IJASRM
International Journal of Advanced Scientific Research and Management, Volume 3 Issue 10, Oct 2018
www.ijasrm.com
ISSN 2455-6378

FUZZY 10^k- BASED GRACEFUL LABELING OF GRAPHS RELATED TO CIRCUITS

A.Solairaju[1], T.Narppasalai Arasu[2]

[1]Associate Professor of Mathematics,
Jamal Mohamed College(Autonomous),
Trichy, India.

[2]Assistant Professor of Mathematics,
Chikkanna Government Arts College ,
Tirupur,641602,India.

Abstract

In this paper the concept of Fuzzy 10^k -based graceful labeling is discussed and the existence of Fuzzy 10^k -based graceful labeling for graphs related to circuits is proved.

Keywords: *Graceful labeling , Fuzzy graceful labeling, Fuzzy 10^k-based graceful labeling*

1. Introduction

Fuzzy relation on a set was defined by Zadeah[8] in 1965.Based on fuzzy relation the first definition of a fuzzy graph was introduced by Rosenfeld and Kaufmann in 1973. The concept of graceful labeling has been introduced by Rosa[6] in 1976.Many remarks on fuzzy graphs and properties were given by Nagoor Gani A. and Rajalaxmi D. (a) Subhashini [5] and Bhattacharya P [1].Many variations of graceful labeling were introduced by Mordeson J.N., and P.S.Nair [4] and also by Vaidya S.K and Lekha Bijukumar [7] Jebesty Shajila and R Vimala S [3] introduced fuzzy vertex labeling of some graphs. Labeling of various graphs were shown by Harary [2]. Fuzzy 10^k-based graceful labeling of $nC_4 * P_2$ and $P_{2n} * nP_2 * P_3$ graphs are discussed in this paper .

2. Basic Definitions.

Definition 2.1 : A fuzzy closed path P_n is called a fuzzy cycle C_n.

Definition 2.2 : A fuzzy graph G =$\langle\sigma,\mu\rangle$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that for all $u, v \in V, we\ have\ \mu(u,v) \le \sigma(u) \land \sigma(v)$

Definition 2.3 : A labeling of a graph is an assignment of values to the vertices and edges of a graph.

Definition 2.4 : A graceful labeling of a graph G with q edges is an injection $f: V(G) \to \{0,1,2,\dots q\}$ such that when each edge is assigned the distinct label $|f(x) - f(y)|$

Definition 2.5 : A fuzzy graph G =$\langle\sigma,\mu\rangle$ is said to be a fuzzy graceful labeling if $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ is bijective such that the membership value of edges and vertices are distinct and

$\mu(u,v) < \sigma(u) \land \sigma(v)$ for all $u, v \in V$

3.Main Results.

Definition 3.1 : A fuzzy graph G =$\langle\sigma,\mu\rangle$ with p vertices and q edges is said to be a fuzzy 10^k-based graceful labeling if k is a least positive integer such that $0 < p + q < 10^k$ and $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ are injective such that the membership value of edges and vertices are distinct and satisfying the following four conditions

(i) $\mu(u,v) < \sigma(u) \land \sigma(v)$ for all $u, v \in V$

(ii) $\mu(u,v) = |\sigma(u) - \sigma(v)|$ for all $u, v \in V$

(iii) $\{\sigma(v) \times 10^k \ for\ all\ v \in V\} \subset \{q+1, q+2, \dots 2q+1\}$

(iv) $\{\mu(v) \times 10^k for\ all\ v \in V\} = \{1,2, \dots q\}$

Definition 3.2 : The fuzzy graph $nC_4 * P_2$ is defined as a connected graph contains n copies of fuzzy circuits with four vertices and a fuzzy path with two

IJASRM
International Journal of Advanced Scientific Research and Management, Volume 3 Issue 10, Oct 2018
www.ijasrm.com
ISSN 2455-6378

vertices whose vertex set is $\{v_1, v_2, \dots v_{3n}, v_0, v_0'\}$ and edge set is

$\{v_i v_{n+i} : \ i = 1 \ to \ n\} \cup \{ v_i v_{2n+i} : \ i = 1 \ to \ n\} \cup$
$\{v_i v_{n+i+1} : \ i = 1 \ to \ n-1\} \cup$
$\{v_i v_{2n+i+1} : i = 1 \ to \ n-1\} \cup$
$\{v_0 v_{n+1}, v_0 v_{2n+1}, v_0 v_0'\}$

Here p = 3n+2 and q = 4n+1.

Theorem 3.3 : The fuzzy graph $G = nC_4 * P_2$ is a Fuzzy 10^k-based graceful graph.

Proof : Consider a fuzzy graph G $= \langle\sigma, \mu\rangle$ with p =3n+2 vertices $\{v_1, v_2, \dots v_{3n}, v_0, v_0'\}$ *and* q = 4n+1 edges

$\{v_i v_{n+i} : i = 1 \ to \ n\} \cup$
$\{v_i v_{2n+i} : \ i = 1 \ to \ n\} \cup$
$\{v_i v_{n+i+1} : i = 1 \ to \ n-1\} \cup$
$\{v_i v_{2n+i+1} : i = 1 \ to \ n-1\} \cup$
$\{v_0 v_{n+1}, v_0 v_{2n+1}, v_0 v_0'\}$ such that

$\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ defined by

$$\sigma(v_0') = \left(\frac{2q+1}{10^k}\right)$$

$$\sigma(v_0) = \sigma(v_0') - \left(\frac{q}{10^k}\right)$$

$$\sigma(v_{n+1}) = \sigma(v_0) + \left(\frac{q-2}{10^k}\right)$$

$$\sigma(v_{n+i}) = \sigma(v_{n+i-1}) - \left(\frac{2}{10^k}\right)$$

$for \ i = \ 2,3, \dots n$

$$\sigma(v_{2n+1}) = \sigma(v_0) + \left(\frac{q-1}{10^k}\right)$$

$$\sigma(v_{2n+i}) = \sigma(v_{2n+i-1}) - \left(\frac{2}{10^k}\right)$$

$for \ i = \ 2,3, \dots n$

$$\sigma(v_i) = \sigma(v_0) + \left(\frac{2i}{10^k}\right) \ for \ i = 1,2, \dots n$$

For example the fuzzy 10^k-based graceful labeling of $nC_4 * P_2$ and $5C_4 * P_2$ are shown in Fig.1 and in Fig 2 respectively

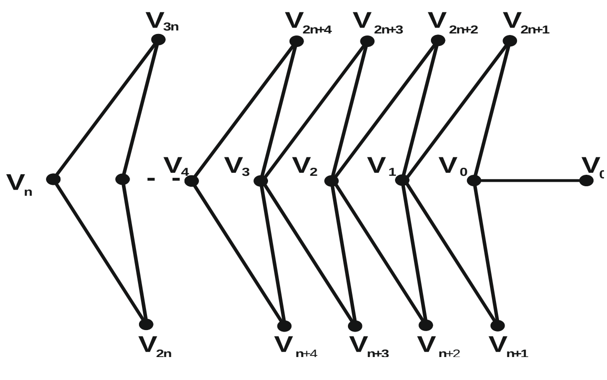

Fig.1.

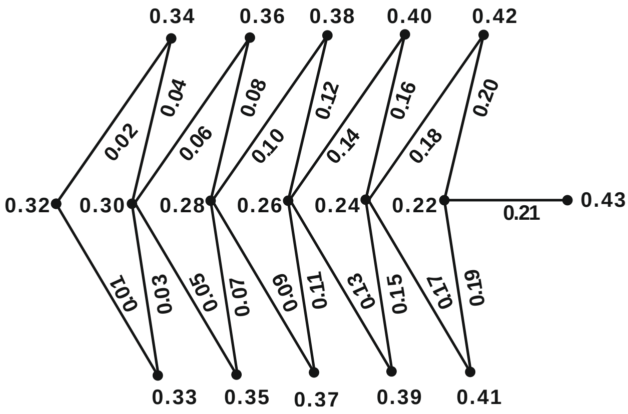

Fig.2.

Definition 3.4 : The fuzzy graph $P_{2n} * nP_2 * P_3$ is defined as a connected graph contains n copies of fuzzy path with two vertices and a fuzzy path with three vertices whose vertex set is $\{v_1, v_2, \dots v_{2n}, v_0\}$ and edge set is

$\{v_i v_{i+1} : i = 1 \ to \ 2n-1\} \cup$
$\{v_n v_0, v_0 v_{n+1}\} \cup$
$\{v_i v_{2n+1-i} : \ i = 1 \ to \ n\}$.

Here p = 2n+1 and q = 3n.

Theorem 3.5 : The fuzzy graph $G = P_{2n} * nP_2 * P_3$ is a Fuzzy 10^k-based graceful graph.

Proof : Consider a fuzzy graphG $= \langle\sigma, \mu\rangle$ with p =2n+1 vertices $\{v_1, v_2, \dots v_{2n}, v_0\}$ *and* q = 3n edges

$\{v_i v_{i+1} : i = 1 \ to \ 2n-1\} \cup \{v_n v_0, v_0 v_{n+1}\} \cup$
$\{v_i v_{2n+1-i} : i = 1 \ to \ n\}$ such that

$\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ defined by

$$\sigma(v_1) = \left(\frac{2q+1}{10^k}\right)$$

$$\sigma(v_i) = \sigma(v_{i-2}) - \left(\frac{3}{10^k}\right)$$

$for \ i = \ 3,5, \dots n \ or \ n-1$
$if \ n \ is \ odd \ or \ even$

$$\sigma(v_2) = \sigma(v_1) - \left(\frac{q-2}{10^k}\right)$$

$$\sigma(v_i) = \sigma(v_{i-2}) + \left(\frac{3}{10^k}\right)$$

$for \ i = \ 4,6, \dots n-1 \ or \ n$
$if \ n \ is \ odd \ or \ even$

$$\sigma(v_{2n}) = \sigma(v_1) - \left(\frac{q}{10^k}\right)$$

$$\sigma(v_{2n-i}) = \sigma\left(v_{2n-(i-2)}\right) + \left(\frac{3}{10^k}\right)$$

$for \ i = \ 2,4, \dots n-1 \ or \ n-2$
$if \ n \ is \ odd \ or \ even$

$$\sigma(v_{2n-1}) = \sigma(v_{2n}) + \left(\frac{q-1}{10^k}\right)$$

$\sigma(v_{2n-i}) = \sigma(v_{2n-(i-2)}) - \left(\frac{3}{10^k}\right)$
$for\ i = 3,5, \ldots n-2\ or\ n-1$
$if\ n\ is\ odd\ or\ even$

$\sigma(v_0) = \sigma(v_n) - \left(\frac{2}{10^k}\right)\ if\ n\ is\ odd$

$\sigma(v_0) = \sigma(v_n) + \left(\frac{2}{10^k}\right)\ if\ n\ is\ even$

For example the fuzzy 10^k-based graceful labeling $P_{2n} * nP_2 * P_3$, $P_8 * 4P_2 * P_3$ and $P_{10} * 5P_2 * P_3$ are shown in Fig.3, Fig 4 and Fig 5 respectively.

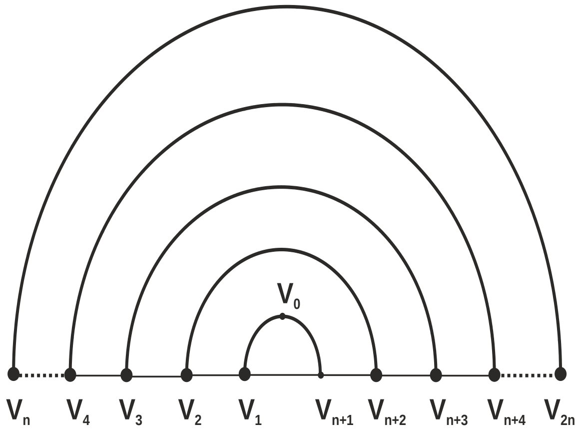

Fig.2.

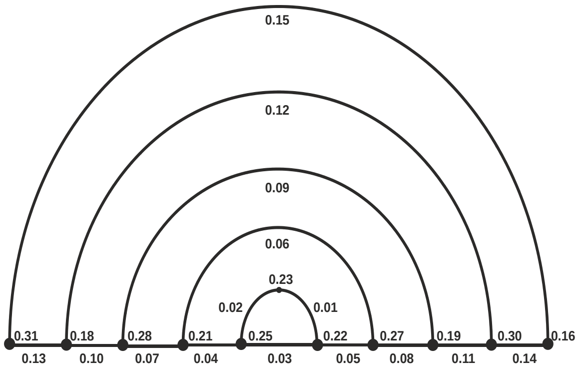

Fig.3.

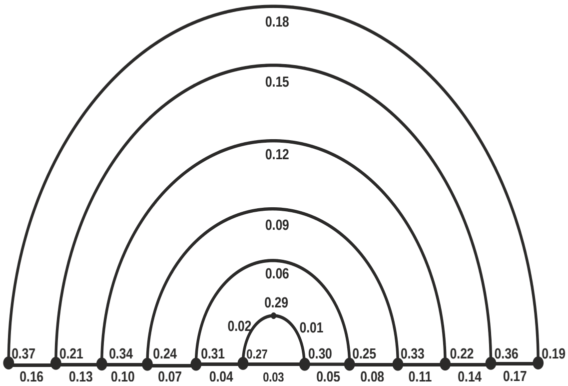

Fig.4.

4. Conclusion

In this paper the concept of fuzzy 10^k-based gracefulness for graphs related to circuits has been introduced

References

[1] Bhattacharya P , Some remarks on fuzzy graphs, Pattern Recognition Letters 6 297-302. (1987)

[2] Harary, Graph theory , Wesley Publication Company (1969)

[3] Jebesty Shajila R. and Vimala S. , Fuzzy Vertex labelling on wheel and fan graphs , IOSR Journal of Mathematics ,Volume 2, 45-49 (2016),

[4] Mordeson J.N., and P.S.Nair, Cycles and cocycles of fuzzy graphs, Inform. Sci.90 39 -49,(1996)

[5] Nagoor Gani A. and Rajalaxmi D. (a) Subhashini, Properties of fuzzy labeling graphs, Applied Mathematical Science,Vol.6,no.69-72,pp.3461-3466, (2012).

[6] Rosa A. On certain valuations of the vertices of a graph, Theory of graphs, International Symposium, Rome, July (1966), Gordon and Breach, New York and Dunod Paris, 349-355. (1967).

[7] Vaidya S.K and Lekha Bijukumar, Some new graceful graphs, International Journal of Mathematics and Soft Computing. Vol 1, No 1, 37-45,(2011)

[8] Zadeh,L. A.,Fuzzy sets, Inform. Control 8, 338-353 (1965).