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Markovian Inventory Model For Deteriorating Items With Bayesian Estimation

N. S. Indhumathy[1] , Dr. P. R. Jayashree[2]

[1]Research Scholar, Department of Statistics, Presidency College, Chennai-600 005, India.

[2]Assistant professor, Department of Statistics, Presidency College, Chennai- 600 005, India.

Abstract

Inventory models are generally classified as deterministic or stochastic. Deterministic models are models where the demand for a time period is known, whereas in stochastic models the demand is a random variable having a known probability distribution. Most of the research papers available in the literature deals with the deterministic demand. However, the demand is usually unknown and probabilistic in nature. Therefore, In this paper, a Markovian inventory model with inter-demand time as exponential distribution for deteriorating items is considered. The model contains the exponential parameter which is unknown and is estimated through MLE and Bayesian estimation under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically. The objective of the paper is to develop an optimum policy that minimizes the total average cost by using the above estimates of the parameter. The sensitivity analysis is also carried out for the model with percentage change in the parameters.

Keywords: *Bayesian Estimation, Deteriorating items, Optimal cycle time, Sensitivity analysis, Stochastic demand.*

1. Introduction

Inventory control is set of policies and operating transactions that are planned to maximize of any organization with the help of inventory, therefore it make the maximum earnings from the smallest amount of inventory investment with not including purchaser satisfaction levels. A good inventory control system offers the benefits of the proper relationship between sales and inventory can better be well maintained. Without inventory control procedures in place, the store or department can become overstocked or under stocked. Here an effort to make a structure for decision making to take the exact decision by using application of models which play an important role in existing life. The main objective of any organization is to maximize the profit function and minimize the cost of inventory.

Inventory models are classified as either deterministic or stochastic. Deterministic models are models where the demand for a time period is known, whereas in stochastic models the demand is a random variable having a known probability distribution. These models can also be classified by the way the inventory is reviewed, either continuously or periodic. In a continuous model, an order is placed as soon as the stock level falls below the prescribed reorder point. In a periodic review, the inventory level is checked at discrete intervals and ordering decisions are made only at these times even if inventory dips below the reorder point between review times. In recent trends businessmen have shown an increasing awareness of the need for precision in the field of inventory control of the deteriorating items. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches to zero. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as blood, fish, strawberry, alcohol, gasoline, radioactive chemical, medicine and food grains (i.e., paddy, wheat, potato, onion, etc.,) deteriorate remarkable overtime. Fundamentally, the problem is one of matching supply and demand by efficiently coordinating the production and the distribution of goods. Recent developments in information technology have equipped managers with the means to obtain better and timely information regarding, for example, demand, lead times, available assets and capacity. Technology has also enabled customers to obtain vast amounts of information about a product, such as its physical attributes and availability.

Several researchers have studied the inventory problems with deteriorating items which are available in the literature. Haiping X.U., et. al., (1990) have given an economic ordering policy model for deteriorating items with time proportional demand. Goswami .A et.al. (1991) have given an EOQ model for deteriorating item with shortages and a linear trend in demand. Shah .N .H., and Shah .Y .K., (2000), have given Literature survey on inventory model for deteriorating items. Li .R et.al (2010) has reviewed on deteriorating inventory study. Singh .T and Pattnayak .H., (2013) have focus on an EOQ model for deteriorating items with linear demand, variable deterioration and partial backlogging. Good inventory management is no longer a competitive advantage.

It is an essential capability to survive in a global market. An important aspect of good inventory management is effective use of information. Inventory control problems have attracted researchers for many years.

In this paper, a stochastic inventory periodic review system with inter demand time as exponential distribution and shortages are not allowed with constant rate of deterioration. The model contains the exponential parameter which is unknown and is estimated through MLE and Bayesian estimation under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically. The objective of the paper is to develop an optimum policy that minimizes the total average cost by using the estimates of the parameter through MLE and Bayesian estimation. The sensitivity analysis is also carried out for the model with percentage change in the parameters.

2. Assumptions and Notations

The following assumptions are used to develop mathematical model:

1. A single item is considered in the inventory system.

The inter demand time is assumed to be exponential distribution with probability density function $f(t) = \lambda e^{-\lambda t}$ at any time t ≥ 0

2. The rate of deterioration is constant.
3. Lead time is zero.
4. Shortages are not allowed.

The following are notations used in this model:

$\frac{1}{\lambda}$: Expected rate of demand

θ : The rate of deterioration per unit time.

c_1 : Ordering cost per unit time

c_2 : Holding cost per unit time

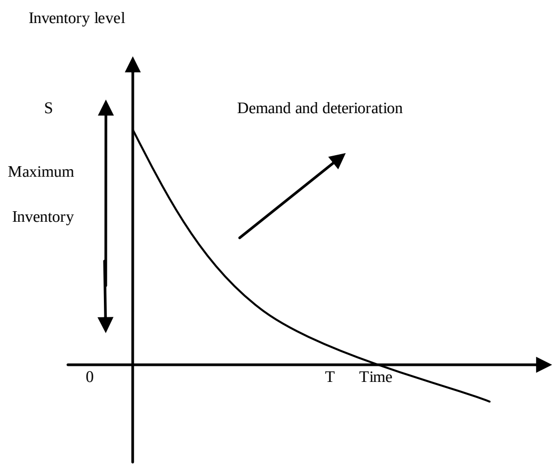
c_3 : Deterioration cost per unit time

TC : The total average cost

3. Description of the model

Let I(t) is inventory level at any instant of time 0 < t < T. The inventory level depletes due to demand and deterioration of items. The differential equation is given by

Figure 1: Schematic diagram represents the inventory level



$$\frac{dI(t)}{dt} + \theta I(t) = -\lambda e^{-\lambda t} \qquad 0 < t < T \quad ----(1)$$

With the initial conditions t = 0, I (0) = 0

The solution of (1) using the condition $I(0) = 0$

$$I(t) = \frac{\lambda e^{-\lambda t}}{\lambda - \theta} + \left[S - \frac{\lambda}{\lambda - \theta} \right] e^{-\theta t} \quad ----(2)$$

With the boundary conditions t = T, I (T) = S

The maximum inventory level is given by,

$$S = \lambda T \quad ----(3)$$

During [0,T] inventory level is $I_1 = \int_0^T I(t)\,dt$

$$I_1 = \int_0^T \frac{\lambda}{\lambda-\theta} e^{-\lambda t} + \left[S - \frac{\lambda}{\lambda-\theta} \right] e^{-\theta t} dt \quad ---- (4)$$

$$I_1 = \lambda T^2 + \frac{\lambda\theta T^2}{2\lambda} - \frac{\lambda\theta T^2}{2\theta} - \frac{\lambda\theta T^3}{2} \quad ----(5)$$

Ordering cost is given by $c_1 \lambda$ ----(6)

Inventory Holding cost is given by

$$c_2 \left[\lambda T^2 + \frac{\theta T^2}{\lambda} - \frac{\lambda T^2}{\theta} - \frac{\lambda\theta T^3}{2} \right] ----(7)$$

Deterioration cost is given by $c_3 \theta$ ----(8)

The total average cost TC is given by

TC = $\frac{1}{T}$ [Ordering cost + Inventory Holding cost + Deteriorating cost]

Totalcost=

$$\frac{1}{T} \left[\begin{array}{l} c_1 \lambda + c_2 \left[\lambda T^2 + \frac{\theta T^2}{\lambda} - \frac{\lambda T^2}{\theta} - \frac{\lambda\theta T^3}{2} \right] + \\ c_3 \theta \end{array} \right] --(9)$$

Using calculus, minimize average total cost. The optimal cycle time for the minimum average total cost is the solution of the equation

$$\frac{\partial TC}{\partial T} = 0 ----(10)$$

By differentiating equation (9) can be written as

$$-c_1 \lambda\theta + c_2 \lambda^2 \theta T^2 + c_2 T^2 \theta^2 - c_2 \lambda^2 T^2$$
$$-c_2 \lambda^2 \theta^2 T^3 - c_3 \theta^2 \lambda = 0 \quad ----(11)$$

Solving the non-linear equation (11) by using the package of 'nleqslv' in 'R - programe', the optimal cycle time T^* is obtained. The average total cost and the maximum inventory level is also obtained by substituting the optimal cycle time T^* in equations (3) and (9) respectively.

4. Parameter Estimation

4.1 Maximum Liklihood Estimation

The probability density function of the exponential distribution is given by,

$$f(x ; \lambda) = \begin{cases} \lambda e^{-\lambda x} & if \quad x \geq 0 \\ 0 & if \quad x < 0 \end{cases}$$

where λ is the parameter to be estimated. The MLE of λ given by $\frac{n}{\sum_{i=1}^{n} x_i}$

4.2 Bayesian Estimation

In this section, we consider the Baye's estimation for the parameter λ of exponential distribution assuming the conjugate of prior distribution for λ as two parameter Gamma distribution given as

$$f(\lambda/\alpha,\beta) = \begin{cases} \frac{\beta^\alpha}{\gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta x} & ,\lambda \geq 0 \\ 0 & ,\lambda < 0 \end{cases}$$

α > 0, β > 0

The likelihood function is assumed as L (λ/x) and the posterior distribution is,

$$p(\lambda/x) \propto L(\lambda/x) \; f(\alpha,\beta)$$

$$p(\lambda / X) \propto \lambda^{n+\alpha-1} \; e^{-\lambda[\beta + \sum_{i=1}^{n} x_i]}$$

This follows Gamma distribution with parameter

$$\gamma(n+\tilde{\alpha} \;, \tilde{\beta} + \sum_{i=1}^{n} x_i)$$

The mean and variance are given by

$$\text{Mean}=\frac{\alpha}{\beta} = \frac{n+\tilde{\alpha}}{\tilde{\beta}+\sum_{i=1}^{n} x_i} \quad \text{Variance}=\frac{\tilde{\alpha}}{\tilde{\beta}^2}$$

5. Numerical simulation

To compare the different estimators of the parameters λ of the exponential distribution, the risks under squared error loss of the estimates are considered. These estimators are obtained by maximum likelihood and Bayesian estimation methods under Expected risk. The MCMC procedure for Bayesian estimation is as follows

A sample of size n is then generated from the density of the exponential distribution, which is considered to be the informative sample.

(i) The MLE and Baye's estimators are calculated with $\alpha = n + \tilde{\alpha} \;, \beta = \tilde{\beta} + \sum_{i=1}^{n} x_i$

(ii) Steps (i) to (ii) are repeated N = 2000 times for different sample sizes and the risks under squared error loss of the estimates are computed by using:

ExpectedRisk

$$(\hat{\lambda}) = \frac{1}{N} \sum_{i=1}^{n} \left(\lambda_i - \hat{\lambda} \right)$$ Where, $\hat{\lambda}_i$ is the estimate at the i^{th} run

Assuming the value of λ = 0.2 the estimated value of $\hat{\lambda}$ using MLE and Baye's along with Expected risk are given in Table 1.

Table-1: Parameter Estimation of and Expected risk

n	Criteria	$\alpha = n + \tilde{\alpha} \;, \beta = \tilde{\beta} + \sum_{i=1}^{n} x_i$			
		MLE	**α=0.5, β=0**	**α=1, β=0.5**	**α=1.5, β=1**
10	Estimated value ER	0.2731 0.00008	0.2544 0.00007	0.2622 0.00006	0.2703 0.00006
25	Estimated value ER	0.2455 0.00006	0.2398 0.00005	0.2361 0.00004	0.2370 0.00005
50	Estimated value ER	0.2421 0.000008	0.2354 0.000006	0.2351 0.000006	0.2348 0.000005
75	Estimated value ER	0.2211 0.000004	0.2169 0.000004	0.2171 0.000004	0.2121 0.000002
100	Estimated value ER	0.2010 0.000002	0.1982 0.000002	0.1970 0.000002	0.1964 0.000002
125	Estimated value ER	0.1962 0.000002	0.1958 0.000002	0.1932 0.000002	0.1911 0.000002
150	Estimated value ER	0.1910 0.000002	0.1906 0.000002	0.1906 0.000002	0.1906 0.000002

It is seen that for small sample sizes the estimators under the Expected Loss function have smaller ER when choosing proper parameters α and β. But for larger sample sizes (n>50), all the estimators have approximately same ER. The obtained results are demonstrated in Table 1 and shown graphically in Figure 2 and 3. The estimated value of $\hat{\lambda} = 0.1906$.

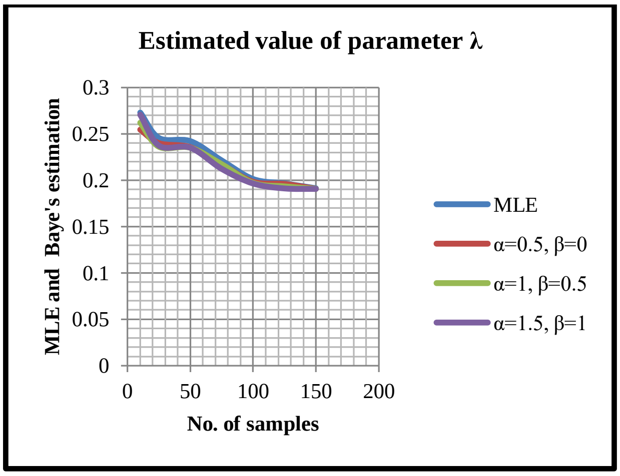
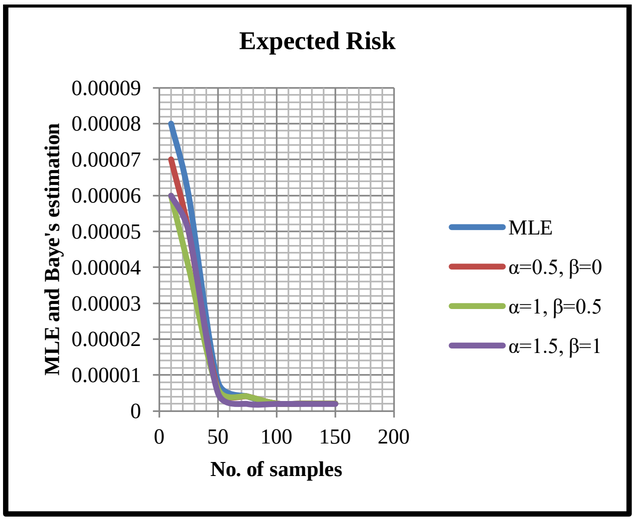


Figure :2 MLE and Bayesian Estimation

The above Figure 2 shows the estimated value of parameter through MLE and Bayesian estimation.

Figure:3 Expected risk under loss function



The above Figure 3 shows the expected risk under loss function for different α and β values through MLE and Bayesian estimation.

6. Numerical Illustration

By using the MLE and Bayesian estimation method for the parameter, it is found that the estimated value of λ = 0.1906. For finding the optimum values of the cycle time, average total cost and maximum inventory level by assuming c_1 = 5, c_2 = 1, c_3 = 0.05, and Θ = 0.01 in appropriate units.

Solving the non-linear equation (11) of the above model using R-Programme, the optimal cycle time T is obtained as T^* = 90.1072 and the optimal average total cost is obtained as TC* = 127.5401, the maximum inventory level is obtained as S^*= 17.1744

7. Sensitivity analysis

The effects of changes in the system parameter λ and θ on the optimum values of T^* and TC^* are studied in the model. The sensitivity analysis is performed by changing each of the parameter by +50%, +25%, +10%, -10%, -25%, -50%. The results are shown in Table-2.

On the basis of the results in Table 2, as the changes of parameter λ decreases, the cycle time period and total cost increases. as the changes of parameter θ decreases, the cycle time increases and total cost decreases.

Table : 2 Variation of the total cost for different cycle time period

Parameter changes in %	Changes in λ		Changes in θ	
	T^*	TC^*	T^*	TC^*
+50%	22.3369	27.5088	89.9035	137.2333
+25%	50.6097	65.1475	89.9926	132.3528
+10%	72.9044	98.8423	90.0583	129.436
-10%	109.0208	162.031	90.1602	125.5569
-25%	140.877	227.658	90.2474	122.653
-50%	203.061	386.131	90.4130	117.8184

Figure :4 Percentage change in the parameter λ and Θ for cycle time

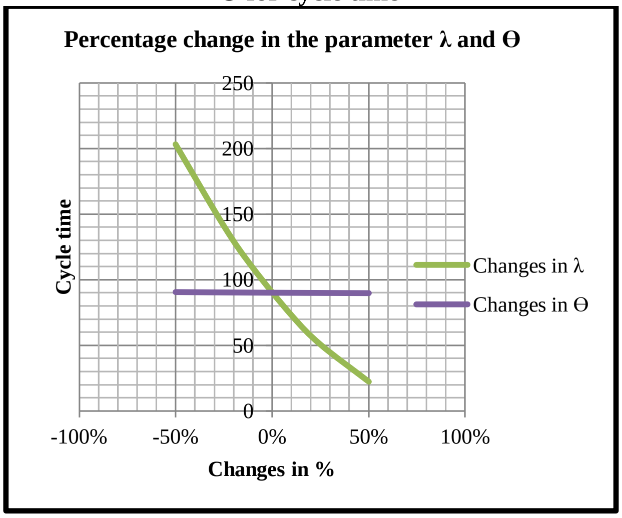


Figure 4 shows the sensitivity analysis in optimal cycle time. It is observed that highly sensitive to changes in the parameter λ and slightly sensitive to changes in the parameter θ

Figure : 5 Percentage change in the parameter λ and Θ for total cost

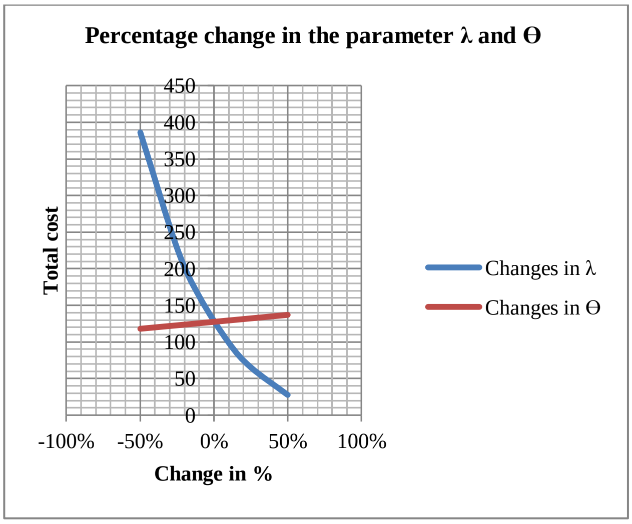


Figure 5 shows the sensitivity analysis in optimal total cost. It is observed that highly sensitive to changes in the parameter λ and slightly sensitive to changes in the parameter θ.

8. Conclusion

A stochastic Inventory periodic review system with inter demand time as exponential distribution for deteriorating items is considered. The constant rate of deterioration is also considered in this paper. The exponential parameter is estimated through MLE and Baye's under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically. The model is described for stochastic demand and the non-linear equations involved in the model is solved using R-program. The model is illustrated numerically and by using the estimates of the parameters involved in the model, the optimal time period and total cost are found. The optimal values are T^* = 90.1072 , TC^* = 127.5401 , and the maximum inventory level is S^* = 17.1744. Sensitivity analysis is also carried out with percentage change in the parameters.

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