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On the Trace of Symmetric bi-Semi derivations on Near-Rings

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Abstract

In this paper N is a 3-prime left near-ring having Z as a multiplicative centre. The centralizer of t in N is denoted by $C(t)$for $t \in N$. In this paper we mainly study the properties of symmetric bi-semi derivations on near-ring N.

Keywords: 2-*torsion free near-ring, prime near-ring, symmetric bi-semi derivation*.

1. Introduction

Symmetric bi-derivations introduced by Maksa .The main aim of this paper is to study some properties of symmetric bi-semi derivations of near-rings. Entire in this paper N will denote zero symmetric left near-ring. A near-ring N will be 3-prime if $tNu = 0 \Rightarrow t = 0$ (or) $u = 0$ .Here Z represents the multiplicative centre of N. Here multiplicative commutator $[t,u]$ and additive commutator (t,u) will be denoted by $tu - ut$ and $t + u - t - u$ respectively $\forall\, t,u$ belongs to N. A function D: $N \times N \to N$ is called symmetric if $D(t,u) = D(u,t)$ $\forall\, t,u$ belongs to N. A function d: $N \to N$ distinct by $d(t) = D(t,t)$ is said to be trace of D where D: $N \times N \to N$ is a symmetric function. The relation

$d(t+u) = d(t) + 2D(t,u) + d(u)$

$\forall\, t,u$ belongs to N is satisfied by the trace of D. A symmetric bi-additive function D: $N \times N \to N$ is said to be a symmetric bi-derivation if $D(tu,v) = D(t,v)u + tD(u,v)$ $\forall\, t,u,v$ belongs to N.

2. Key Results

Lemma 2.1: Suppose N is a 3-prime near-ring.

(i) Suppose $v \in Z - \{0\}$, $\Rightarrow v$ is not a zero divisor.

(ii) Suppose $Z - \{0\}$ contains an element v for which $v + v \in Z$, $\Rightarrow$ N is abelian with respect to addition.

Proof: [G.Maksa ,1987]

Lemma 2.2: Suppose N is a 2-torsion free near-ring, D is a symmetric bi-additive function of N and d be the trace of D. If

$$d(t) = 0\ \forall\, t \in N, \Rightarrow D = 0.$$

Proof: $\forall\, t,u$ belongs to N

$d(t+u) = d(t) + 2D(t,u) + d(u)$, also from hypothesis $2D(t,u) = 0$ $\forall\, t,u$ belongs to N, because N is a 2-torsion free, then we have $D = 0$.

Lemma2. 3: Suppose N is a 2-torsion free 3-prime near-ring, D be a symmetric bi-semi derivation of N and d be the trace of D. If $td(u) = 0, \forall\, t,u$ belongs to $N \Rightarrow t = 0\ (or)\ D = 0$.

Proof: $\forall\, u, v$ belongs to N, $d(u+v) = d(u) + 2D(u,v) + d(v)$, also from given hypothesis $\forall\, t, u, v$ belongs to N

Since N is 2-torsion free

We get $tD(u,v) = 0$, Eq.(1)

Replace uw in the place of u in the above equation then we have

$$0 = tD(uw, v)$$

$$0 = t(D(u,v)g(w) + uD(w,v))$$

$$0 = tD(u,v)g(w) + tuD(w,v)$$

Using equation (1) we have $tuD(w,v) = 0\ \forall\, t, u, v, w$ belongs to N

By the 3-primeness of N we have $t = 0\ (or) D(w,v) = 0$

Therefore $t = 0\ (or)\ D = 0$

Lemma 2.4: Suppose N is a near-ring, D be a symmetric bi-additive function of N then the following are true.

(i) $D(tu, v) = D(t,v)g(u) + tD(u,v)$ $\forall\, t, u, v$ belongs to N

(ii) $D(tu, v) = tD(u,v) + D(t,v)\, g(u)$ $\forall\, t, u, v$ belongs to N .

Proof: (i) $\Longrightarrow (ii)$

Let $D(tu, v) = D(t,v)g(u) + tD(u,v)$ $\forall\, t, u, v$ belongs to N

We have $D(t(u+u), v) = D(t,v)g(u + u+tD(u+u,v)$

$= D(t,v)g(u) + D(t,v)g(u) + tD(u,v) + tD(u,v)$ Eq.(2)

On the other side we have

$D(t(u+u), v) = D(tu+tu, v) = D(tu,v) + D(tu,v)$

$= D(t,v)g(u) + tD(u,v) + D(t,v)g(u) + tD(u,v)$ Eq. (3)

From equations (2) and (3) we have

$$D(t,v)g(u) + tD(u,v) = tD(u,v) + D(t,v)g(u)$$

$D(tu, v) = tD(u,v) + D(t,v)g(u)$ $\forall\, t, u, v$ belongs to N .

(ii) $\Longrightarrow (i)$ can be proved in same manner.

Lemma 2.5: Suppose N is a near-ring, D be a symmetric bi-additive function of N and d be the trace of D, then $\forall\, t, u, v, w$ belongs to N we have

(i) $\big(D(t,v)g(u) + tD(u,v)\big)w = D(t,v)g(u)w + tD(u,v)w$

(ii) $\big(d(t)g(u) + tD(t,u)\big)w = d(t)g(u)w + tD(t,u)w$

(iii) $(tD(u,v) + D(t,v)g(u))w = tD(u,v)w + D(t,v)g(u)w$

(iv) $(tD(t,u) + d(t)g(u))w = tD(t,u)w + d(t)g(u)w$

Proof: (i) suppose $D(tu, v) = D(t,v)g(u) + tD(u,v)$ $\forall\, t, u, v$ belongs to N

We have $D\big((tu)w, v\big) = D(tu,v)g(w) + tuD(w,v)$

$= \big(D(t,v)g(u) + tD(u,v)\big)g(w) + tuD(w,v)$ Eq. (4)

And also we have

$$D(t(uw),v) = D(t,v)g(uw) + tD(uw,v)$$

$= D(t,v)g(u)g(w) + t(D(u,v)g(w) + uD(w,v))$

$= D(t,v)g(u)g(w) + tD(u,v)g(w) + tuD(w,v)$ Eq. (5)

Comparing equations (4) and (5) we get

$\big(D(t,v)g(u) + tD(u,v)\big)g(w) = D(t,v)g(u)g(w) + tD(u,v)g(w)$

Since g is onto we have

$\big(D(t,v)g(u) + tD(u,v)\big)w = D(t,v)g(u)w + tD(u,v)w$
$\forall\ t,u,v,w$ belongs to N

(ii) Substitute t in the place of v in (i)

$$\big(D(t,t)g(u) + tD(u,t)\big)w = D(t,t)g(u)w + tD(u,t)w$$

Since D is symmetric,

$$\big(d(t)g(u) + tD(t,u)\big) = d(t)g(u)w + tD(t,u)w$$

(iii) Suppose D$(tu,v) = tD(u,v) + D(t,v)\ g(u)\ \forall\ t,u,v$ belongs to N

We have D$\big((tu)w,v\big) = tuD(w,v) + D(tu,v)\ g(w)$

$= tuD(w,v) + (tD(u,v) + D(t,v)g(u))g(w)$ Eq.(6)

And also we have D$(t(uw),v) = tD(uw,v) + D(t,v)g(uw) = t(uD(w,v) + D(u,v)g(w)) + D(t,v)g(u)g(w)$

$= tuD(w,v) + tD(u,v)g(w) + D(t,v)g(u)g(w)$ Eq.(7)

From equations (6) and (7) we have

$(tD(u,v) + D(t,v)g(u))g(w) = tD(u,v)g(w) + D(t,v)g(u)g(w)$

Since g is onto
$(tD(u,v) + D(t,v)g(u))w = tD(u,v)w + D(t,v)g(u)w$

(iv) Substitute t in the place of v in the above equation then we have

$(tD(u,t) + D(t,t)g(u))w = tD(u,t)w + D(t,t)g(u)w$

$(tD(u,t) + \mathrm{d(t)}g(u))w = tD(u,t)w +$ d(t) $g(u)w$

Since D is symmetric, we have
$(tD(t,u) + d(t)g(u))w = tD(t,u)w + d(t)g(u)w$

Theorem 2.6: Suppose N is a 3-prime near-ring, D be a nonzero symmetric bi-semi derivation of N and d be the trace of D. If $d(N) \subseteq Z$ and N is a 2-torsion free $\Rightarrow$ N is a commutative ring.

Proof: Suppose t is a variable and c is an arbitrary constant, we have

$$d(t+c) = d(t) + 2D(t,c) + d(c) \in Z.$$

Since$d(t) \in Z - \{0\}$, N is a 2-torsion free, we have $D(t,c) \in Z - \{0\}$ Eq.(8)

Replace ct in the place of t in equation (8) then we have

$$D(ct,c) \in Z - \{0\}$$

$$D(c,c)g(t) + c\,D(t,c) \in Z - \{0\}$$

Then we have $c\,D(t,c) \in Z - \{0\}$

From equation (8) we have $c \in Z - \{0\}$

Since $d(c + c) = 0, \forall$ c

From lemma (2.1) (ii) we have N is abelian with respect to addition.

In above case, suppose 0 is the only constant,

We have $d(t) \in Z, \forall$ $t \in N$,then$\forall$ t, u, v belongs to N we have $D(t, u) \in Z$ Eq.(9)

Let us consider e is not a zero divisor for $e \in N$, and suppose $t, u \in N$, we have

$$D(e(t + e), u) = D(e, u)g(t + e) + eD(t + e, u)$$

$$= D(e, u)g(t) + D(e, u)g(e) + eD(t, u) + eD(e, u) \qquad \text{Eq.(10)}$$

And also $D(e(t + e), u) = D(et +$ *e2,u=Det,u+D(e2,u)*
$= D(e, u)g(t) + eD(t, u) + D(e, u)g(e) + eD(e, u)$ Eq.(11)

From equation (10) and (11) we have $D(e, u)g(e) + eD(t, u) = eD(t, u) + D(e, u)g(e)$

From Equation (9) and using the above equation we get $\forall$ t, u belongs to N, since g is onto

$$0 = e(D(e, u) + D(t, u) - D(t, u) - D(e, u))$$

$$0 = eD(e + t - e - t, u)$$

Since e is not zero divisor we have $\forall$ t, u belongs to N
$0 = D((e, t), u)$ Eq.(12)

Substitute (e, t) in the place of u in equation (12) then

$$0 = D((e, t), (e, t))$$

$0 = d((e, t))$ $\forall$ $t \in N$ and then (e, t) is a constant

That is $0 = (e, t)$ $\forall$ t, u belongs to N

$\therefore e \in C(N)$ which is the centre of (N, +).

Suppose t is a non-zero element of N and by lemma (2.2), from the hypothesis

$d(t)$ is not a zero divisor,we have $d(t) \in C(N)$ $\forall$ $0 \neq t \in N$.

Since $d(t + v) = d(t) + 2D(t, v) + d(v) \in C(N)$,where $0 \neq v \in N$,

We have $D(t, v) \in C(N)$, $\forall$ $0 \neq t \in N$ and $0 \neq v \in N$

Then $0 = D(t, v) + D(u, v) - D(t, v) - D(u, v) = D((t, u), v)$ and then we have

$(t, u) = 0$ $\forall$ t, u belongs to N

Therefore N is abelian with respect to addition.

Replace tw in the place of t in equation (9) then we have

$D(tw, u) \in Z$ then

$v\big(D(t, u)g(w) + tD(w, u)\big) = \big(D(t, u)g(w) + tD(w, u)\big)v$
$\forall$ t, u, v, w belongs to N.

By lemma 2.3 and 2.4 and from equation (9) and also from g is onto we have $D(t, u)[v, w]$

$= D(w, u)[v, t]$ Eq.(13)
Substitute $d(w)$ in the place of w in equation (13)

by the given hypothesis we get $\forall$ t, u, v, w belongs to N.

$D(d(w),u)[v,t] = 0$ Eq.(14)

Replace un for u, where $n \in N$ in equation (14) and also from equation (14) we have$\forall\ t,u,v,w$ belongs to N.

$D(d(w),u)N[v,t] = 0$ Eq.(15)

Now let us assume that N is not commutative,

By the 3-primeness of N and from equation (14) we have $\forall\ u,w$ belongs to N

$D(d(w),u) = 0$ Eq.(16)

Replace $t+w$ for w in equation (16), since N is a 2-torsion free and from (16) we have$\forall\ t,u,v,w$ belongs to N.

$D(D(t,w),u) = 0$ Eq.(17)

Replace tw in the place of t in equation (17) and from equations (16), (17) we have $\forall\ t,u,v,w$ belongs to N.
$D(t,w)D(w,u) +$

$D(t,u)d(w) = 0$ Eq.(18)

Now replace t for w in equation (18) and since N is a 2-torsion free, we have $\forall\ t,u$ belongs to N.

$D(t,u)d(t) = 0$ Eq.(19)
Substituting tu instead of u in equation (19) we have for all $t,u \in N$ and using equation (19)

$$d(t)ud(t) = 0$$

By the 3-primeness of N we have $D = 0$ by lemma 2.2

Which is contradiction to $D \neq 0$

Therefore N is commutative.

Theorem 2.7: Suppose N is a 3-prime near ring, D be a non zero symmetric bi-semi derivation of N and d be the trace of D. If $d(u), d(u)+d(u) \in C(D(t,u))$ $\forall\ t,u,v,w$ belongs to N and N is a 2-torsion free then N is a commutative ring.

Proof: suppose e and $e+e$ commutate element wise with $D(t,v)$ $\forall\ t,u,v,w$ belongs to N.then

$$\big(D(t,v)+D(u,v)\big)(e+e) = \big(D(t,v)+D(t,v) + D(u,v)+D(u,v)\big)e$$

And also we have

$$\big(D(t,v)+D(u,v)\big)(e+e) = \big(D(t,v)+D(u,v)+D(t,v)+D(u,v)\big)e$$

From the above two equations we get $\forall\ t,u$ belongs to N

$D\big((t,u),v\big)e = 0$ Eq.(20)

Let us take $e = d(n)$ in equation (20) we have
$D\big((t,u),v\big)d(n) = 0\ \ \forall\ t,u,v$ belongs to N

Then by lemma 2.2 we have
$D\big((t,u),v\big) = 0\ \forall\ t,u,v$ belongs to N

Since $v(t,u)$ is an additive commutator $\forall\ v$ belongs to N then

$$0 = D(v(t,u),v) = d(v)(t,u)$$

From lemma 2.2 we have$(t,u) = 0$,then

N is abelian with respect to addition.

From $d(u) \in C(D(t,u))$ $\forall\ t,u,v$ belongs to N we have

$[D(t,v),d(u)] = 0$ Eq.(21)

Now substitute ve in the place of v in equation (21) and also from equation (21) we have $\forall\ t, u, v, e$ belongs to N

$$0 = D(t, ve)d(u) - d(u)D(t, ve)$$

$$0 = (D(t, v)g(e) + vD(t, e))d(u) - d(u)(D(t, v)g(e) + vD(t, e))$$

And from equation (21) and by lemma 2.5(1), N is abelian with respect to addition.

We have $\forall\ t, u, v, e$ belongs to N

$$0 = D(t, v)g(e)d(u) - D(t, v)d(u)g(e) + vd(u)D(t, e) - d(u)vD(t, e)$$

Substitute $d(v)$ for v in the above equation and using the given hypothesis, we have $\forall\ t, u, v, e$ belongs to N

$$D(t, d(v))[e, d(u)] = 0 \qquad \text{Eq.(22)}$$

Substitute tn instead of t in equation (22) and also from equation (22) we have $\forall\ t, u, v, e$ belongs to N

$$D(t, d(v))N[e, d(u)] = 0$$

By the 3-primeness of N, we have $[e, d(u)] = 0\ \forall\ u, e$ belongs to N (or) $D(t, d(v)) = 0\ \forall\ t, v$ belongs to N .

Suppose $d(N) \subseteq Z, \Rightarrow$ N is commutative ring by theorem 2.6

Suppose $D(t, d(v)) = 0$ $\forall\ t, v$ belongs to N, then the procedure used in the proof of theorem 2.6 [from equation (16)], gives that $D = 0$.

Which is contradiction to $D \neq 0$

N is a commutative ring.

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