_{i=2} i(i-1)a_i X^{i-2}$$

Then the equation (28) becomes

$$\sum_{i=2} i(i-1)a_i X^{i-2} - \lambda^2 \sum_{i=0} a_i X^{i+1} = 0$$

This can be written as

$$\sum_{i=2} i(i+2)(i+1)a_{i+2}X^i - \lambda^2 \sum_{i=0} a_{i-1}X^i = 0$$

$$2.1a_2 + [\textstyle\sum_{i=2}(i+2)(i+1)a_{i+2} - \lambda^2 a_{i-1}]X^i = 0$$

Thus we have

$a_2 = 0$, $(i+2)(i+1)a_{i+2} - \lambda^2 a_{i-1} = 0$, i≥ 1

which implies

$a_{i+2} = \frac{\lambda^2 a_{i-1}}{(i+2)(i+1)}$ for, i≥ 1

Thus we get

$$Y = a_0\left[1 + \frac{\lambda^2X^3}{2.3} + \frac{\lambda^4X^6}{2.3.5.6} + \frac{\lambda^6X^9}{2.3.5.6.8.9} + \cdots\right] + a_1[X + \frac{\lambda^2X^4}{3.4} + \frac{\lambda^4X^7}{3.4.6.7} + \frac{\lambda^6X^{10}}{3.4.6.7.9.10} + \cdots] \quad (30)$$

which can be written as

$$Y = a_0A(X) + a_1B(X) \ldots \quad (31)$$

Where $A(X) = 1 + \sum_{j=0} \lambda^{2j+2}Q_j \frac{X^{3j+3}}{(3j+3)!}$,

$B(X) = \sum_{j=0} \lambda^{2j}R_j \frac{X^{3j+1}}{(3j+1)!}$,

Further from $P_1 + c = Y$ we have

$$P_1 = a_0A(X) + a_1B(X) - c \quad (32)$$

Where

$$a_0 = \frac{cB(1+\beta) - (c-1)B(1)}{A(1)B(1+\beta) - A(1+\beta)B(1)}, \; a_1 = \frac{c - a_0A(1)}{B(1)}$$

Substituting equation (32) in equations (25) and (26) we get

$$V_1 = \frac{\beta\delta}{4}[a_0A_1 + a_1B_1](y^2 - 1) \quad (33)$$

$$U_1 = \frac{\beta^2}{\mu^2}[a_0A_1 + a_1B_1](2y - y^3) \quad (34)$$

Where $A_1 = \sum_{j=0} \lambda^{2j+2}Q_j \frac{X^{3j+2}}{(3j+2)!}$

$$A_2 = 1 + \sum_{j=0} \lambda^{2j+2}Q_j \frac{X^{3j+1}}{(3j+1)!}$$

$$B_1 = \sum_{j=0} \lambda^{2j}R_j \frac{X^{3j}}{3j!}$$

$$B_2 = \sum_{j=1} \lambda^{2j}R_j \frac{X^{3j-1}}{(3j-1)!}$$

On using the equation (33) and(34)and integrating the equations (22-24) by applying the appropriate boundary conditions, we get

$$V_2 = \frac{\beta^3\delta^3}{32}\left[a_0A_3(X)a_1B_3(X)\right](y^4 - 1) + \frac{f(X)}{4}(y^4 - 1) \quad (35)$$

Where

$$A_3(X) = \lambda^2 \sum_{j=0} \lambda^{2j}Q_j \frac{X^{3j}}{(3j)!}$$

$$B_3(X) = \sum_{j=1} \lambda^{2j} R_j \frac{X^{3j-2}}{(3j-2)!}$$

$f(X) = D_1F(X) + D_2G(X) + D_3H(X)$ in which

$$F(X) = 1 + \sum_{j=1} \lambda^{2j} R_j \frac{X^{3j}}{(3j)!}$$

$$G(X) = X^2 + 2\sum_{j=1} \lambda^{2j} Q_j \frac{X^{3j+2}}{(3j+2)!}$$

$$H(X) = X \sum_{j=0} X^j c_j$$

$$D_3 = \frac{\beta^5 \delta^3 \lambda^2}{12\mu^2}$$

$$c_0 = -a_1\lambda^2 d_0, \quad c_1 = 0, \quad c_2 = \frac{\lambda^4 a_0 e_2}{18},$$

$$c_{3j} = \frac{\lambda^2}{(3j-1)(3j+1)}\left[c_{3j-3} + \frac{R_{3j-1} + \lambda^{2j} d_{3j} a_1}{(3j+1)!}\right]$$ where

$j = 1,2,3,4,\ldots \quad c_{3j+1} = 0$

$$c_{3j+2} = \frac{\lambda^2}{(3j+1)(3j+3)}\left[c_{3j-1} + \frac{Q_{3j+1}}{(3j+3)!} a_0 \lambda^{2j+2} e_{3j+2}\right]$$

In which

$$U_2 = -\frac{\beta^4\delta^2}{12\mu^2}[a_0A_4(X) + a_1B_4(X)](y^5 - 3y) - \frac{\beta}{\mu^2\delta}[D_1F_1(X) + D_2G_1(X) + D_3H_1(X)](y^3 - 2y) \quad (36)$$

where

$$A_4 = \lambda^2 \sum_{j=0} \lambda^{j+2} Q_j \frac{X^{3j-1}}{(3j-1)!}$$

$$B_4(X) = \sum_{j=1} \lambda^{2j} R_j \frac{X^{3j-3}}{(3j-3)!}$$

$$F_1(X) = \sum_{j=1} \lambda^{2j} R_j \frac{X^{3j-1}}{(3j-1)!}$$

$$G_1(X) = 2\sum_{j=0} \lambda^{2j} Q_j \frac{X^{3j+1}}{(3j+1)!}$$

$$H_1(X) = \sum_{j=0} c_j (j+1) X^j$$

$$P_2 = -\frac{\beta^2\delta^2}{12\mu^2}[a_0A_2(X) + a_1B_2(X)](y^2+1) + \frac{1}{\beta\lambda^2\delta X}[D_1F_1(X) + D_2G_1(X) + D_3H_1(X) - D_3(a_0T_0(X) + a_1T_1(X)] - c \quad (37)$$

Where $T_0(X) = \frac{e_2\lambda^4X^2}{3!} + \sum_{j=0} e_{3j+5}\lambda^{2j+6} Q_j \frac{X^{3j+6}}{(3j+6)!}$

$$T_1(X) = -d_0 + \sum_{j=0} d_{3j+3}\lambda^{2j+6} R_j \frac{X^{3j+3}}{(3j+4)!}$$

In which

$$d_j = [(-2j-4) - 6i](j^2 - 1), j = 0,3,6,9,\ldots$$

$$e_j = [(-2j-4) - 6i](i^2 - 1), i = 2,5,\ldots$$

the constants

$$D_1 = \frac{\gamma_1 G_1(1+\beta) - \gamma_2 G_1(1)}{F_1 G_1(1+\beta) - F_1(1+\beta) G_1(1)}$$

$$D_2 = \frac{\gamma_1 - D_1F_1(1)}{G_1(1)}$$

Where

$$\gamma_1 = \frac{\beta^3\lambda^2\delta^3}{8}[a_0A_2(1) + a_1B_2(1)](1+2s) + \beta\lambda^2 c\delta + D_3T(1) - D_3H_1(1)$$

$$\gamma_2 = \frac{\beta^3\lambda^2\delta^3}{8}\left[a_0A_2(1+\beta) + a_1B_2(1+\beta)\right](1+\beta)(1+2s) + (1+\beta)\lambda^2\beta c\delta + D_3T(1+\beta) - D_3H_1(1+\beta) - \lambda^2(1+\beta)$$

$$T(1) = a_0T_0(1) + a_1T_1(1)$$

Consider the flow rate

$$\bar{Q} = \int_0^R 2\pi r v dr$$

The volume of the blood flowing per unit time across the cross section is given by

$$Q = \int_0^l (Vy)dy$$

Where $Q = \frac{\bar{Q}\eta}{2\pi R^3 \Delta p}$

By using equation (35)we get

$$Q = \frac{\beta^3\delta^3}{96}[a_0A_3(X) + a_1B_3(X)] + \frac{1}{16}[D_1F(X) - D_2G(X) + D_3H(X)]$$

It may be noted that the solutions $U_2, V_2, and P_2$ are good approximation for *U*, *V* and *P*.

Let us now introduce a set of non dimensional variables (related to the earlier non-dimensional variables) which are quit suitable for studying the underlying physical phenomena. These variables are

$$U^{\sim} = \frac{uR}{v} = \varepsilon NU, V^{\sim} = \frac{vR}{v} = NV, \qquad P^{\sim} = \frac{pR^2\rho}{\eta^2} = N_a + NP$$

$$C = \frac{\Delta\alpha R^2\rho}{\eta^2} = N_a - N_0 = cN, N = N_a - N_v$$

where

$$N_0 = \frac{\alpha R^2\rho}{\eta^2}, N_v = \frac{R^2\rho P_v}{\eta^2}, N_a = \frac{R^2\rho P_a}{\eta^2}$$

Further we take $Q^{\sim} = QN$

Which represent the effect of β on the physical variables occurring in the problem.

5. Result and Discussion

The velocity components $U^{\sim}, V^{\sim}$, hydrostatic pressure $P^{\sim}$ and the plug flow rate $Q^{\sim}$ are computed by using the experimental physiological data considered by Oka and Murata describing the blood flow through a capillary with variable permeability have taken the average values of p_a, p_v and α as p_a =35mmHg, $p_v = 15mmHg$ and α=24mmHg referred the atmospheric pressure in the capillaries of a man at heart level under resting condition. whereas the values of β, ε ,η, R and L have been taken as $10x10^{-6}$ cm ps.Cm.H_2O a $4.9x10x10^{-6}$, $5x10x10^{-4}$cm, $4x10^{-2}$ poise and 100-300 μm. Taking the values of β from 0-90.

It is noticed that the radial distribution (U) and axial velocity (V) for different values of variable wall permeability parameter (β) at different position is significant.

Radial Velocity

It demonstrate that the variation in radial velocity (U) at different values of permeability parameter (β) at different positions.From fig. 1 (a) to fig. 1 (e) .It is observed that at the entrance of the tube the increase in the value of permeability parameter (β) effects the radial velocity(U) such that it is moving from positive to negative in the entire cross section. For relatively small variation in permeability parameter (β) causes a significant change in radial velocity (U). Also it is noted that in the downstream the variation of radial velocity (U) with permeability parameter (β) is a non linear .Further it is observed that except in the beginning the radial velocity (U) is found to be exclusively negative in the middle position of the tube. It is quite interesting to note that at the discharging end the radial velocity(U) is moving from negative to positive for relatively higher values of permeability parameter (β). Thus this situation is exactly contrast to the one which is found at x=0.25.

Axial Velocity

It demonstrate that the variation in axial velocity (V) at different values of permeability parameter (β) at different axial positions. From fig. 2(a) to fig. 2(e) it observed that the variation in axial velocity (V) decreasing throughout the tube. Hence it is clearly observed that for relatively higher values of permeability parameter (β) the axial velocity significantly decreases and move from positive to negative.

Pressure

From fig. 3(a) to fig. 3(e) it observed that the pressure increases with increase in permeability parameter (β) throughout the channel. However this

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increase is relatively more significant for the smaller values of (β). While comparing with the higher values permeability parameter (β). Also it is observed that for all values of (β) pressure P decreases with axial distance and this decrease is significant for smaller values of (β). Hence the effect of permeability parameter (β) on pressure (P) is only significant when (β) is relatively small.

6. Conclusion

From the above analysis it is understood that the variation in permeability at the wall plays an important role on the blood flow through the capillary.

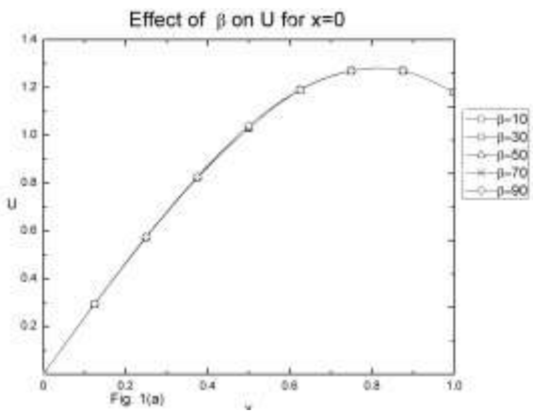

Fig. 1(a)

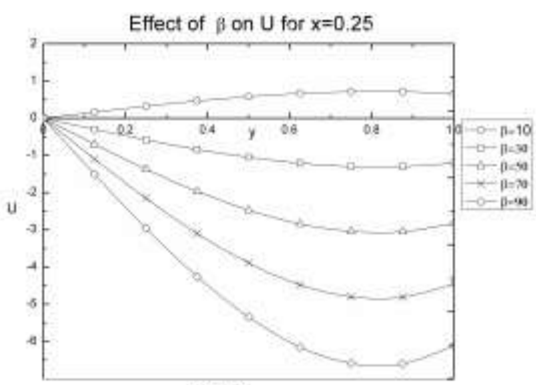

Fig. 1(b)

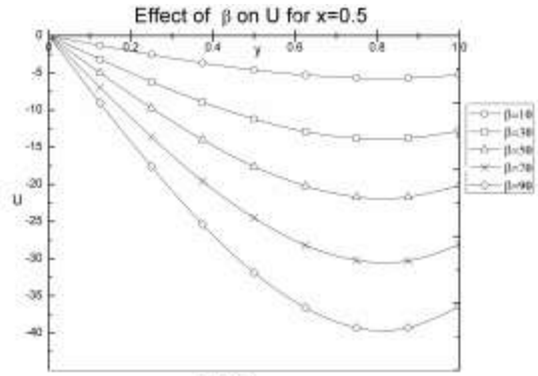

Fig. 1(c)

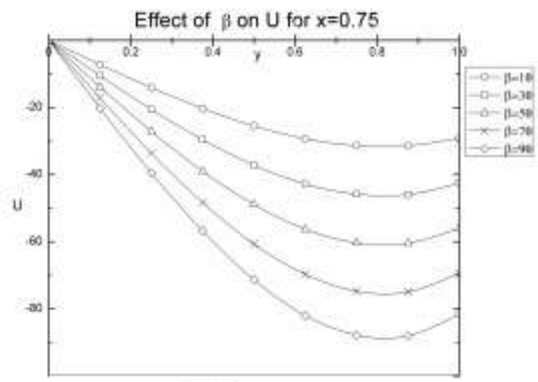

Fig. 1(d)

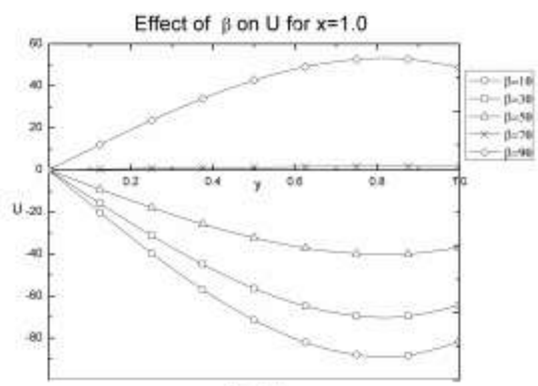

Fig. 1(e)

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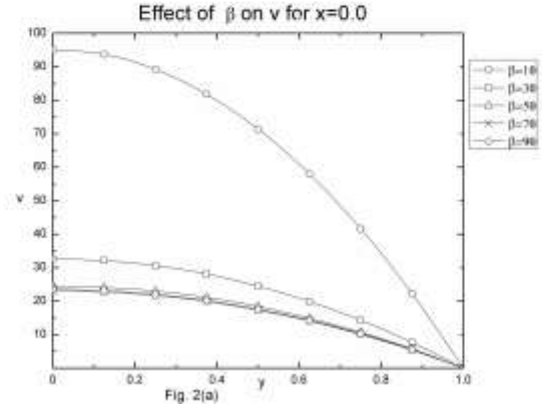

Fig. 2(a)

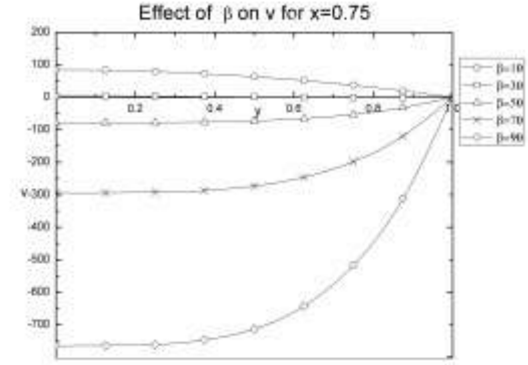

Fig. 2(d)

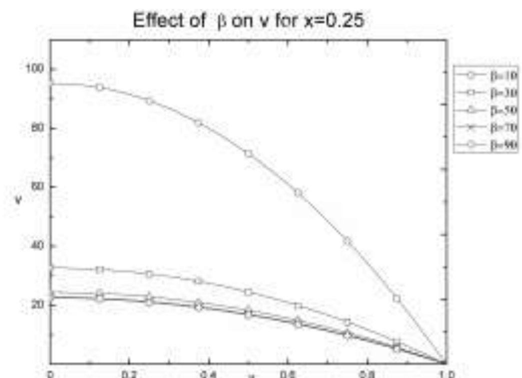

Fig. 2(b)

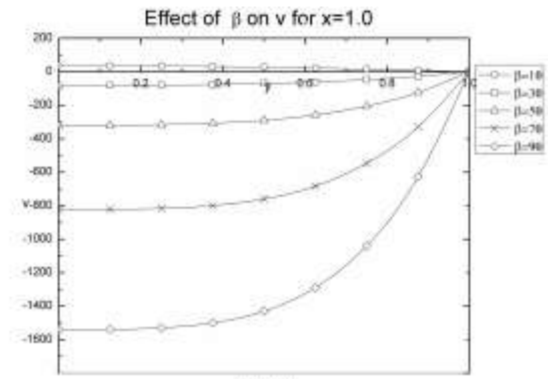

Fig. 2(e)

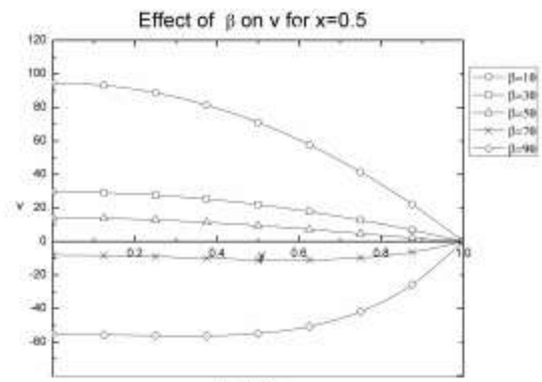

Fig. 2(c)

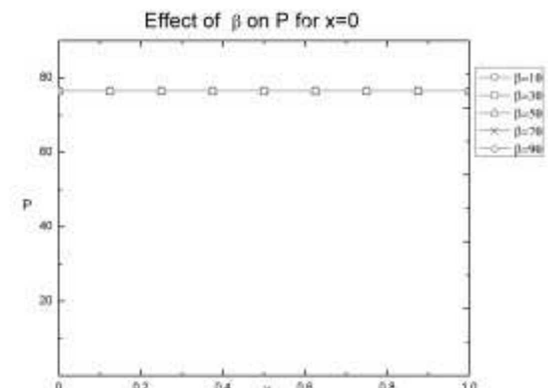

Fig. 3(a)

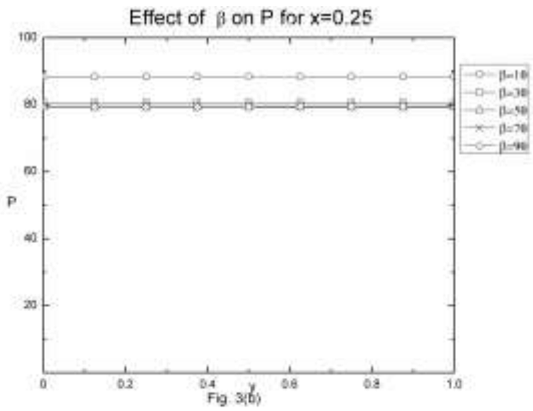

Fig. 3(b)

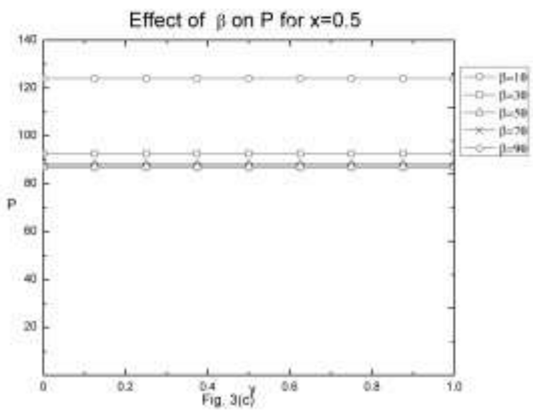

Fig. 3(c)

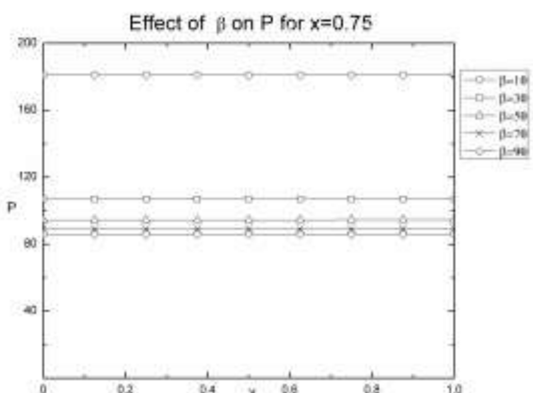

Fig. 3(d)

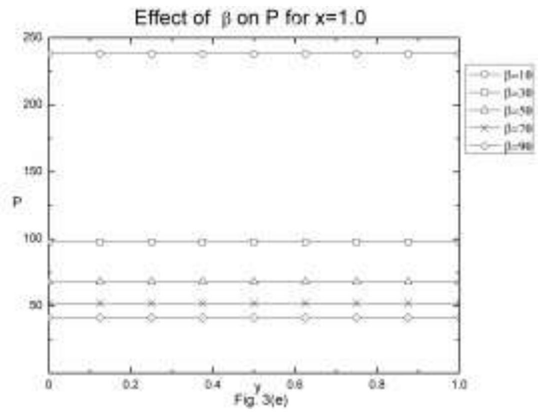

Fig. 3(e)

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