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Chromatic Number of Sierpriński Wheel Graph

Nirbhay Singh[1] and Dr. Mohit James[2]

[1]Department of Mathematics and Statistic, SHUATS, Allahabad, (U.P.), India
[2]Assistant Professor, Department of Mathematics and Statistic, SHUATS, Allahabad, (U.P.), India

Abstract

The minimum number of colors needed to color the vertices of a graph such that no two adjacent vertices have the same color In this paper we found the chromatic number of Sierpriński Wheel Graph.
Keywords: *Chromatic Number*, *Sierpriński Graph*, *Wheel Graph*

1. Introduction

In the past some years intensely those Graphs studied whose drawings can be viewed as approximations to the famous Sierpriński triangle. The curiosity for these graphs comes from many diverse sources such as games like the Chinese ringsor the Hanoi Tower, physics, topology the study of interconnection systems, and elsewhere. In 1997 Klavžar and Milutinovic were defined generalized Sierpriński graphs, S(n,k) . S(1,k) graphis the simply complete graph Kk and S(n,3) are the graphs of problem of Hanoi Tower. HinzandSchief[1, Section 2], found the isomorphism between Hanoi graphs and a sequence of graphs obtained from approximations to The Sierpriński triangle was constructed. Hinz, Klavzar, Milutinovic and Peter, gave the definition and properties of these graphs. [2, Sections 2.3 and 5.5].

In Section 2 we described a definition of generalized Sierpriński and chromatic number and Wheel graphs.In Section3, we study the chromatic number of Sierpriński Wheel graph. In Section 4, we gave the result between chromatic number of Sierpriński Wheel graph and chromatic number of Wheel graph.

2.1 Generalized Sierpriński Graph

From Gravier,Kovšeand Aline [2011] let k be an integer and G be a finite undirected graph on a vertex set $\{1,2,....k\}$. In the following, vertices of graphs will be identified with words on integers. We denote by $\{1,2,....k\}^n$ the set of words of size n on alphabet $\{1,2,....k\}$ The letters of a word u of $\{1,2,....k\}^n$ are denoted by $u = u_1u_2u_3...u_n$. The concatenation of two words u and v is denoted by uv.

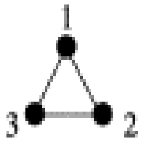
The generalized Sierpriński graph of G of dimension n denoted by $S(n,G)$ is the graph with vertex set $\{1,2,....k\}^n$ and edge set defined by: $\{u,v\}$ is an edge if and only if there exists $i \in \{1,2....,n\}$ such that:

i.) $u_i = v_j$ if $j < i$,

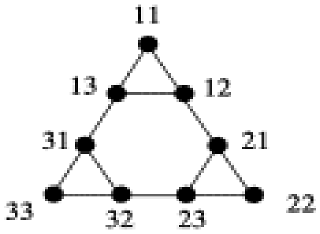
ii.) $u_i \neq v_i$ and $(u_i v_i) \in E(G)$,

iii.) $u_i = v_i$ and $v_j = u_i$ if $j > i$.

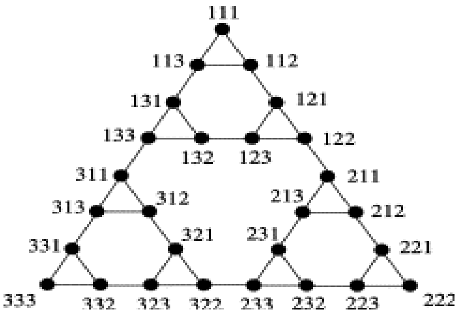
In other words, if $\{u,v\}$ is an edge of $S(n,G)$ there is an edge $\{x, y\}$ of G and a word such that $u = wxy....y$ and $v = wyx...x$. We says that edge $\{u,v\}$ is using edge $\{x, y\}$ of G .Graphs $S(n,G)$ can be constructed recursively from G with the following process: $S(1,G)$ is isomorphic to G . To construct $S(n,G)$ for $n > 1$, copy k times $S(n-1,G)$ and add to labels of vertices in copy x of $S(n-1,G)$ the letter x at the beginning. Then for any edge $\{x, y\}$ of G , add an edge between vertex $xy....y$ and vertex $yx...x$. For any word u of length d , with $1 \leq d < n$, the subgraph of $S(n,G)$ induced by vertices with label beginning by u , is isomorphic to $S(n-d,G)$. For a vertex x of G , we call extreme vertex x of $S(n,G)$ the vertex with label $x.....x$.



$S(1, C_3)$



$S(2, C_3)$



$S(3, C_3)$

Fig.2.1

2.2 Chromatic Number

The minimum number of colors needed to color the vertices of a graph such that no two adjacent vertices have the same color.This is denoted by the

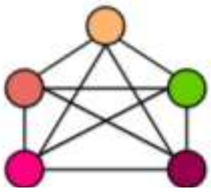


Fig.2.2

The chromatic number of above graph is 5

2.3 Wheel Graph

A wheel graph W_n contain an additional vertex to the cycle, W_n forn ≥ 3, and connect this new vertex to each of the n vertices in C_n ,by new edges. The wheel W_3 , W_4, W_5 are displayed below.

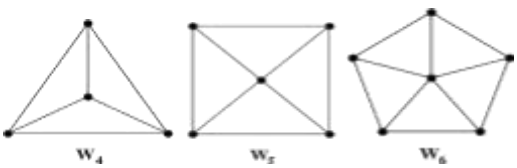
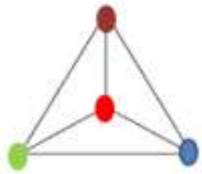
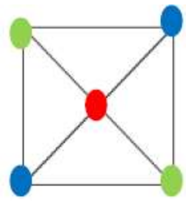


Fig.2.3 Wheel Graph

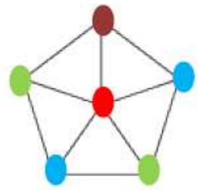
2.4 CHROMATIC NUMBER IN WHEEL GRAPH



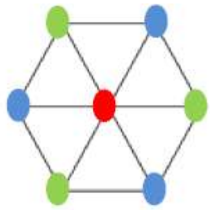
$\chi(W_3) = 4$



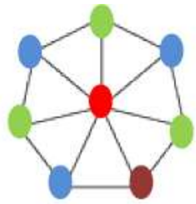
$\chi(W_4) = 3$



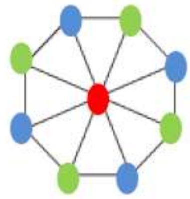
$\chi(W_5) = 4$



$\chi(W_6) = 3$

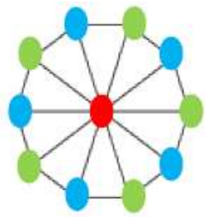


$\chi(W_7) = 4$



$\chi(W_8) = 3$

$\chi(W_9) = 4$

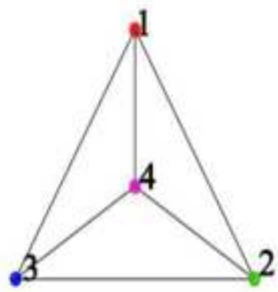


$\chi(W_{10}) = 3$

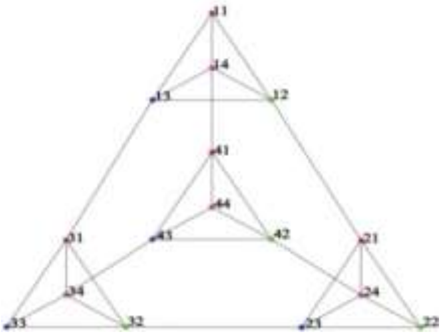
$$\chi(W_n) = \begin{cases} 3 \text{if } n \text{is even} \\ \\ 4 \text{if } n \text{is odd} \end{cases}$$

3. Results and Discussion

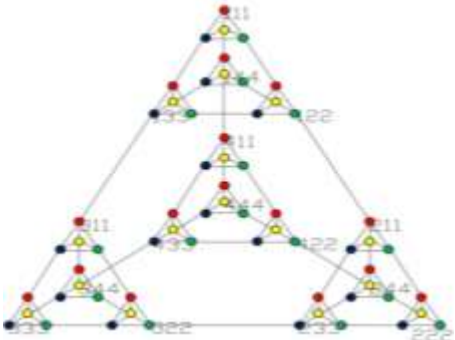
CHROMATIC NUMBER IN SIERPINSKI WHEEL GRAPH



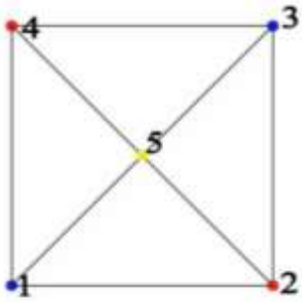
$\chi\left(S(1, W_3)\right) = 4$



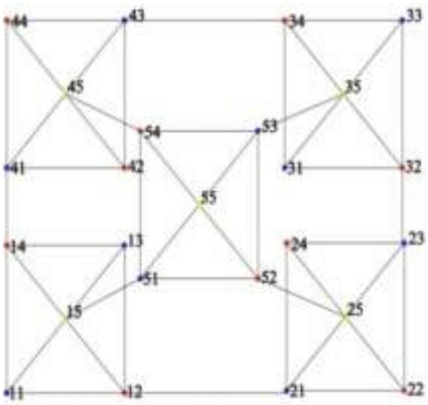
$\chi(S(2,W_3)) = 4$



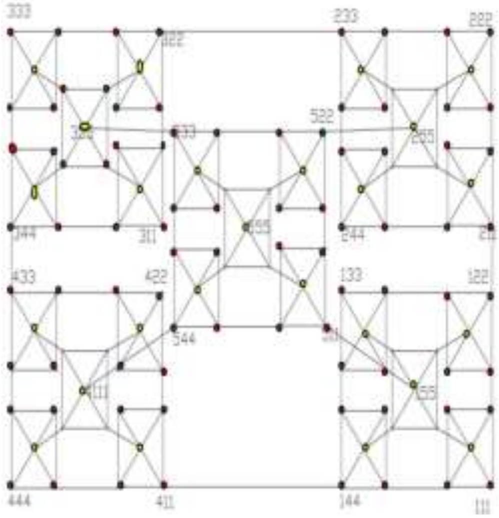
$\chi(S(3,W_3)) = 4$



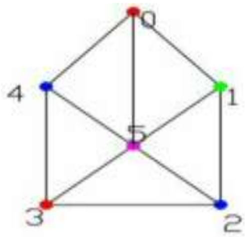
$\chi(S(1,W_4)) = 3$



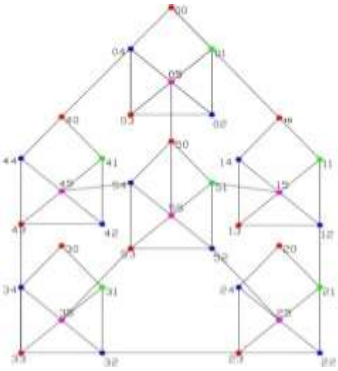
$\chi(S(2,W_4)) = 3$



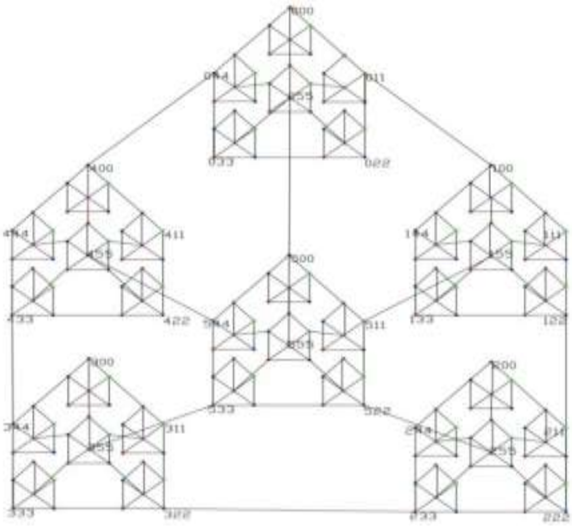
$\chi(S(3,W_4)) = 3$



$\chi(S(1,W_5)) = 4$



$\chi(S(2,W_5)) = 4$



$\chi(S(3,W_5))=4$

S N	$\chi(S(k,W_n))$		
	$S(k,W_3)$	$S(k,W_4)$	$S(k,W_5)$
1	$\chi(S(1,W_3))=$	$\chi(S(1,W_4))=$	$\chi(S(1,W_5))=$
2	$\chi(S(2,W_3))=$	$\chi(S(2,W_4))=$	$\chi(S(2,W_5))=$
3	$\chi(S(3,W_3))=$	$\chi(S(3,W_4))=$	$\chi(S(3,W_5))=$
	Ν Ν	Ν Ν	Ν Ν
k	$\chi(S(k,W_3))=$	$\chi(S(k,W_4))=$	$\chi(S(k,W_5))=$

4. Conclusions

From above table we conclude that

$$\chi(S(1,W_3))=\chi(S(2,W_3))=\chi(S(3,W_3))=\Lambda=\chi(S(k,W_3))=4=\chi(W_3)$$
$$\chi(S(1,W_4))=\chi(S(2,W_4))=\chi(S(3,W_4))=\Lambda=\chi(S(k,W_4))=3=\chi(W_4)$$
$$\chi(S(1,W_5))=\chi(S(2,W_5))=\chi(S(3,W_5))=\Lambda=\chi(S(k,W_5))=4=\chi(W_5)$$

Hence,
$$\chi(S(k,W_n))=\begin{cases}3 \text{ if } n \text{ is even}\\ 4 \text{ if } n \text{ is odd}\end{cases}$$

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