

Seasonal Effect in Volatility-Volume Relationship at an Index Futures Market with High-Hedging Activities

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Abstract

This study discusses Granger causality of daily return volatility and trading volume of the index futures with high-hedging or speculative trading activity at Taiwan futures market. We use Flexible Fourier Form (FFF) proxy to seasonal factor's component into the volatility and volume in a bivariate vector autoregressive (VAR) framework. This differs from Luu and Martens (2003) who took into account the effect of the seasonal factor's component on the volume only but excluding its effect on volatility. It has been proved by empirical studies that the seasonal factor's component exists concurrently in the volume and the return volatility. Such results are apparently different from the hypothesis in the mixture of distributions hypothesis (MDH) proposed by Andersen (1996) (the return volatility is presumed not to be in the seasonal factor's component). Finally, we find out that both Taiwan index futures market's return volatility and trading volume exhibit bidirectional causality.

Keywords: *index futures, seasonal effect, volatility-volume relationship, MDH*

1. Introduction

The relationship between financial asset price volatility (return) and trading behavior has been one of the focuses of many studies (Huang et al., 2012; Huang and Zhang, 2015; Lai and Wang, 2016; Chang. and Yu, 2017; Gökçen and Post, 2018). An important theory, the mixture of distributions hypothesis (MDH), was proposed by Clark (1973) in the discussion of the relationship between the futures return volatility and the trading volume. Meanwhile, Harris (1987), Tauchen and Pitts (1983), and Lamoureux and Lastrapes (1990) have elaborated and supported the theory. Tauchen and Pitts (1983) and Andersen (1996) proposed modified MDH versions regarding the return volatility and the trading volume are subordinated stochastic processes

of the information arrival rate or the news. And the daily return volatility and the trading volume are influenced by the latent information arrival or information flow.

These MDH versions hypothesize that the daily return r_t is the sum of each equilibrium return (δ_{it}) on intraday where $\delta_{it} \stackrel{i.i.d.}{\sim} N(0, \sigma^2)$; σ^2 is a constant variance. The daily trading volume V_t is the sum of the intraday trading volume of each transaction. $r_t = \sum_{i=1}^{n_t} \delta_{it}$ and V_t are two random variables subject to the joint distribution of (r_t, V_t) , which is a bivariate normal distribution. Given the numbers of the daily information n_t , we can obtain the following:

$$r_t | n_t \sim N(0, \sigma^2 n_t) \quad (1)$$

$$V_t | n_t \sim N(a + b n_t, c n_t) \quad (2)$$

(1) and (2) represent two dynamic stochastic processes of the return volatility and the trading volume respectively, both depending on the behavior of the information numbers n_t . Tauchen and Pitts (1983) presume a is zero in equation (2). However, Andersen (1996) allows the trading volume has a component of an additional parameter a , which has uncorrelated to the information flow. Meanwhile, such part of the trading volume is caused by the noise or the liquidity trading activities. In Tauchen and Pitts (1983), the information arrival process is presumed to be a process subject to an i.i.d. (identically and independently distributed) and regarding both the return volatility and the trading volume as normal distribution as well as both having same subordinated latent information arrival rate at the same time. While in Andersen (1996), it is presumed that the information arrival rate takes on as serially correlated, hence, leading to marked

relationship between the lagged volume (volatility) and the current volume (volatility).

Luu and Martens (2003) apply the aforementioned viewpoints to verify the daily squared return and the trading volume of S&P 500 index futures by the vector autoregressive model, finding out that there is only unidirectional causality from volatility to volume. Meanwhile, they use more precise data to verify and discover that the sum of intraday squared return and the trading volume have bidirectional causality in further support of MDH. We can learn from this study why the test results of weighing futures return volatility by the daily squared return are only unidirectional causality from volatility to trading volume if the futures return volatility and the trading volume have the bidirectional causality. These tests and verification results seem to demonstrate that other components may exist in the daily return volatility, leading to verification results' differences.

The main reason for this study to select Taiwan index futures market as the empirical research subject is that there are relatively more speculative or hedging activities in this futures market. Take Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) futures; TX futures, in Table 1 for an example, the mean of trading volume divided by open interest (VO_t) is 1.0527. Such a mean is fairly big in general futures markets such as the Japanese Nikkei 225 stock index futures in Watanabe (2001)'s study, or the Malaysian Kuala Lumpur Composite Index Futures Contracts (KLSE CI futures) verified by Pok and Poshakwale (2004), and even the Short Sterling futures verified by Kalotychou and Staikouras (2006) as well as the 4 electricity futures verified by Hadsell (2006). The results of these empirical studies demonstrate that the mean of trading volume divided by open interest (VO_t) is less than 0.4.

Luu and Martens (2003) believe that the value of the trading volume divided by open interest (VO_t) can reflect the impact of the speculative trading activities and hedging activities in the market. When there are relatively more speculative or hedging activities in the market, the value of the trading volume divided by open interest (VO_t) will be relatively larger. If there is relatively higher percentage of speculative or hedging activities in the futures market, these trading activities seem to have relatively greater impact on the volume or the return volatility intuitively. Hence, we take three index futures contracts of relatively longer trading history and higher trading volume issued by Taiwan Futures Exchange, TAIFEX, as our study's subjects. The remainder of this paper can be divided into the following three sections: Section II describes the data and methodologies, section III

reports the empirical results, and the final section provides the conclusions of this study.

2. Data and Methodologies

The empirical data of this study cover the nearby contracts of TX futures, Taiwan Stock Exchange Electronic Sector Index futures (TE) and Taiwan Stock Exchange Finance Sector Index futures (TF) during the period from February 1, 2002 to February 9, 2007 in a total of 1249 observations of daily trading data. The main reason for us to adopt the futures nearby contracts is that the majority of the futures trading activities took place during this period. Meanwhile, we take the daily squared return to measure the daily volatility and the daily squared return as the rate of daily squared return of the continuous compounding as $r_t = 100 \times (\ln P_t - \ln P_{t-1})$ when P_t is the futures settlement price on day t. In addition, our measurement of the trading volume mainly refers to Chatrath et al. (1996). In addition to using the daily trading volume for measuring, we also use the daily trading volume divided by open interest (VO_t) as measure for the volume as Bessembinder and Seguin (1993) believe that the daily trading volume divided by open interest (VO_t) can be regarded as the total numbers for measuring the daily trading activities.

Proposed by Andersen and Bollerslev (1998), FFF is used by Martens, Chang and Taylor (2002) to verify the high-frequency data. It is a model able to precisely capture the time trend. We use FFF as an extraneous variable to integrate it directly into the dynamic structure of bivariate vector autoregressive composed by the return volatility and the trading volume. Our method is different from the one used by Luu and Martens (2003) who adjust the trading volume by making FFF as a seasonal factor to form the de-trended futures volume and discuss the relationship between the trading volume and the volatility by the two-step estimation method. However, this study adds FFF directly into the VAR model, and this method can verify the time trend or the relationship between the seasonal factor and the return volatility or the trading volume. FFF is shown as the following:

$$\begin{aligned}
 FFF_t = & a_0 + \sum_{i=2}^5 \delta_i \cdot D_{i,t} + a_1 t + a_2 t^2 + b_1 \cdot TTM_t + b_2 \cdot TTM_t^2 \\
 & + \sum_{j=0}^q \left(\phi_j \cos\left(\frac{2\pi \cdot j \cdot TTM_t}{TTM_{\max}}\right) + \theta_j \sin\left(\frac{2\pi \cdot j \cdot TTM_t}{TTM_{\max}}\right) \right)
 \end{aligned}
 \tag{3}$$

In the above equation, FFF_t refers to the total effect of the seasonal factor on the t day, D_{it} refers to the dummy variable, and $i = 2, 3, 4, 5$, represent Tuesday through Friday. Suppose the day t is a Tuesday, we define $D_{2t} = 1$, and the other dummy variables are defined as $D_{it} = 0$; where $i = 3, 4, 5$. We use t and t^2 to capture the market time trend, defining the start day's initial values as $t = 1$ with number of days increased subsequently. TTM_{max} refers to the farthest day from the expiration date by the futures contract during the study period. q is the number of sinoids. To simplify the problem in equation (3) we overlook other calendar effects on Taiwan index market. Meanwhile, we input equation (3) directly into the dynamic structure of the autoregressive VAR-cum-FFF model made up of bivariate variables to capture the effect of the seasonal factor on the index futures volatility and the trading volume. The model is as shown in the following:

$$r_t^2 = \sum_{k=1}^p \alpha_{1k} r_{t-k}^2 + \sum_{k=1}^p \beta_{1k} Vol_{t-k} + FFF_t + \varepsilon_{1t}$$

$$Vol_t = \sum_{k=1}^p \alpha_{2k} r_{t-k}^2 + \sum_{k=1}^p \beta_{2k} Vol_{t-k} + FFF_t + \varepsilon_{2t}$$

(4)

The r_t^2 in the above equation refers to the daily squared return on the t th day and the Vol_t represents the futures volume. We use the daily trading volume and the ratios of daily trading volume to open interest (VO_t) to measure for volume (Vol_t). ε_{it} is the residual where $i = 1, 2$. ε_{it} may be contemporaneously correlated. However, their own lagged values (ε_{it-1}) is uncorrelated and uncorrected as well as the right-hand side variables in equation (4). p is the lag length, including the lag length of the return volatility (volume) and the volume (return volatility) variables. q is the number of sinoids. The selection of the optimal lag lengths both p and q are determined by Schwarz's information criterion (the minimum SIC). The model in equation (4) can verify the futures return volatility and the volume Granger causality as well as verify the relationship between the seasonal factor and the futures return volatility or the volume.

3. Empirical results

Table 1 illustrates the summary statistics of the nearby contracts of TX, TE and TF futures. It can be seen from Table 1 that the mean of daily return rate

of the three futures is positive with the mean return rate of TE at 0.063% as the minimum. In terms of the daily squared return, the mean of TX is 2.1695 with standard deviation at 5.2538, being the smallest one among the three futures while the mean number of TE is 3.1627 with standard deviation at 6.8899, being the maximum. Secondly, as regards the daily trading volume (V_t) and the daily trading volume divided by open interest (VO_t), TX is the biggest with its mean at 26,509 and 1.0527 respectively while TF is the smallest with its mean at 3,754 and 0.6111. We also find out that TX's mean of VO_t (1.0527) is more than the mean of other futures' VO_t (TE is 0.7192; TF is 0.6111). It is apparent that TX in Taiwan index market has higher percentage of speculative trading activity or hedging activity than other two index futures.

Table 1: Summary statistics, TX, TE and TF futures returns, volatility, and volume

Variables	Return(r) %	Squared Return	Volume	Volume/Open Interest
Panel A: TX				
Mean	0.0242	2.1695	26509	1.0527
Median	0.0254	0.4864	25354	0.9939
Std. Dev.	1.4733	5.2538	11792	0.4270
Q(12)	30.573	225.84	3768.1	3004.1
A.D.F.	0.0000	0.0000	0.0000	0.0000
Panel B: TE				
Mean	0.0063	3.1627	4262	0.7192
Median	0.0266	0.7373	3987	0.6680
Std. Dev.	1.7791	6.8899	1885	0.3238
Q(12)	31.950	239.62	3134.4	1501.8
A.D.F.	0.0000	0.0000	0.0000	0.0000
Panel C: TF				
Mean	0.0294	2.5462	3754	0.6111
Median	0.0809	0.5729	2781	0.5346
Std. Dev.	1.5961	5.9491	3283	0.3469
Q(12)	18.358	149.01	7586.1	3004.1
A.D.F.	0.0000	0.0000	0.0045	0.0000

NOTE: The 5% critical value for $Q(12)$ is 21.03; A.D.F. is the unit root test of Augmented Dickey-Fuller test

The test results of Augmented Dickey-Fuller (A.D.F.) in Table 1 show that the 12 time series verified in Table 1 all have p-values less than 0.005 in the unit root tests. Hence, it can be found that all the three futures' rate of daily return, daily squared return, daily trading volume (V_t) and daily trading volume divided by open interest (VO_t) reject unit root, and are stationary. Ljung-Box's 1 % level of significance, the $Q(12)$ values for all the series of the three index futures' daily squared return, daily trading volume (V_t) and daily trading volume divided by open interest (VO_t). The TF future's daily squared return series is $Q(12) = 149.01$, less than other future's $Q(12)$. Therefore, the verification results indicate that the series of the daily squared

return, daily trading volume (V_t) and the daily trading volume divided by open interest (VO_t) are autocorrelated.

Table 2: Pairwise Granger Causality Tests for volatility and volume with seasonal effect

Futures	TX		TE		TF	
	H_{01}	H_{02}	H_{01}	H_{02}	H_{01}	H_{02}
$\sigma_t^2 = r_t^2$ and $V_t = V_t$	(3,1)		(3,0)		(3,0)	
	0.0001 ^a	0.0004 ^a	0.0019 ^a	0.0225 ^a	0.0000 ^a	0.0000 ^a
$\sigma_t^2 = r_t^2$ and $V_t = VO_t$	(5,0)		(3,1)		(2,1)	
	0.0029 ^a	0.0000 ^a	0.0614 ^a	0.0007 ^a	0.0000 ^a	0.0000 ^a

NOTE: p and q are optimal lag length in the parentheses (p,q);^a are p-value for null hypotheses of $H_{02} : \alpha_1 = \dots = \alpha_p = 0$ and $H_{01} : \beta_1 = \dots = \beta_p = 0$, respectively.

Table 2 reports the pairwise Granger causality test results of the return volatility and the volume of TX, TE and TF futures in Taiwan futures market after taking into consideration of the seasonal effect. The optimal lag lengths of p and q in it is determined by SIC. In equation (3), if p and q are of the optimal lag lengths, the then Min SIC. Inside each parentheses in Table 2 are the optimal lag length of p and q in the VAR-cum-FFF model of the index futures return volatility and the volume. After having selected optimal lag length, we conduct, by equation (4), Taiwan index futures return volatility and the trading volume pairwise Granger Causality tests. In case of the null hypothesis as $H_{02} : \alpha_1 = \dots = \alpha_p = 0$, by the Wald statistics results, it can be found out that all the p-values of the three futures are less than 0.005, no matter using the daily trading volume (V_t) or the daily trading volume divided by open interest (VO_t) as the volume (Vol_t). Therefore, the null hypothesis is rejected, and we support that the three index futures TX, TE and TF all have the return volatility Granger causing volume relationship. For the same reason, when the null hypothesis is $H_{01} : \beta_1 = \dots = \beta_p = 0$, by the Wald statistics verification results, it can be found out that the both the p-values of index futures TX and TF are less than 0.05. Therefore, the null hypothesis is rejected while the TE future's p-value at 0.0614 is slightly more than 0.05, although it is still less than 0.10. Hence, we still reject the null hypothesis. From the above results we find out that the daily return volatility and the volume of Taiwan index futures have bidirectional causality. Finally, we only verify whether the seasonal factor's component exists in the futures return volatility or the daily trading volume (daily trading volume divided by open interest). Table 3 illustrates the

estimated results by VAR-cum-FFF model of three index futures TX, TE and TF in Taiwan futures market. We set the null hypothesis as FFF do not exist and be zero for each verification in the model. We can tell from the verification results in Table 3 that the index futures' FFF concurrently exists in the futures return volatility or the daily trading volume (daily trading volume divided by open interest).

Table 3: F-tests examining TX, TE and TF futures with seasonal factor's component existing in VAR-cum-FFF model

Futures		TX	TE	TF
Models	Variables	F-value	F-value	F-value
VAR1	r_t^2	7.312*	7.701*	6.076*
	V_t	38.285*	22.864*	7.969*
VAR2	r_t^2	3.581*	4.388*	3.424*
	VO_t	19.695*	45.391*	42.830*

NOTE:*** indicates significance at the 1% level; null hypothesis: FFF is not exists, i.e. $H_0 : a_i = b_i = \phi_j = \theta_j = \delta_k = 0 \forall i, j, k$

At 1% level of significance, the F-statistic values of the three futures are all significant. Therefore, the null hypothesis is rejected again, meaning the seasonal factor's component may exist in the daily return volatility (r_t) and the daily volume (V_t or VO_t). That implies that FFF exist in the VAR-cum-FFF model.

4. Conclusions

This study is to verify the Granger causality of three Taiwan index futures' return volatility and trading volume between February 1, 2002 and February 9, 2007. We employ the variables of the daily return volatility and the volume to conduct the vector autoregressive model. An extraneous variable FFF is added in the model to test the impact of the seasonal factor on the return volatility or the volume of the futures. Our empirical results indicate that with seasonal factor being considered, the return volatility and the volume of the three index futures in Taiwan futures market TX, TE and TF all have bidirectional causality. In addition, we also find out that the seasonal factor's component exists concurrently in the volume and the return volatility. Therefore, the results of this study demonstrate that it is supported that the MDH version proposed by Andersen (1996) allows an seasonal factor's component be in the trading volume. We also find out that the index futures' daily return volatility has an seasonal factor's component at the same time. Such verification results seem to be of more relevance than the return volatility hypothesis by MDH.

In the end, we make comparison between the verification results of this study with the empirical findings by Luu and Martens (2003). Luu and Martens have only the single direction cause and effect relationship in support of daily squared return Granger causing trading volume. However, we support that the daily squared return and the trading volume have the bidirectional causality. The difference may be due to the different futures market being verified (we take the Taiwan market while Luu and Martens take the American market) However, from the perspective of a futures market with high hedging activities in this study, if we ignore the hidden seasonal factor's component in the daily return volatility and its effects on the return volatility, we may obtain different empirical verification results.

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