IJASRM
International Journal of Advanced Scientific Research and Management, Volume 3 Issue 11, Nov 2018
www.ijasrm.com
ISSN 2455-6378

Algorithm and Matlab Program for Software Reliability Growth Model Based on Weibull Order Statistics Distribution

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Abstract

In this paper, a new software reliability growth model based on Weibull Order Statistics distribution is developed based on Non-Homogeneous Poisson Process. A set of data is allowed to follow Weibull Order Statistics distribution and it tested with the model. Algorithm and Matlab program are also given in this paper to execute this model.

Keywords: *Weibull Order Statistics distribution, Software Reliability Growth Model (SRGM), Non-Homogeneous Poisson Process (NHPP), Unconstrained Optimization Technique, Statistics Process Control.*

1. Introduction

Reliability of a Software [6] is defined as the probability that a software system works without failure occurring on specified operating conditions for a specified amount of time. Assessing the reliability of a software and thereby maintaining software quality during software development and software usage is most mandatory. Software Reliability Growth Models (SRGM) can be applied to analyze reliability of software. These models detect the software failure which can be eradicated and therefore the life time of the software ascends which hence increases the reliability of the software too. If a random variable X is allowed to follow Weibull distribution, it's density function is given by

$$f(x)=\frac{\delta}{\eta}\left(\frac{x}{\eta}\right)^{\delta-1} e^{-(x/\eta)^{\delta}}$$

where $x\in[0,\infty)$, $\delta>0$ is the shape parameter, $\eta>0$ is the scale parameter and whose corresponding cumulative distribution function is $F(x)=(1-e^{-(x/\eta)^{\delta}})$. Let us suppose that (X_1, ..., X_n) are n jointly distributed random variables. The X_i's are arranged in increasing order is its corresponding order statistics. Thus $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$. An independent and identically distributed sample from an absolutely continuous distribution with density $f(x)$ has the joint density function of the order statistics [5] as

$$f_{X_{1:n},\ldots,X_{n:n}}(x_1,x_2,\ldots,x_n)=n!\prod_{i=1}^{n} f(x_i), -\infty<x_1<x_2<\ldots<x_n<\infty$$

Non-Homogeneous Poisson Process (NHPP) models or fault counting models can be categorized as finite and infinite failure models depending on the specification. The number of failures in this model follows NHPP distribution. The intensity function of failure $\lambda(x)$ is defined as $\lambda(x)=af(x)$ where 'a' is the number of failures expected and $f(x)$ is the probability density function of X. Based on NHPP assumptions, Mean value function is $m(x) = aF(x)$ where $F(x)$ is the cumulative distribution function of X and $a=\frac{n}{F(x_n)}$.

Monitoring the failure occurrence process using the time chart is straightforward [7]. The exact probability limits are used to calculate the control limits. The upper control limit, UCL_r, the central line, CL_r and lower control limit, LCL_r can be easily calculated using

$$F(UCL_r,r,\lambda)=1-\alpha/2$$
$$F(CL_r,r,\lambda)=0.5$$
$$F(LCL_r,r,\lambda)=\alpha/2$$

where α is the accepted false alarm risk, if the random variable is taken as representing inter failure time of a device, a control chart for such a data would be based on 0.9973 probability limits of the failure times. These limits and the central line are respectively the solutions of

$$F(UCL_r, r, \lambda) = 0.99865$$
$$F(CL_r, r, \lambda) = 0.5$$
$$F(LCL_r, r, \lambda) = 0.00135$$

If the plotted point falls below the LCL, it indicates that the process average or the failure occurrence that may have increased which results in a decrease in the failure time. This means that process may have deteriorated and thus actions should be taken to identify the causes, which may be removed.

Goel and Okumoto [9] presented Goel and Okumoto imperfect debugging model which is a stochastic model based on Poisson Process with Non-Homogeneity (NHPP). Akilandeswari V.S., Poornima R. and Saavithri V. developed a reliability growth model of a software based on Lehmann-type Laplace distribution-I[2]. To test Reliability of a Software Akilandeswari V.S., Poornima R. and Saavithri V. used Lehmann-Type Laplace distribution Type II (LLD-II)[1] SRGM which had a better fit for software failure data than Goel-okumoto, Weibull, Exponential Geometric, Pareto III, Lehmann-Type Laplace distribution Type I (LLD-I)distributions. They also developed Lehmann-Type Laplace distributions –Type I and Type II[3,4] software reliability growth models too.

In this paper, reliability growth model of a software is developed in section 2 with its parameter estimation. Algorithm and Matlab program are also given in this section. In section 3, software failure data analysis is performed and paper is concluded in section 4.

2.Weibull Order Statistics Growth Model

Let X_1, X_2, ..., X_n be the random variables representing a sample of cumulative time between failures. Let $X_{1:n}$, $X_{2:n}$, ..., $X_{n:n}$ be the original random variables so that $X_{1:n} \leq X_{2:n} \leq$ ..., $\leq X_{n:n}$ called the order statistics.

The probability density function of Weibull r^{th} order statistics is given by

$$f_{r:n}(x) = r\binom{n}{r}\frac{\delta}{\eta}\left(1-e^{-\left(x/\eta\right)^{\delta}}\right)^{r-1}\left(\frac{x}{\eta}\right)^{\delta-1} e^{-(n-r+1)\left(x/\eta\right)^{\delta}} \quad \text{... (2.1)}$$

where $x \in [0,\infty),\ \delta > 0,\ \eta > 0,\ 1 \leq r \leq n$

The cumulative distribution function is

$$F_{r:n}(x) = \sum_{i=r}^{n}\binom{n}{i} e^{-(n-i)(x/\eta)^{\delta}}\left(1-e^{-(x/\eta)^{\delta}}\right)^{i} \quad \text{... (2.2)}$$

2.1. Parameter estimation

Method of Maximum likelihood is used to estimate η and δ.

The likelihood function of Weibull order statistics is

$$l = \prod_{i=1}^{n} r\binom{n}{r}\frac{\delta}{\eta}\left(1-e^{-\left(x_i/\eta\right)^{\delta}}\right)^{r-1}\left(\frac{x_i}{\eta}\right)^{\delta-1} e^{-(n-r+1)\left(x_i/\eta\right)^{\delta}} \quad \text{... (2.3)}$$

The log-likelihood function is

$$\log l = \log\left(\prod_{i=1}^{n} r\binom{n}{r}\frac{\delta}{\eta}\left(1-e^{-\left(x_i/\eta\right)^{\delta}}\right)^{r-1}\left(\frac{x_i}{\eta}\right)^{\delta-1} e^{-(n-r+1)\left(x_i/\eta\right)^{\delta}}\right) \quad \text{... (2.4)}$$

Using unconstrained optimization technique, the minimum of $\log l$ is found.

2.2 NHPP model for Weibull order statistics SRGM

The mean value function for this SRGM, using (2.2), is

$$m(x) = a\sum_{i=r}^{n}\binom{n}{i} e^{-(n-i)(x/\eta)^{\delta}}\left(1-e^{-(x/\eta)^{\delta}}\right)^{i} \quad \text{... (2.5)}$$

The intensity value function, using (2.1), is

$$\lambda(x) = ar\binom{n}{r}\frac{\delta}{\eta}\left(1-e^{-(x/\eta)^{\delta}}\right)^{r-1}\left(\frac{x}{\eta}\right)^{\delta-1} e^{-(n-r+1)(x/\eta)^{\delta}} \quad \text{... (2.6)}$$

a, the expected number of failures

$$= \frac{n}{\sum_{i=r}^{n}\binom{n}{i} e^{-(n-i)(x_n/\eta)^{\delta}}\left(1-e^{-(x_n/\eta)^{\delta}}\right)^{i}} \quad \text{... (2.7)}$$

2.3 Algorithm for Weibull Order Statistics SRGM

Step 1: Find the cumulative data of the time between failures

Step 2: Choose the value of r

Step 3: Using minimization techniques of non-linear unconstrained objective function, find - $\log l$ in (2.4).

Step 4: $Max\, f(z) = -Min(-f(z))$, using this, find the maximum of $\log l$ multiplying the value by (-1). The values of η and δ that gives the maximum of

IJASRM International Journal of Advanced Scientific Research and Management, Volume 3 Issue 11, Nov 2018
www.ijasrm.com
ISSN 2455-6378

$\log l$ are the optimum values of η and δ.

Step 5: Calculate the expected number of failures in (2.7) using these parameters

Step 6: Find the control limits UCL, LCL and CL

Step 7: Estimate the mean value function in (2.5) at all failure numbers.

Step 8: Then, find the successive differences of mean value functions

Step 9: Plot the mean value chart taking failure numbers along X-axis and successive differences along Y-axis

Step 10: The failure numbers at which the mean value function is below LCL, detects the failure of the software.

2.4 MATLAB Program for Weibull Order Statistics SRGM

```
global r n x
r=value of r
y0=[initial value for parameters];
d=0;sd=0;
options=optimoptions(@fminunc,'Algorithm','quasi-newton');
[y,fval,exitflag,output] = fminunc(@weibull_os,y0,options);
F=1-exp(-(x(n)/y(2))^y(1));
sum1=0;
for i=r:n
   sum1=sum1+nchoosek(n,i)*(F^i)*((1-F)^(n-i));
end
a=n/sum1;
for j=1:n
   osF=0;
   F=1-exp(-(x(j)/y(2))^y(1));
   for i=r:n
         osF=osF+(nchoosek(n,i)*(F^i)*((1-F)^(n-i)));
    end
    m=a*osF;
    d=[d m];
    dummy=d(j+1)-d(j);sd=[sd dummy];
end
UCL=0.99865*a;LCL=0.00135*a;CL=0.5*a;
hold on
plot(sd(:,3:n+1));
lcl=LCL;flag1=lcl;ucl=UCL;flag2=UCL;cl=CL;flag3=cl;
for i=1:n
   lcl=[lcl flag1];
   ucl=[ucl flag2];
   cl=[cl flag3];
end
plot(lcl);plot(ucl);plot(cl);

function fun=weibull_os(y)
global n x r
u=[data]
n=length(u);
cum=u(1);sum1=u(1);
for i=2:n
   sum1=sum1+u(i);
   cum=[cum sum1];
end
x=cum;
n=length(x);
prod=1;
for i=1:n
    F=1-exp(-(x(i)/y(2))^y(1));
    f=(y(1)/y(2))*((x(i)/y(2))^(y(1)-1))*exp(-(x(i)/y(2))^y(1));
    os=(factorial(n)/(factorial(r-1)*factorial(n-r)))*(F^(r-1))*((1-F)^(n-r))*f;
    prod=prod*os;
end
fun=-log(prod);
```

3. Weibull order statistics SRGM Dataset

3.1 Cumulative Time between Failures

Failure Number	Time between failure times in CPU units	Cumulative time between failures
1	5.5	5.5
2	1.83	7.33
3	2.75	10.08
4	70.89	80.97
5	3.94	84.91
6	14.98	99.89
7	3.47	103.36
8	9.96	113.32
9	11.39	124.71
10	19.88	144.59
11	7.81	152.4
12	14.59	166.99
13	11.42	178.41
14	18.94	197.35
15	65.3	262.65
16	0.04	262.69
17	125.67	388.36
18	82.69	471.05
19	0.45	471.5
20	31.61	503.11
21	129.31	632.42
22	47.6	680.02

The following result was obtained for the dataset when tested using Weibull order statistics SRGM. Table 3.2 gives the maximum likelihood values of dataset when program 2.4 is run for all possible values of *r* for the above dataset.

ISSN 2455-6378

Table 3.2
Maximum likelihood values for Weibull Order Statistics distribution at all possible values of *r*

r	Maximum Likelihood Values
1	**-142.2293**
2	-142.5666
3	-142.9677
4	-143.2090
5	-143.4069
6	-143.6939
7	-144.1952
8	-145.0353
9	-146.3475
10	-148.2857
11	-151.0381
12	-154.8448
13	-160.0170
14	-166.9553
15	-164.8549
16	-157.7849
17	-144.3617
18	-144.4507
19	-144.5651
20	-144.7287
21	-145.0048
22	-145.6452

From Table 3.2, it is found that maximum likelihood value for dataset1 is -142.2293 which is obtained at I order statistics of Weibull. Thus the SRGM program 2.4 is run for $r = 1$ to test the failure detection. Table 3.1 gives the cumulative data between failures.
Parameters,

$$\delta = 0.0009 \times 10^3$$

$$\eta = 6.6076 \times 10^3$$

Expected number of failures, $a = 23.7552$
Table 3.3 gives the mean value function and its successive differences.

Control limits

$$UCL = 23.7231$$

$$LCL = 0.0321$$

$$CL = 11.8776$$

Table 3.3
Successive differences of mean value function

Failure Number	Mean value function $m(x)$	Successive differences of $m(x)$
1	0.6644	0.2017
2	0.8661	0.2943
3	1.1604	5.9173
4	7.0777	0.2668
5	7.3445	0.9698
6	8.3143	0.2151
7	8.5293	0.5985
8	9.1278	0.6519
9	9.7798	1.0613
10	10.8411	0.3922
11	11.2332	0.6980
12	11.9312	0.5166
13	12.4478	0.8035
14	13.2513	2.3319
15	15.5832	**0.0012**
16	15.5844	3.0791
17	18.6635	1.3416
18	20.0050	**0.0062**
19	20.0112	0.4100
20	20.4212	1.2504
21	21.6717	0.3283
22	22	-

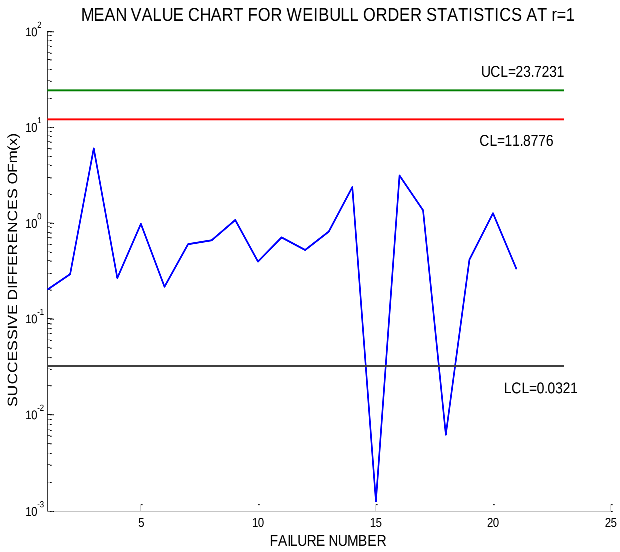

Figure 3.4

Figure 3.4 gives the mean value chart at I order statistics of Weibull SRGM. It is found from the graph that the failure numbers are detected at failure points 15 and 18.

4. Conclusion

Here reliability growth model of software is developed based on Order Statistics of Weibull distribution and it is tested for a set of data. Using unconstrained optimization technique, the parameters are estimated and evaluation of maximum likelihood values at all orders is done. At the I order, the maximum out of this is found and hence the software failure detection is done for I order of Weibull order statistics distribution and it is detected at two failure points 15 and 18.

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