

Application of Coefficient of range for solving Transportation Problem

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Abstract

There are several methods for obtaining initial basic feasible solution (IBFS) of transportation problem (TP) such as north west corner(NWC) rule, least cost method (LCM) and Vogel approximation method (VAM) etc. In this paper we have used coefficient of range for finding intial solution of transportation Problem. Numerical examples are given in support of the result.

1. Introduction

Transportation problem was first formulated in 1941 by Hitchcock [2], is a special case of linear programming problem in which our objective is to satisfy the demand at destinations from the supply at the minimum transportation cost.It was further developed in 1949 by Koopman [3] and in 1951 by Dantzig [1]. A certain class of linear programming problem knows as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contacts of determining optimum shipping pattern. For example: A product may be transported from factories to retail stores. The factories are the sources and the store are the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origin and it is required that the given quantities of the product be shipped to each of n destinations. The minimum cost of ship- ping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. The problem can be formulated as:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij}$$

subject to $\sum_{j=1}^n x_{ij} = a_i, \sum_{i=1}^m x_{ij} = b_j$ for $i = 1, 2, \dots, m$ & $j = 1, 2, \dots, n$ and $x_{ij} \geq 0 \forall i, j$

For each supply point $i, (i = 1, 2, \dots, m)$ and demand point $j, (j = 1, 2, \dots, n)$

c_{ij} =unit transportation cost from i^{th} source to j^{th} destination

x_{ij} =amount of homogeneous product transported from i^{th} source to j^{th} destination

a_i = amount of supply at i^{th} source

b_j = amount of demand at j^{th} destination. where a_i and b_j are given non-negative numbers and assumed that total supply is equal to total demand, i.e $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$, then transportation problem is called balanced otherwise it is called unbalanced. The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost. Because of the special structure of the transportation model, the problem can also be represented as Table 1.

Table1: Tabular representation

Destination → source ↓	D ₁	D ₂	D _n	Supply(a _i)
S ₁	c ₁₁	c ₁₂	c _{1n}	a ₁
S ₂	c ₂₁	c ₂₂	c _{2n}	a ₂
...
S _m	c _{m1}	c _{m2}	...	c _{mn}	a _m
Demand(b _j)	b ₁	b ₂	...	b _n	

Definition

Coefficient of range = $\frac{l-s}{l+s}$ where l is largest and s is smallest cost.

Proposed Algorithm

1. Balance the tp if it is unbalanced by adding dummy row or column whichever is necessary
2. Find Coefficient of range of each row and each column.
3. Do the allocation at minimum cost in that row or column in which Coefficient of range is maximum.
4. Cross out the row or column whichever is satisfied.
5. Continue in this way until whole demand is satisfied.

Numerical Examples

Input data and initial solution obtained by applying DSM method for different examples is given in tables 2-3.

Table 2: Input data and Initial solution

Ex.	Input Data	Obtained Allocations by coefficient of range.	Initial solution	Optimal solution
1	$[c_{ij}]_{3 \times 3} = [7 \ 3 \ 4; \ 2 \ 1 \ 3; \ 3 \ 4 \ 6];$ $[a_i]_{3 \times 1} = [2, 3, 5];$ $[b_j]_{1 \times 3} = [4, 1, 5]$	$x_{13}=2,$ $x_{21}=2,$ $x_{22}=1,$ $x_{31}=2,$ $x_{33}=3,$	37	33

Table 3: Input data and Initial solution

Ex.	Input Data	Obtained Allocations by coefficient of range.	Initial solution	Optimal solution
2	$[c_{ij}]_{3 \times 3} = [50 \ 30 \ 220; \ 90 \ 45 \ 170; \ 250 \ 200 \ 50];$ $[a_i]_{3 \times 1} = [1, 3, 4];$ $[b_j]_{1 \times 3} = [4, 2, 2]$	$x_{12}=1,$ $x_{21}=2,$ $x_{22}=1,$ $x_{31}=2,$ $x_{33}=2$	855	820

Conclusions

This method is easy to understand and simple to apply. The initial feasible solution obtained is near to optimal solution.

References

- [1] Dantzig, G. B. (1951). Linear Programming and Extensions. Princeton, NJ: Princeton University Press.
- [2] Hitchcock FL (1941). The Distribution of a product from several sources to numerous localities. Journal of Mathematics and Physics 20: 224- 230.
- [3] Koopman, T.C. (1949) Optimum utilization of transportation system. Econometrica, Supplement vol 17.