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# Establishing the calculating program for rock stresses around a petroleum wellbore.

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**Abstract**

The stress model around boreholes, which is associated with the in-situ stresses, rock properties as well as the wellbore pressure and configuration, is developed. The new approach uses transformation formula of a full stress tensor including its orientations and magnitudes. The general solution for three-dimensional (3D) space is derived to the arbitrarily inclined wells. A calculating program for the stress analysis of wellbores (SAoWB), which is written in Matlab language, has described and calculated all components of the stress tensor at the wellbore wall as well as around boreholes. The developed stress model around boreholes has presented fully, quickly and precisely all components of the stress tensor around boreholes. It helps to understand better the stress state around boreholes as well as to solve wellbore problems such as wellbore stability analysis, optimization of well trajectory or drilling mud.

***Keywords:*** *Stress Model, Boreholes, Failure Observations, Wellbore Problems, Stability Analysis.*

## 1. Introduction

The stressed solid material is removed when drill a well into formations. The rock surrounding boreholes must support the stresses previously carried by the removed material. This causes an alteration of the stress state surrounding boreholes because the fluid pressure in the hole generally does not match the in-situ formation stresses. There will be the stress redistribution and concentration in the vicinity of boreholes (Fjaer et al, 2008).This can also result in wellbore failures if induced stresses around boreholes are over the rock strength. Consequently, knowledge of the stresses around boreholes is essential for wellbore problems.

Wellbore failures may be compressive failures known as borehole breakouts (BOs) and/or tensile failures as drilling-induced tensile fractures (DITFs) at the wellbore wall (Zoback et al., 1985, Peska and Zoback, 1995, Khanh, 2013, etc.).Today, many petroleum drilling wells have complex trajectories, either horizontal or highly deviated from vertical axis. It is necessary to understand the stresses around an arbitrarily deviated wellbore as well as the factors controlling the occurrence of compressive and tensile failures in this well with the arbitrary orientation. Therefore, the stress model around boreholes, which is associated with the in-situ far field stresses, rock properties as well as the wellbore pressure and configuration, is developed to describe fully and exactly stress components for the three dimensional (3D) space.

In this work, the general solution for a three-dimensional (3D) space is derived to the arbitrarily inclined wells. The new approach uses transformation formula of a full stress tensor including its orientations and magnitudes. A calculating program for the stress analysis of wellbores (SAoWB), which is written in Matlab language, has described and calculated all components of the stress tensor at the wellbore wall as well as around boreholes. Case studies are presented using the program SAoWB based on the new approach. The first one is to cross-check Barton's study on the compressive failure and breakout width analysis in KTB wellbore, Germany. The field case studies are applied for boreholes at different interest depths of the studied wellbore at Cuu Long basin, offshore Vietnam. The obtaining results from our program SAoWB are in good agreement with the failure observations from high solution image logs of the studied wellbore.

## 2. Stress model around boreholes

The calculation of the stresses around an arbitrarily inclined wellbore requires that the far field in-situ stress tensor is transformed into the borehole coordinate system. In this coordinate system, the stress tensor may no longer be represented by the principal stress magnitudes and directions for a vertical wellbore. The shear stress components may be non-zero and the transformed stress tensor must be represented. The transformed stress tensor is required to calculate the stress concentration around boreholes.

In a deviated well, the principal stresses acting in the vicinity of the wellbore wall are generally not aligned with the wellbore axis (Figure 1).

To consider failure in a well of arbitrary orientation, we must define three coordinate systems (Figure 2) as:

- A geographic coordinate system, X, Y and Z oriented north, east and vertical (down);
- A stress coordinate system, $x_s$, $y_s$ and $z_s$ corresponding to the orientations $S_1$, $S_2$ and $S_3$;
- A wellbore coordinate system $x_b$, $y_b$ and $z_b$ where $x_b$ is radial, pointing to the bottom of the well, $z_b$ is down along the wellbore axis and $y_b$ is orthogonal in a right-hand coordinate system.

To most easily visualize wellbore failures we will always look down deviated wells and evaluate wellbore failures as a function of angle Θ from the bottom of the well in a clockwise direction. We also consider stress variations as a position function of position of angle Θ around the wellbore going clockwise from the bottom.

## 3. Coordinate Transforms

Following Peska and Zoback (1995), the tensor transformations are used to evaluate stresses in the three coordinate systems of interest. It is useful to choose a reference coordinate system with respect to which both the stress tensor and wellbore trajectory can be measured.

In general, the principal stress tensor can be written:

$$S_s = \begin{pmatrix} SHmax & 0.0 & 0.0 \\ 0.0 & Shmin & 0.0 \\ 0.0 & 0.0 & Sv \end{pmatrix}$$

To rotate these stresses into a wellbore coordinate system ($x_b$, $y_b$, $z_b$), we need two coordinate transforms in succession.

Firstly, we need to know how to transform the stress field into a geographic coordinate system. We use the transform as:

$$\begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} = R_s \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

Mathematically, the matrix $R_s$ required to transform the stress tensor into the geographic coordinate system will be:

$$R_s = \begin{pmatrix} cos\alpha.cos\beta & sin\alpha.cos\beta & -sin\beta \\ cos\alpha.sin\beta.cos\gamma - sin\alpha.cos\gamma & sin\alpha.sin\beta.sin\gamma + cos\alpha.cos\gamma & cos\beta.sin\gamma \\ cos\alpha.sin\beta.cos\gamma + sin\alpha.sin\gamma & sin\alpha.sin\beta.cos\gamma - cos\alpha.sin\gamma & cos\beta.cos\gamma \end{pmatrix}$$

where α defines the clockwise rotation about the vertical axis from geographic north to the orientation of the maximum horizontal stress, β defines the rotation about the minimum horizontal stress direction towards the vertical down, and γ defines the rotation about the maximum horizontal stress direction.

Next, to transform the stress field from the geographic coordinate system to the wellbore system, we use the transform as:

$$\begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = R_b \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

The matrix $R_b$ required to transform the stress tensor in the geographic coordinate system into the borehole coordinate system is:

$$R_b = \begin{pmatrix} -cos\delta.cos\varphi & -sin\delta.cos\varphi & sin\varphi \\ sin\delta & -cos\delta & 0.0 \\ cos\delta.sin\varphi & -sin\delta.sin\varphi & cos\varphi \end{pmatrix}$$

where δ is the azimuth of the horizontal projection of the borehole measured clockwise from geographic north and φ is the angle between the borehole and the vertical. The trajectory of a borehole can also be described in the geographic coordinate system by δ and φ.

With matrices $R_s$ and $R_b$ defined, we can define the stress tensors $S_g$ and $S_b$ in the geographic and wellbore coordinate systems.

The stress tensor in the geographic coordinate system $S_g$ can be described by:

$$S_g = R_S^T \, S_S \, R_S$$

The stress tensor in the wellbore coordinate system $S_b$ can be described by:

$$S_b = R_b \, (R_S^T \, S_S \, R_S) \, R_b^T$$

When considering effective stresses, the effective stress tensor becomes:

$$\sigma_{ij} = S_{b,ij} - \delta_{ij} P_p$$

where $S_{b,ij}$ is the i, j component of the stress tensor $S_b$ and $\delta_{ij}$ is the Kronecker.

(3.17)

Mathematically, the general solution of effective stresses around an arbitrarily inclined well of radius R will be described in terms of a cylindrical coordinate system by the following equations:

$$\sigma_{\Theta\Theta} = \frac{1}{2}(\sigma_{11} + \sigma_{22} - 2P_p)\left(1 + \frac{R^2}{r^2}\right) - \frac{1}{2}(\sigma_{11} - \sigma_{22})\left(1 + 3\frac{R^4}{r^4}\right)\cos 2\theta - \Delta P\,\frac{R^2}{r^2}$$

$$\sigma_{rr} = \frac{1}{2}(\sigma_{11} + \sigma_{22} - 2P_p)\left(1 - \frac{R^2}{r^2}\right) + \frac{1}{2}(\sigma_{11} - \sigma_{22})\left(1 - 4\frac{R^2}{r^2} + 3\frac{R^4}{r^4}\right)\cos 2\theta + \Delta P\,\frac{R^2}{r^2} \quad (3.18)$$

$$\tau_{r\Theta} = \frac{1}{2}(\sigma_{11} - \sigma_{22})\left(1 + 2\frac{R^2}{r^2} - 3\frac{R^4}{r^4}\right)\sin 2\theta$$

Using the effective stresses described above, the effective stresses at the wellbore wall (r=R) will become:

$$\sigma_{\theta\theta} = \sigma_{11} + \sigma_{22} - 2(\sigma_{11} - \sigma_{22})\, cos2\theta - 4\sigma_{12}\, sin2\theta - \Delta \mathrm{P}$$

$$\sigma_{zz} = \sigma_{33} - 2\upsilon(\sigma_{11} - \sigma_{22})\, cos2\theta - 4\upsilon\sigma_{12}\, sin2\theta$$

$$\tau_{\theta z} = 2(\sigma_{23} cos\theta - \sigma_{13} sin\theta) \quad (3.19)$$

$$\sigma_{rr} = \Delta \mathrm{P}$$

For a borehole arbitrarily inclined with respect to the principal stresses, $\tau_{\theta z}$ is non-zero i.e. the axial and circumferential stresses are not principal stresses. In this case, the three principal stresses at the wellbore wall can be calculated using:

$$\sigma_{tmax} = \frac{1}{2}\left(\sigma_{zz} + \sigma_{\theta\theta} + \sqrt{(\sigma_{zz} - \sigma_{\theta\theta})^2 + \tau_{\theta z}^2}\right) \tag{3.29}$$

$$\sigma_{tmin} = \frac{1}{2}\left(\sigma_{zz} + \sigma_{\theta\theta} - \sqrt{(\sigma_{zz} - \sigma_{\theta\theta})^2 + \tau_{\theta z}^2}\right) \tag{3.30}$$

$$\sigma_{rr} = \Delta P \tag{3.31}$$

where $\sigma_{tmax}$ and $\sigma_{tmin}$ are the maximum and minimum effective stresses in the plane tangential to the wellbore wall (Figure2).

Moreover, the angle ω between $\sigma_{tmax}$ and the wellbore axis in the plane tangential to the wellbore wall also is defined by:

$$\tan 2\omega = \frac{\tau_{\theta z}}{\sigma_{zz} - \sigma_{\theta\theta}} \tag{3.32}$$

Applying these above formula and theory, a calculating program for the stress analysis of wellbores (SAoWB) written in Matlab language has described and calculated all components of the stress tensor at the wellbore wall as well as around boreholes form the vertical wells to the arbitrarily inclined wells.

## 4. Results and Discussion

***Case study 1: Cross-checking Barton's study (1998) on compressional failure and breakout width analysis at the KTB wellbore, Germany***

Results of Barton's study (1998) showed that for the observed breakout width $40^0$ and known value of rock strength C of 350 MPa, the magnitude of $S_{Hmax}$ is approximately 205 MPa in the KTB wellbore at the depth of 5390 m with given parameters (the orientation of $S_{Hmax}$ oriented in $170^0$N; $S_v$ = 151 MPa; $S_{hmin}$ = 105 MPa; $P_p = P_m$ = 54 MPa).

The program SAoWB was also used to model the in-situ stress state and apply it for compressional failure and breakout width analysis. This approach used the full 3D stress tensor in the computation, providing a more complete solution than previous 2D methods (Barton et al, 1998; Vernik and Zoback, 1992).

Results obtained from our program SAoWB for the KTB wellbore are shown in following Figure 3.

Both the results from our program RAoWB also showed that under the full in-situ stress tensor (Azi_$S_{Hmax}$ = $170^0$N; $S_v$ = 151 MPa; $S_{hmin}$ = 105 MPa; $P_p = P_m$ = 54 MPa and $S_{Hmax}$ = 205 MPa) of the KTB wellbore at the depth of 5390 m, the breakout width will approximate $40^0$ if the given rock strength of 350 MPa. This breakout width is in excellent agreement with other methods and the failure observations in the KTB wellbore at the depth of 5390 m. Moreover, the results from program RAoWB could display the stress distribution of around the KTB wellbore. Especially, the program RAoWB could analyze the risk of the occurrence of both BOs and DITFs and display the variation of orientations of both BOs and DITFs (if they occur) of any wellbore trajectories by the stereographic diagrams.

In summary, comparing the results obtained from our program SAoWB with those studied earlier of thewell-known case of Barton'study (1998) confirmed their degree of accuracy, reliability. The program SAoWB has not only described the stress distribution around boreholes but also it could analyse the risk of the occurrence of both DITFs and BOs at the wellbore wall. Moreover, through these well-known investigations earlier also prove that our program SAoWB may be user-friend, attractive and easy to develop for other implications.

Therefore, we could use the program SAoWB written in Matlab language to investigate the field case studies for boreholes at different interest depths of the studied wellbore at Cuu Long basin, offshore Vietnam.

***Case study 2: The field case studies for boreholes at different interest depths of the studied wellbore at Cuu Long basin, offshore Vietnam***

The observation of BOs and/or DITFs at the depths in the basement reservoir of White Tiger field, Cuu Long basin, offshore Vietnam combined with $S_v$ from density logs, $S_{hmin}$ from the hydraulic fracturing tests, and $P_p$ from DSTs and WFITs indicate the far field in-situ stress tensor at these interest depths in the basement reservoir of White Tiger field are shown in the following Table 1.

The values summarized in Table 1 are used in considering the implications of the full tensor of in-situ stress for the choice of optimum drilling trajectories and for wellbore stability. From the full tensor of in-situ stress at the interest depths, we will input these data in the program SAoWB (Stress Analysis of Wellbore) to calculate in detail the wellbore stresses around the wellbore to check and constrain with the available data and obtained information on the failure images of the wellbores. We can see the distribution of all stress components around the wellbore subjected to the full tensor of in-situ stress in the radius range from r = 1.0R to 1.5R (R is the wellbore radius).

At the depth of 3900 m, the obtaining results from the program SAoWB are shown in Figure 4. From the figure 4, we can see that the highest possibility of the BO occurrence is along the direction of $S_{hmin}$ due to the concentration of the compressive stresses. If the compressive stress over the rock strength the BO formation will appear. Therefore, under the full stress tensor at the depth of 3900 m, the occurrence of the BOs is fairly clear because the granite rock strength measured (110 MPa) is lower than the rock strength required (120 MPa) to prevent the occurrence of BOs. Moreover, the width of BOs can be predicted about 320. However, the occurrence of DITFs along the direction of $S_{Hmax}$ is almost impossible because the minimum principal stress is still positive at the wall of the wellbore.

At the depth of 4100 m, the obtaining results from the program SAoWB are shown in Figure 5. Similarly, under the full stress tensor at the depth of 4100 m, the occurrence of the BOs is fairly clear because the granite rock strength measured (110 MPa) is lower than the rock strength required (130 MPa) to prevent the occurrence of BOs but the predicted width of BOs can increase to 500. Moreover, the occurrence of DITFs along the direction of $S_{Hmax}$ is also almost impossible because the minimum principal stress does still not reach zero at the wall of the wellbore.

From the figure 6, the highest possibility of the BO occurrence is also along the direction of $S_{hmin}$ due to the concentration of the compressive stresses. Furthermore, from the depth of 4300m the weathered basement rock maybe disappears. The basement rock strength measured from the depth of 4300 m should be increased from 110 MPa to 155 MPa. Therefore, under the full stress tensor at the depth of 4300 m, the occurrence of the BOs is impossible because the maximum rock strength required (148 MPa) to prevent the occurrence of BOs is lower than the rock strength measured (155 MPa) from the depth of 4300 m. However, the occurrence of DITFs along the direction of $S_{Hmax}$ occurs because the minimum principal stress may reach zero at the wall of the wellbore.

At the depth of 4500 m, the obtaining results from the program SAoWB are shown in figure 7. Similar to figure 6, the occurrence of the BOs is also impossible because the maximum rock strength required to prevent the occurrence of BOs is still lower than the rock strength measured (155 MPa) at the depth of 4500 m. Moreover, the occurrence of DITFs along the direction of $S_{Hmax}$ occurs because the minimum principal stress may still reach zero at the wall of the wellbore.

## 5. Tables, Figures

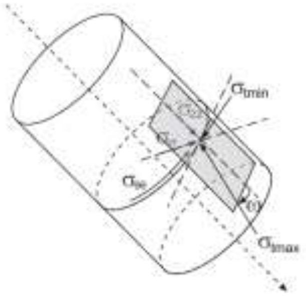


**Figure 1An arbitrarily deviated wellbore with the orientations of the cirumferential ($\sigma_{\Theta\Theta}$), axial ($\sigma_{zz}$), radial ($\sigma_{rr}$), minimum ($\sigma_{tmin}$) and maximum ($\sigma_{tmax}$) stresses, where ω is the angle between τtmax and the wellbore axis (after Peska and Zoback, 1995).**

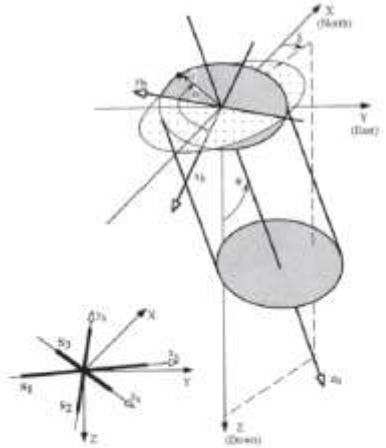
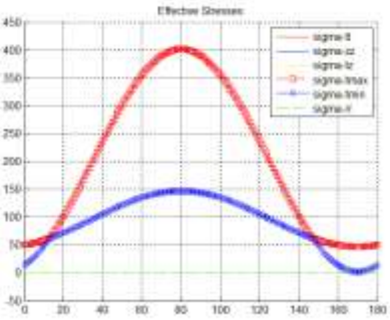


**Figure 2 Three coordinate systems used to transform for an arbitrarily deviated wellbore: Wellbore coordinate system (xb, yb, zb) and (r, Ѳ, zb) and a stress coordinate system (xs, ys, zs ) with respect to the geographic coordinate(X, Y, Z). The system (Xs,Ys,Zs) coincide with the far-field principal stresses S1, S2, S3.The wellbore orientation relative to the geographic coordinate is described by the azimuth δ and inclination φ (after Peska and Zoback, 1995).**



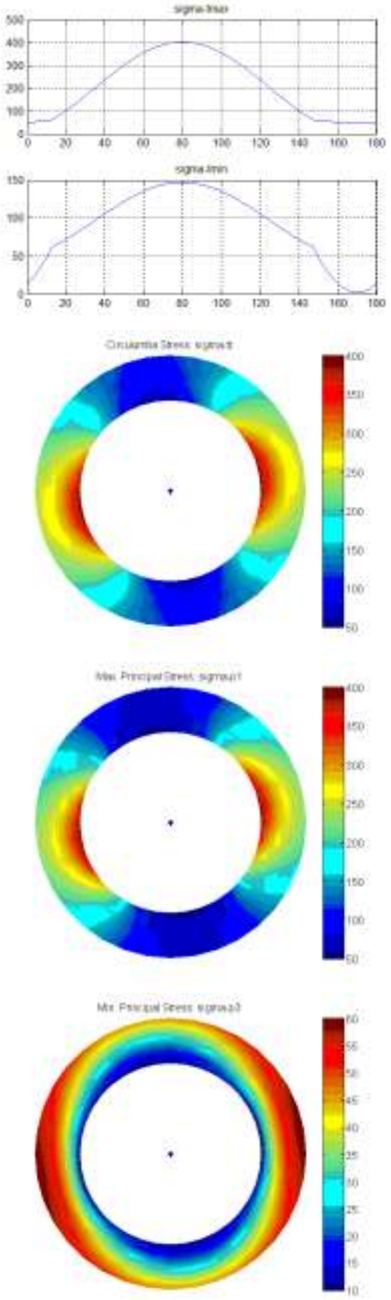


**Figure 3 Stress distribution of case study 1 from program SAoWB**

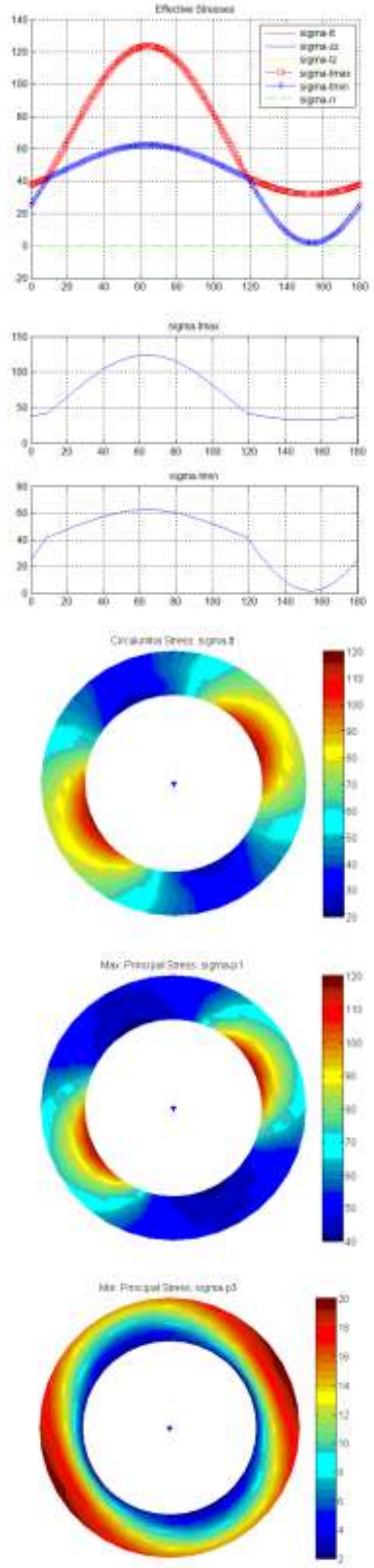


**Figure 4 Stress distribution at the basement depth of 3900 m at White Tiger field.**

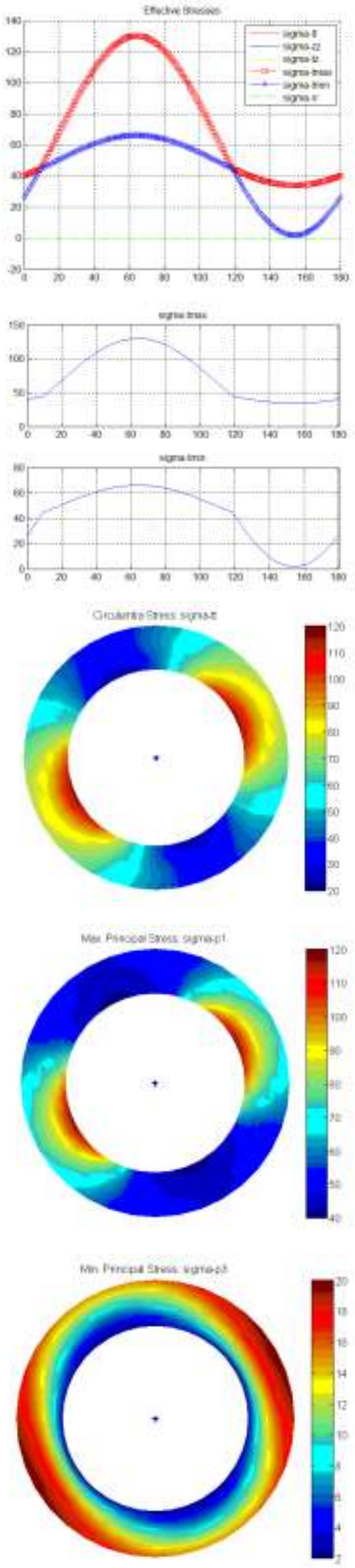


Figure 5 Stress distribution at the basement depth of 4100m at White Tiger field.

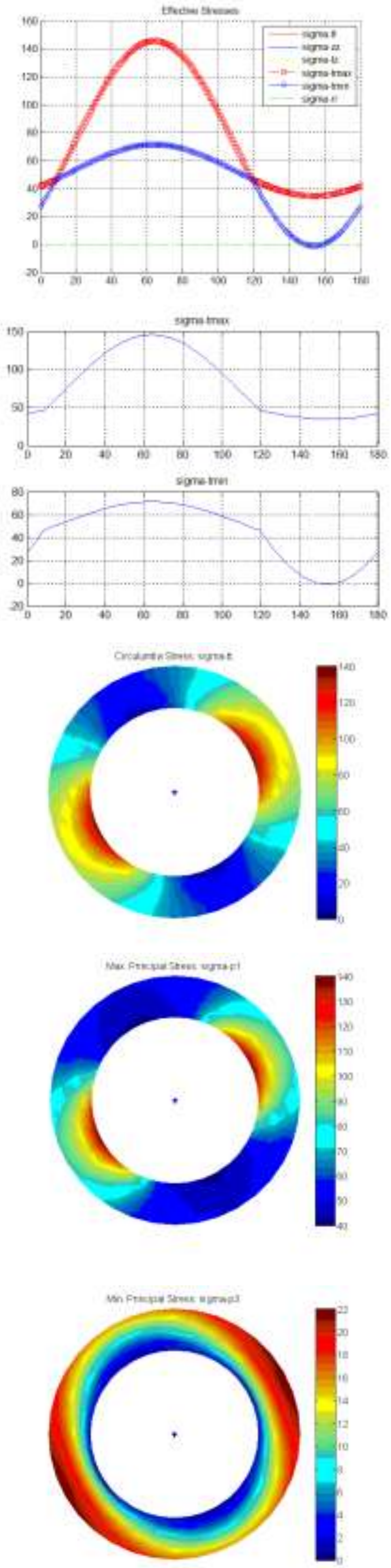


Figure 6 Stress distribution at the basement depth of 4300m at White Tiger field.

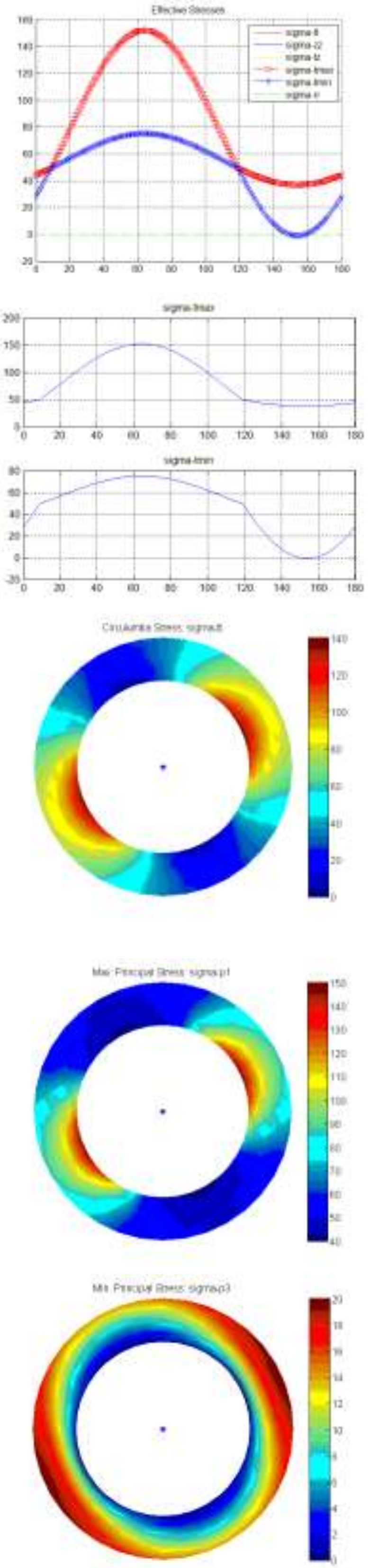


Figure 7 Stress distribution at the basement depth of 4500 m at White Tiger field.

Table 1 The full tensor of in-situ stress at basement depths at White Tiger field.

| Stress Components | At 3900m with C=110 MPa, BOs but no DITFs | At 4100m with C=110 MPa, BOs, & no DITFs | At 4300m with C=155 MPa, no BOs but DITFs | At 4500m with C=155 MPa, no BOs but DITFs |
|---|---|---|---|---|
| Azi_SHmax, $^{0}$N From FMI,CAST-V | 154$^{0}$N (148$^{0}$N ~ 160$^{0}$N) | 154$^{0}$N (148$^{0}$N ~ 160$^{0}$N) | 154$^{0}$N (148$^{0}$N ~ 160$^{0}$N) | 154$^{0}$N (148$^{0}$N ~ 160$^{0}$N) |
| Sv, MPa | 87.57 | 92.55 | 97.56 | 102.60 |
| S$_{hmin}$, MPa From hydralic fracturing tests with gradient of 0.63 psi/ft | 56.51 | 59.41 | 62.31 | 65.21 |
| Pp, MPa From DSTs and WFITs with gradient of 0.45 psi/ft | 40.37 | 42.44 | 44.51 | 46.58 |
| S$_{Hmax}$, MPa | 87.00 | 91.50 | 99.00 | 103.50 |
| | | | | |

## 6. Conclusions

The developed stress model around boreholes, which uses transformation formula in three coordinate systems, has presented fully, quickly and precisely all components of the stress tensor around boreholes for 3D space. It is associated with the in-situ stresses, rock properties as well as the wellbore pressure and configuration.

The program SAoWB written in Matlab language has not only described the stress distribution around boreholes but also it could analyse the risk of the occurrence of wellbore failures. Moreover, it has also proved that this program may be user-friend, attractive and easy to develop for other implications.

The obtaining results from our program SAoWB for the field case studies at Cuu Long basin, offshore Vietnam are in good agreement with the failure observations from high solution image logs of the studied wellbore. It helps to understand better the stress state around boreholes as well as to solve wellbore problems such as wellbore stability analysis, optimization of well trajectory or drilling mud.

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