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Performance of CUSUM chart under Three Delta Control Limits and Six Delta Initiatives

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Abstract

A control chart is a graphical device for representation of the data for knowing the extent of variations from the expected standard. The technique of control chart was suggested by W.A. Shewhart of Bell Telephone Company based on three sigma limits. M. Harry, the engineer of Motorola has introduced the concept of six sigma in 1980. In six sigma initiatives, it is expected to produce 3.4 or less number of defects per million of opportunities. Moderate distribution proposed by Naik V.D and Desai J.M, is a sound alternative of normal distribution, which has mean and mean deviation as pivotal parameters and which has properties similar to normal distribution. Naik V.D and Tailor K.S. have suggested the concept of 3-delta control limits and developed various control charts based on this distribution. K.S.Tailor has introduced the concept of six-delta initiatives. In this paper an attempt is made to construct a control chart based on three delta limits and six delta initiatives for Cumulative sum(CUSUM). Suitable Table for mean deviation is also constructed and presented for the engineers for making quick decisions.

Keywords: *Cumulative, Process control, Moderate distribution, Mean deviation, Six Delta*.

1. Introduction

The technique of quality control was developed by W. A. Shewhart (1931). It was based on three sigma(3σ) control limits. The concept of six-sigma was introduced by Motorola by the engineer Mikel Harry in 1980. He developed methods for problem solving that combined formal techniques, particularly relating to measurement, to achieve measurable savings in millions of dollars. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities.

R.Radhakrishnan and P.Balamurugan (2010, 2011, and 2016) have developed six sigma based control charts for mean, Exponentially Weighted Moving Average (EWMA), X bar using standard deviation, sample standard deviation, range and moving averages and CUSUM.

Naik V.D and Desai J.M (2015) have proposed an alternative of normal distribution called moderate distribution, which has mean(μ) and mean deviation(δ) as pivotal parameters and which has properties similar to normal distribution. V.D.Naik and K.S.Tailor (2015, 2016) have suggested 3δ(3 mean deviation) control limits based on moderate distribution. On the basis of 3δ control limits, they have developed $\overline{X}$ -chart, R-chart, s-chart and d-chart. K.S.Tailor (2016) has also developed moving average and moving range chart and exponentially moving average chart under moderateness assumption.

Similar to six sigma concept, K.S.Tailor (2017) has introduced the concept of six delta initiatives. The six sigma concept is based on normality assumption and the control limits are determined by using standard deviation (σ -sigma) of the statistic, whereas the six delta concept is based on moderateness assumption and the control limits are determined by using mean deviation (δ -delta) of the statistic. In six sigma initiatives, it is expected to produce 3.4 or less number of defects per million of opportunities whereas in six delta initiatives, it is expected to produce 1.7 or less number of defects per million of opportunities. If the companies practicing Six Delta initiatives use the control limits, then no point fall outside the control limits because of the improvement in the quality of the process. K.S.Tailor (2017, 2018) has proposed X-bar chart associated with mean deviation, sample standard deviation (s) chart, sample mean deviation (d) chart and exponentially weighted moving average (EWMA) chart based on six delta initiatives.

Here an attempt is made to construct a control chart based on three delta limits and six delta initiatives for Cumulative sum (CUSUM) specially designed for the companies who want to apply Six Delta initiatives in their organization. Suitable Table for mean deviation is also constructed and presented for the engineers for making quick decisions.

2. CUSUM - Chart

Page (1954) introduced the CUSUM chart. The moving average and moving range charts take into account part of the previous data, but the CUSUM chart technique uses all the information available. It is one of the most powerful management tools available for the detection of trends and slight changes in data. This chart has an entirely different appearance from any Shewhart control chart. In place of the centerline and symmetrical control limits, a mask is constructed that incorporates a location pointer and two decision lines that angle away from it.

The calculation of the CUSUM score S_r is very simple and may be represented by the formula:

$$S_r = \sum_{i=1}^{r}(x_i - t),$$

Where S_r is the cusum score of the r^{th} sample, x_i is the result from the individual sample i (x_i may be a sample mean, $\overline{x_i}$) and t is called the target mean.

There are two basic techniques to represent CUSUM control chart; one is tabular or algorithmic CUMSUM and second is V-mask. Here an attempt is made to present CUSUM chart using V-mask technique under the assumption of moderateness instead of normality.

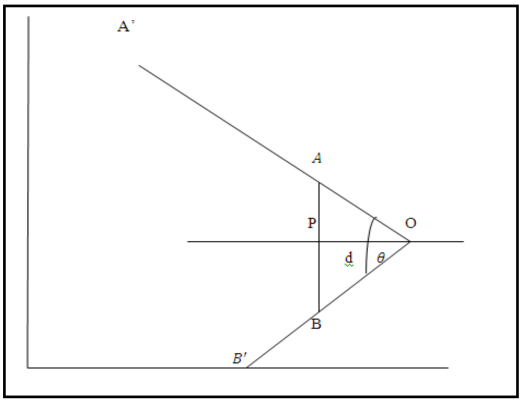
3. V- Mask under Moderateness Assumption

A. Barnard (1959) described a V-shaped mask which is useful to present the CUSUM chart. A V-mask is a V-shape overlay superimposed on the graph of the cumulative sum. The origin point of the V-mask is placed on top of the latest cumulative sum point so that past points can be examined to see if any point falls below or above the V-shape overlay. If all the past points lie inside the V-shape overlay, the process is said to be in control. Otherwise, even with one point lies outside the V-shape overlay, the process is suspected of being out of control.

Following are the parameters of V- mask. Suppose that the main variable of the process x follows **moderate distribution**. The mean of x is E(x) = μ and mean deviation of x is δ_x.

The design of the V-mask depends upon the lead distance d and the angle θ. This may be made empirically by drawing a number of masks and testing out each one on the past data. The lines AA' and BB' are considered to be L.C.L and U.C.L. respectively. The construction of V-mask is based on the mean error of the plotted points and its distribution and the average number of samples up to the point at which a signal occurs, i.e the ARL properties

Figure 1: V-mask



The formulas for d, the distance OP from the vertex of the angles between OP and the locator point P, and θ, the angle that each control limit makes with OP, are defined as

$$d = Op = E(\alpha)\left(\frac{\delta}{\sqrt{n}\,D}\right) = \frac{E(\alpha)}{\varepsilon^2}$$

$$\theta = tan^{-1}\left(\frac{D}{2y}\right),$$

where,

$E(\alpha)$ is a factor which is a function of the acceptable Type I error probability. Values of this factor for various levels of α are given in table 1. E(0.0167) corresponds to the Type I(α) error probability associated with standard 3-delta limits and E(1.7×10^{-6}) to the Type I(α) error probability associated with standard 6-delta limits.

Table 1: Function of the acceptable Type I error probability

Type I Error	0.10	0.05	0.02	0.01	0.0167	1.7×10^{-6}
$E(\alpha)$	5.991	7.378	9.210	10.597	9.571	27.956

$D = \epsilon \frac{\delta}{\sqrt{n}}$ is the actual value of the shift magnitude, either plus or minus, that must be detected with virtual certainty.

$\frac{\delta}{\sqrt{n}}$ is the mean deviation of sample averages which is estimated from $\bar{R}$ or $\bar{s}$

y is a scaling factor related to the geometry of the control chart and the mask dimensions.

4. CUSUM - chart based on Six Delta Initiatives:

Let us assume that the main variable of interest follows moderate distribution. We can establish control limits equivalent to six delta limits by stipulating that a shift of magnitude D should be detected with probability 1.7×10^{-6}. The scaling factor ($V_{6\delta}$) and magnitude of shift ($D_{6\delta}$) in six delta initiatives are defined as,

$V_{6\delta} = 2.0\delta_{6\delta}$, where $\delta_{6\delta} = \frac{\delta}{\sqrt{n}}$ and $D_{6\delta} = \epsilon\delta_{6\delta}$

Two parameters are required to construct the V-mask, one is lead distance d and the other is angle θ, which can be calculated by using six-delta based factors. Fix the tolerance level (TL) and process capability (C_p) to determine the process mean deviation (δ). With the help of this δ, various parameters of V-mask are calculated. For a specified TL and C_p of the process, the values of δ is calculated and presented in table 2.

5. An Empirical Study for CUSUM chart and Comparison of Three Delta Limits against Six Delta Initiatives

To illustrate CUSUM chart with three delta and six delta limits, a data set is taken from E .L. Grant and R. S. Leavenworth (1988) which is given in table 2. Various parameters of V-mask are computed under three sigma and six sigma initiatives from this data set, and CUSUM-charts are plotted under these two limits.

Table 2: Data set

Subgroup Number	Numbers of drawings	Average $\bar{X}$	Range R	$(\bar{X_i} - \bar{X_0})$	$\sum(\bar{X_i} - \bar{X_0})$
1	1-4	39.50	15	9.50	9.50
2	5-8	33.50	1	3.50	13.00
3	9-12	33.25	3	3.25	16.25
4	13-16	26.00	35	-4.00	12.25
5	17-20	34.00	17	4.00	16.25
6	21-24	28.50	18	-1.50	14.75
7	25-28	32.75	22	2.75	17.50
8	29-32	29.25	31	-0.75	16.75
9	33-36	29.25	15	-0.75	16.00
10	37-40	26.00	19	-4.00	12.00
11	41-44	24.75	29	-5.25	6.75
12	45-48	27.25	18	-2.75	4.00
13	49-52	30.25	9	0.25	4.25
14	53-56	30.00	34	0.00	4.25
15	57-60	24.00	10	-6.00	-1.75
16	61-64	20.00	29	-10.00	-11.75
17	65-68	29.50	22	-0.50	-12.25
18	69-72	27.00	6	-3.00	-15.25
19	73-76	28.75	16	-1.25	-16.50
20	77-80	24.25	19	-5.75	-22.25
21	81-84	27.25	18	-2.25	-24.50
22	85-88	32.75	34	2.75	-21.75
23	89-92	30.00	28	0.00	-21.75
24	93-96	33.25	19	3.25	-18.50
25	97-100	33.00	20	3.00	-15.50

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(a) Three delta control limits for CUSUM-chart:

The CUSUM-chart is constructed by using CUSUM score S_r and the sample number. From table 1, it can be seen that $S_r = \sum(\overline{X_i} - \overline{X_0})$, where $\overline{X_i}$ is the sample mean calculated from 4 given sample, $\overline{X_0}$ is the target mean, which is taken as 30. $\bar{R} = 19.48$.

Now the process mean deviation is $\hat{\delta} = \sqrt{\frac{2}{\pi}}\frac{\bar{R}}{d_2} = \sqrt{\frac{2}{\pi}}\frac{19.48}{2.059} = 7.55$

Hence $\delta_{\bar{x}} = \frac{\hat{\delta}}{\sqrt{n}} = \frac{7.55}{2} = 3.77$. If the desired shift in mean deviation is taken as 1.0, then

Lead distance

$$d = \frac{E(\alpha)}{\varepsilon^2} = E(0.0167)\left(\frac{\delta_{\bar{x}}}{D}\right)^2 \text{mm}$$

$$= 9.571\left(\frac{3.77}{7.5}\right)^2 = 2.42$$

Now, the angle

$$\theta = tan^{-1}\left(\frac{D}{2y}\right) = tan^{-1}\left(\frac{7.5}{2\times4}\right)$$

$$= tan^{-1}\left(\frac{7.5}{8}\right)$$

$$= 43^0$$

(b) Control limits based on six delta initiatives for CUSUM-chart:

For a given data set USL = 39.50, LSL = 20.00, TL = 19.50 and C_p = 2.5, then the value of δ = 0.73, which is found from the table 3. And $\delta_{6\delta} = \frac{\hat{\delta}}{\sqrt{n}} = \frac{0.73}{2} = 0.37$. Hence the parameters for V-mask based on six delta initiatives are calculated as:

$$d = \frac{E(\alpha)}{\varepsilon^2} = E(1.7\times10^{-6})\left(\frac{\delta_{6\delta}}{D_{6\delta}}\right)^2$$

$$= 27.956\left(\frac{0.37}{1.03}\right)^2$$

$$d = 3.61$$

And, the angle

$$\theta = tan^{-1}\left(\frac{D_{6\delta}}{2v_{6\delta}}\right) = tan^{-1}\left(\frac{1.03}{2\times0.73}\right)$$

$$= tan^{-1}\left(\frac{1.03}{1.46}\right)$$

$$= 35.2^0$$

Table 3: Values of δ for a specified C_p and TL

C_p \ TL	15.5	16.5	17.5	18.5	19.5
1.0	1.46	1.55	1.65	1.74	1.83
1.1	1.32	1.41	1.50	1.58	1.67
1.2	1.21	1.29	1.37	1.45	1.53
1.3	1.12	1.19	1.27	1.34	1.41
1.4	1.04	1.11	1.18	1.24	1.31
1.5	0.97	1.03	1.10	1.16	1.22
1.6	0.91	0.97	1.03	1.09	1.15
1.7	0.86	0.91	0.97	1.02	1.08
1.8	0.81	0.86	0.91	0.97	1.02
1.9	0.77	0.82	0.87	0.92	0.96
2.0	0.73	0.78	0.82	0.87	0.92
2.1	0.69	0.74	0.78	0.83	0.87
2.2	0.66	0.71	0.75	0.79	0.83
2.3	0.64	0.67	0.72	0.76	0.80
2.4	0.61	0.65	0.69	0.72	0.76
2.5	0.58	0.62	0.66	0.70	0.73

(c) s-charts for data set given in table 1 based on three delta and six delta limits:

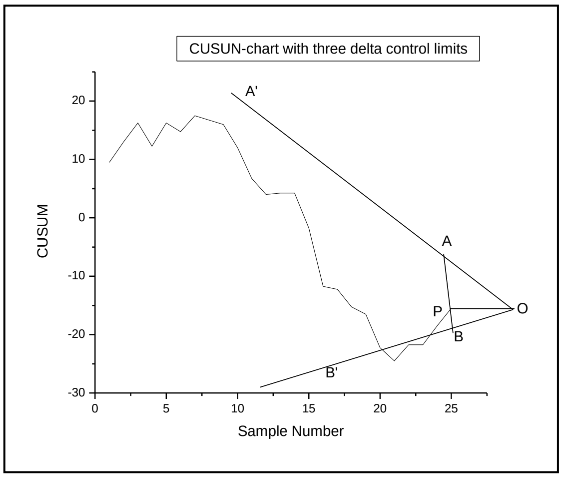

Figure 2: CUSUM-chart with 3-delta limits

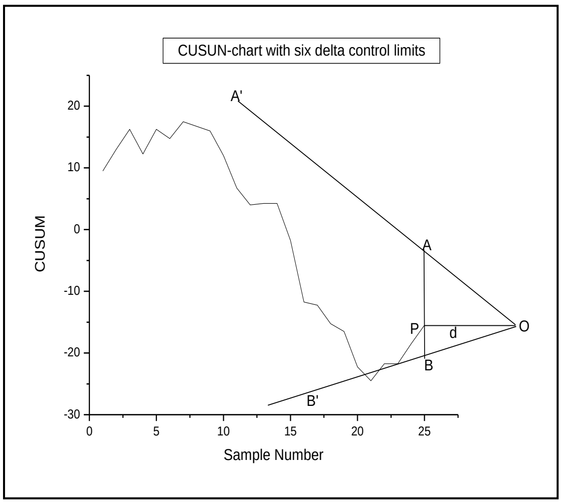

Figure 3: CUSUM-chart with 6-delta limits

6. Summary and Conclusion

In this paper, CUSUM-chart using V-mask is discussed under three delta and six delta control limits with an illustration. From figure 2, it can be seen that many points are falling outside the V-mask and hence the production process is out of statistical control when we are applying 3-delta control limits. Also from figure 3, it can be seen that the process is out of the statistical control when we are using six-delta based control limits. It is very clear from the comparison that there are less points falling out the control limits in six delta initiatives than three delta control limits. So it can be concluded that the chart under six delta control limits are more effective towards detecting the shift in the value of sample mean deviation than the charts under three delta control limits. This is a next generation control chart technique and it can replace existing six sigma technique. So it is recommended that the control charts under six delta control limits should be used for the best results.

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