Investigation on MHD radiative flow of Powell-Eyring Nanofluid through non-linear stretching sheet

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Abstract

In this Article, We have considered the Powell-Eyring nanofluid over a nonlinear stretching sheet with variable thickness. Powell-Eyring nanofluid MHD boundary layer system has been modelled to form continuity equation, momentum equation, energy equation and concentration equation. The governing boundary layer Powell-Eyring flow equations can be transformed into ordinary differential equations by similarity transformations. Then reduced ODE equations under the boundary condition are solved by implicit finite difference scheme. In order to study the physical characteristics of non-dimensional parameters such as Magnetic parameter (M), thickness parameter (α), fluid parameters (λ) and (N), Thermophoresis parameter (Nt), Brownian motion parameter (Nb), Radiation parameter (R), Prandtl number (Pr), Lewis number (Le), nonlinear stretching parameter (n) on the velocity, temperature and concentration profiles of Powell-Eyring nanofluid analyzed and discussed with the help of graphical results.

Keywords - MHD, Nanoparticle, Powell-Eyring fluid, stretching sheet, Variable thickness.

1.Introduction

MHD boundary layer flow of Powell-Eyring nanofluid in a stretching sheet have been investigated by many researchers during the past few decades owing their numerous applications in industrial manufacturing, for example, drawing of plastic films, artificial fibres, wire drawing, glass fibre, and so on. With attention to the many more application of nanofluids, the researchers have been regard to improve heat transfer using nanofluids. Patel and Timol (2009) investigated themethod of asymptotic boundary conditions for Powell-Eyring fluid. Nadeem et al.(2013) examined the boundary layer flow and heat transfer of oldroyd-B nanofluid towards a stretching sheet by the solution of numerical study. Anwar et al. (2013) explored the MHD stagnation-point flow of a nanofluid over a porous sheet on chemical reaction and uniform heat generation or absorption effects. Jalil et al. (2013) discussed the heat transfer flow of Powell-Eyring fluid over a moving surface in a parallel free stream. Ganesh et al.(2014) analysed the magnetic field effects on free convective flow of a nanofluid over a semi-infinite stretching sheet. Araa et al.(2014) studied theEyring-Powell fluid over an exponentially shrinking sheet on boundary layer flow of Radiative effect. Numerical and analytical solution for the MHD Eyring-Powell nanofluid flow of mixed convection boundary layer flow and axisymmetric flow of nanofluid due to non-linearly stretching sheet can be found in (2015). Das (2015) studied the Nanofluid flow over a non-linear permeable stretching sheet with partial slip. Govindaraju et al. (2015) examined the Entropy generation analysis of MHD flow of a nanofluid over a stretching sheet. Ali et al.(2015) explored the heat and mass transfer flow over an inclined stretching sheet with viscous flux dissipation and constant heat flux in presence of magnetic field. Raju et al.(2015) presented the Radiation, inclined magnetic field and cross-diffusion effects on flow over a stretching surface. Gireesha et al.(2015) analysed the three-dimensional MHD Eyring- Powell fluid flow of heat and mass transfer forradiating effectsuspended nanoparticles over a stretching sheet. Akbar et al.(2015) examined the Eyring-Powell fluid flow on magnetic field effect towards

2. Mathematical Formulation

Let us consider the Powell-Eyring nanofluid over a nonlinear stretching sheet with variable thickness. Powell-Eyring nanofluid MHD boundary layer system has been modeled to form continuity equation, momentum equation, energy equation and concentration equation. A Cartesian coordinate system is chosen that x-axis is considered along the direction of the sheet and y-axis is considered along normal to it. Assume nonlinear stretching surface velocity is

$$U_w = U_0(x + b)^n$$

where $U_0$ = initial velocity and $b$ = constant, and nonlinear stretching surface variable is at $y = A(x + b)^{\frac{1-n}{2}}$. If $n=1$, stretching surface has the same thickness. A non-uniform magnetic field is as $B(x) = B_0(x + b)^{\frac{1-n}{2}}$. Induced magnetic and electric field are not taken into account.

The governing equations are given as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \nu + \frac{1}{\rho_0 df} \right) \frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_0 f d^2} \left( \frac{\partial w}{\partial y} \right)^2 \frac{\partial^3 u}{\partial y^3} - \frac{\sigma B^2(x)}{\rho}$$  \hspace{1cm} (2)

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_e \frac{\partial^2 T}{\partial y^2} + \left( D_f \frac{\partial C}{\partial y} \right)^2 \frac{\partial^2 C}{\partial y^2} + D_p \frac{\partial^2 \left( \frac{\partial C}{\partial y} \right)}{\partial y^2} + \frac{1}{\rho_e c_p} \frac{\partial}{\partial y} \left( \rho_e c_p (T - T_e) \right)$$  \hspace{1cm} (3)
\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2} - K \rho (C - C_y) 
\] (4)

By Rosseland approximation, the Radiative flux is follows

\[
q_r = -\frac{4\sigma T^4}{3k} \left( \frac{\partial T}{\partial y} \right) 
\] (5)

By assuming very small temperature differences within the flow, \( T^4 \) expressed in terms of linear combination of temperature and expand \( T^4 \) about \( T_e \) in Taylor’s series and neglecting higher order terms yields:

\[
T^4 \approx 4T_e^3T - 3T_e^4 
\] (6)

By (6),

\[
\frac{\partial q_r}{\partial y} = -16\sigma T^3 \frac{\partial T}{\partial y} 
\] (7)

Using the equations (5) and (6), the equation (3) can be written as follows

\[
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \left( 1 + \frac{D_T}{D_B} \right) \frac{\partial T}{\partial y} - \frac{Q}{\rho c} = 0 
\]

Follow with the boundary conditions:

\[
\begin{align*}
&u = U_e(x), v = 0, y = 0, T = T_e, D_B \frac{\partial C}{\partial y} + D_T \frac{\partial T}{\partial y} = 0 \text{ at } y = A(x + b) \\
&y \to 0, v \to 0, T \to T_e, C \to C_e \text{ as } y \to \infty 
\end{align*}
\] (8)

Where \( k \) = thermal conductivity, \( \alpha_{eff} = \frac{k}{(\rho c)_f} \) the thermal diffusivity of liquid, \( \mu = \) Dynamic viscosity, \( \nu = \frac{\mu}{\rho} \) =kinematic viscosity, \( \rho = \) the fluid density, \( \tau \) =Ratio of the heat capacity of fluid of the nanoparticles material to the effective heat capacity of the base fluid.

\( D_B \) = Brownian diffusion coefficient, \( D_T \) = Thermophoretic Diffusion

The corresponding transformation is as follows:

\[
\begin{align*}
&u = U_e(x + b)F(q), v = \frac{n+1}{2}U_e(x + b)^{n+1} \left[ F'(q) + qF''(q) \frac{n+1}{2} \right] \\
y = A(x + b)^{n+1} \left[ \frac{n+1}{2} \left[ U_e(x + b)^{n+1} \right] \right]
\end{align*}
\] (9)

Using (7), the equations (2), (6), becomes

\[
(1 + N)F' + FF' - N \left( \frac{n+1}{2} \right) F'^2 - \left( \frac{2n}{n+1} \right) M^2F = 0 
\] (10)

\[
\frac{1}{Pr_{eff}}\Theta + F\Theta + N \beta \phi + q\phi + Q\phi = 0 
\] (11)

\[
\Phi' + Pr Le F\Phi' + \left( \frac{N}{N_b} \right) \phi' - K \phi' = 0 
\] (12)

By using (7), reduced boundary conditions are

\[
F'(\eta) = \alpha \left( \frac{1-n}{1+n} \right), F(\eta) = 0, \Theta(\eta) = 0, \phi(\eta) = 0, \left( \frac{N}{N_b} \right) \theta(\eta) = 0 
\] (13)

\[
F'(\infty) \to 0, \Theta(\infty) \to 0, \Phi(\infty) \to 0 
\] (14)

Where \( \alpha = A \left( \frac{n+1}{2} \right) \) wall thickness parameter= the plate surface. Taking

\[
F(\eta) = f(\eta - \alpha) = f(\zeta), \Theta(\eta) = \theta(\eta - \alpha) = \theta(\zeta), \phi(\eta) = \phi(\eta - \alpha) = \phi(\zeta). 
\]

Equations (12) - (16) are as follows:

\[
(1 + N)F' + FF' - N \left( \frac{n+1}{2} \right) F'^2 - \left( \frac{2n}{n+1} \right) M^2F = 0 
\] (15)

\[
\frac{1}{Pr_{eff}}\Theta + F\Theta + N \beta \phi + q\phi + Q\phi = 0 
\] (16)

The dimensionless parameters are

\[
Pr = \frac{\nu}{\alpha_f}, M = \frac{\sigma B^2}{\mu U_0}, N = 1 \beta \mu, \lambda = \frac{U_0}{4\nu \gamma}, \quad \text{Le} = \frac{\alpha_f N_f}{D_b}, \quad \text{Le} = \frac{\alpha_f N_f}{D_b} \quad \text{Le} = \frac{\alpha_f N_f}{D_b} 
\]

Where \( Pr = \) Prandtl number, \( M = \)Magnetic parameter, \( N = \) fluid parameters, \( Le = \) Lewis number, \( N_b = \) Brownian motion parameter, \( N_f = \) thermophoresis parameter

3. Results and Discussions

The governing boundary layer Powell-Eyring flow equations can be transformed into ordinary differential equations by similarity transformations. Then the equations (13) to (15) under the boundary condition (16) solved by implicit finite difference scheme. In order to study the physical characteristics of non-dimensional parameters such as Magnetic parameter (M), thickness parameter (\( \alpha_f \)),fluid parameters(\( \lambda \)) and (N), Thermophoresis parameter (Nt), Brownian motion parameter (Nb), Radiation parameter (R),...
Prandtl number (Pr), Lewis number (Le), nonlinear stretching parameter (n) on the velocity, temperature and concentration profiles of Powell-Eyring nanofluid analyzed and discussed with the help of graphical results.

Figures 1-15 investigated the novel characteristics of non-dimensional parameters on velocity profile, concentration profile and temperature profile respectively. Figure 1 demonstrates the behaviour of velocity profile for various values of Magnetic parameter M. Increasing values of magnetic parameter M decrease the velocity profile. Figure 2 explores the effect of thickness parameter α on velocity profile. Enhancing the values of thickness parameter reduces the velocity profile. Figure 3 and 4 show the characteristic of fluid parameters λ and N. Larger values of fluid parameter λ diminishes the velocity profile and velocity profile enhances by increasing values of fluid parameter N. Figure 5 examined the characteristics of non-linear stretching sheet parameter n. Increasing values of n increase the velocity profile.

Figures 6-11 display the dimensionless temperature field for different non-dimensional parameters Magnetic parameter M, thickness parameter α, Thermophoresis parameter Nt, Brownian motion parameter Nb, Radiation parameter R, Prandtl number Pr. Figure 6 depicts that temperature profile increases by increasing values of magnetic parameter. It can be noticed that increasing values of thickness parameter α diminishes the temperature profile in figure 7. Also figures 8 and 9 explain the variations of Thermophoresis parameter Nt and Brownian motion parameter Nb. The influence of Thermophoresis parameter Nt on temperature distribution increases when it rises and vice versa for the Brownian motion parameter Nb. In Figure 10, thermal boundary layer diminishes when Prandtl number increases. Increasing values of radiation parameter R increase the temperature distribution as shown in the figure 11.

Figures 12-16 demonstrates the behaviour of Concentration profile. Figure 12 explains that increment in magnetic parameter M increases the concentration profile. Figure 13 explores the influence of increasing Brownian motion parameter Nb on concentration profile. The concentration profile reduces with an increasing value of Brownian motion parameter Nb. An enhancing value of Thermophoresis parameter Nt increase the concentration profile in figure 14. Figure 15-16 display the concentration profile for various values of Prandtl number and Lewis number. Both non-dimensional parameters decrease the concentration distribution.
4. Conclusion

The impact of thermal radiation and nth order chemical reaction due to MHD Radiative Powell-Eyring fluid flow through non-linear stretching sheet with variable thickness have investigated in this Article. We conclude the main outcome of this investigation as follows:

- Increasing values of magnetic parameter $M$ decrease the velocity profile.
- Enhancing the values of thickness parameter reduces the velocity profile.
- Larger values of fluid parameter $\lambda$ diminish the velocity profile and velocity profile enhances by increasing values of fluid parameter $N$.
- Increasing values of $n$ increase the velocity profile.
- It can be noticed that increasing values of thickness parameter $\alpha$ diminishes the temperature profile.
- The influence of Thermophoresis parameter $N_t$ on temperature distribution increases when it rises and vice versa for the Brownian motion parameter $N_b$.
- Increasing values of radiation parameter $R$ increase the temperature distribution.
- Increment in magnetic parameter $M$ increases the concentration profile.
- The influence of increasing Brownian motion parameter $N_b$ on concentration profile.
- The concentration profile reduces with an increasing value of Brownian motion parameter $N_b$.
- An enhancing value of Thermophoresis parameter $N_t$ increase the concentration profile.
- Both non-dimensional parameters Prandtl number and Lewis number decrease the concentration distribution.

References


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