Super Sum Vertex Labeling of Binary Tree

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Abstract

In this paper, we give optimal super vertex sum labeling of super subdivision of a crown, t-thorny ring and Perfect Binary Tree.

Keywords: *Sum labeling, sum number, sum graph, isolates*

1. Introduction

The graphs considered in the study are simple, finite and undirected. For all terminologies and notations which are not explained in detail, we refer [2] and [4]. In [5], Joseph and Kureethara introduced *super vertex sum labeling* and proved that the lower bound of $\sigma_{sv}(G)=2$. They also provided the optimal super vertex sum labeling for super subdivision of path, cycle, star and spider with $\sigma_{sv}(G)=2$. Further in [6], they obtained optimal super vertex sum labeling scheme for super subdivision of bi-star, path union of spider with $\sigma_{sv}(G)=2$ and algorithm to construct super vertex sum labeling of super subdivision of Caterpillar.

2. Review of Literature

Let G(p, q) be a graph and $f:V(G) \rightarrow \{1,2,\ldots,|V(G)|\}$ be a bijective mapping. For every pair of adjacent vertices u, v $\in$ V(G), let f(u)+f(v)=f(w) for some vertex w $\in$ V(G). Let $\mu_f(G)$ be the maximum of f(u) for vertex $\forall$ u $\in$ V(G). If $\mu_f(G)=|V(G)|$ then f is called *super vertex sum labeling*[5]. The least number of isolates needed to super vertex sum label the graph G is called *super vertex sum number* of the graph, denoted by $\sigma_{sv}(G)$[5]. The labeling with minimum number of isolates is called *optimal*. A graph that admits *super vertex sum labeling* is called as *super vertex sum graph* [5].

Let G be a graph with q edges. A graph H is called a *super subdivision* of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph K_{2, m_i} for some m_i, $1 \leq i \leq q$ in such a way that the end vertices of each e_i are identified with the two vertices of the 2-vertex part of K_{2,m_i} after removing the edge e_i from graph G. If m_i is varying arbitrarily for each edge e_i then super subdivision is called an *arbitrary super subdivision* of G [7].

A binary tree is a tree where each node has at most two children. A perfect binary tree is a tree where all nodes have exactly two children and all leaves are at the same depth. A perfect binary tree has $2^{n+1}-1$ nodes, where *n* is the height.

Any cycle with a pendent edge attached at each vertex is called crown [1]. The thorn graph of a graph G is obtained by attaching new vertices of degree one to each vertex of the graph G. If exactly t new vertices of degree one is attached to each vertex of G then it is called t-thorn graph [3].

3. Optimal Super Vertex Sum Labeling Scheme for Super Subdivision of Crown and t-thorny ring

In this section, we obtain optimal super vertex sum labeling for super subdivision of a crown and t-thorny ring.

Theorem 1 Super subdivision of crown is a super vertex sum graph with $\sigma_{sv}(sv)(G)=2$.

Proof

Let G(p, q) be the crown graph. Let 'n' be the number of vertices in the cycle of the graph G. Let H be the super-subdivision of G which is obtained by replacing every edge of G with $K_{2,m}$. The resulting

graph H has p+mq vertices and 2mq edges. Let u_1, u_2 be the isolates.
Define f: V(H)→{1, 2, 3, …, p+mq+2}.
Choose a vertex, say v_1, of degree 3m as the first vertex and label it as $f(v_1)=1$.
For i=1 to n
{if ($i \neq n$),
{for any unvisited (unlabelled) vertex 'v' at $d(v, v_i)$ = 2 and deg(v) = 3m,
$f(v_{i+1})$ =i+1.
for any unvisited (unlabelled) vertex 'v' at d (v, v_i) = 1 and $d(v, v_{i+1})$ = 1 and deg(v) = 2,
$f(v_{ij})$ = i+j+ (m+1) (q-i); j = 1to m;
i=i+1}
if (i=n),
{for any unvisited (unlabelled) vertex 'v' at $d(v,v_1)$ = 1 and $d(v,v_i)$ = 1 and deg(v) = 2
$f(v_{ij})$ = i+j+ (m+1) (q-i); j = 1 to m}
}.
For i = (n+1) to p,
{for any unvisited (unlabelled) vertex 'v' at $d(v, v_{i-n})$ = 2 and deg (v) = m,
$f(v_i)$ =i;
for any unvisited (unlabelled) vertex 'v' at $d(v, v_i)$=1 and deg (v) = 2,
$f(v_{ij})$ = i+j+ (m+1) (q-i); j = 1 to m;
i=i+1}
To label the isolates,
$f(u_1)$=p+mq+1 and
$f(u_2)$=p+mq+2.

Example 1
Super vertex sum labeling of super-subdivision of crown (with n=4 and m=4) is given in figure 1.

Theorem 2 Super subdivision of t - thorny ring is a super vertex sum graph with $\sigma_{sv}(G)=2$.

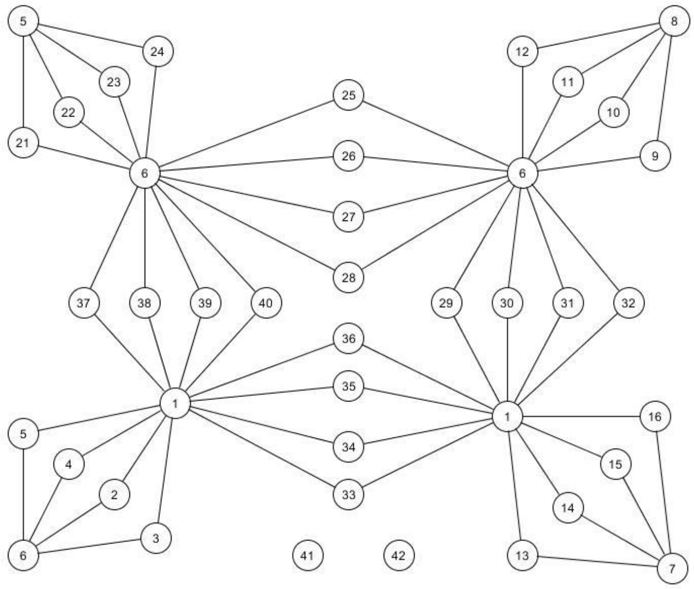

Fig. 1 Super vertex sum labeling of super-subdivision of crown (with n=4 and m=4).

Proof
Let G(p, q) be a t-thorny ring. (i.e., cycle with n vertices and t pendent vertices attached to each vertices of the cycle). G has n(t+1) vertices and edges. Let H be the super-subdivision of G which is obtained by replacing every edge of G with $K_{2,m}$. The resulting graph H has p+mq vertices and 2mq edges. Let u_1, u_2 be the isolates.
Define f: V(H) → {1,2,3,… , p+mq+2}.
Choose a vertex, say v_1, of degree (t+2)m as the first vertex and label it as $f(v_1)=1$.
For i = 1 to n,
{if ($i \neq n$)
{for any unvisited (unlabelled) vertex 'v' at $d(v, v_i)$=2 and deg (v) = 3m,
$f(v_{i+1})$ =i+1.
for any unvisited (unlabelled) vertex 'v' at $d(v, v_i)$= 1 and d (v,v_{i+1}) = 1 and deg (v) = 2,
$f(v_{ij})$ = i+j+ (m+1) (q-i); j = 1 to m;
i=i+1}.
if (i = n)
{for any unvisited (unlabelled) vertex 'v' at $d(v, v_1)$ = 1 and d (v, v_i) = 1 and deg (v) = 2,
$f(v_{ij})$ = i+j+ (m+1) (q-i); j = 1 to m}
}
i=n+;
For c = 1 to n ,
{for r = 1 to t
{for any unvisited (unlabelled) vertex 'v' at $d(v,v_c)$=2 and deg(v) = m,
$f(v_i)$ =i.
for any unvisited (unlabelled) vertex 'v' at $d(v,v_i)$=1 and deg(v) =2,
$f(v_{ij})$ = i+j+ (m+1) (q-i); j = 1 to m;
i=i+1}
}.

IJASRM
International Journal of Advanced Scientific Research and Management, Volume 3 Issue 12, Dec 2018
www.ijasrm.com
ISSN 2455-6378

To label the isolates,
$f(u_1)=p+mq+1$ and
$f(u_2)=p+mq+2$.
Hence, super subdivision of t - thorny ring is a super vertex sum graph with $\sigma_{sv}(G)=2$.

4. Algorithm for Optimal Super Vertex Sum Labeling of Super Subdivision of Perfect Binary Tree

In this section, we provide the algorithm for optimal super vertex sum labeling for super subdivision of Perfect Binary Tree.

Theorem 3 Super subdivision of Perfect Binary Tree is a super vertex sum graph with $\sigma_{sv}(G)=2$.

Proof

Let G(p, q) be a perfect binary tree with height(level) 'n'. It has $p=2^{n+1}-1$ vertices and $q=2^{n+1}-2$ edges. The number of vertices with degree 1 is 2^n, degree 2 is 1 and degree 3 is 2^n-2. By the definition of super subdivision, each edge of G is replaced by complete bipartite graph $K_{2,m}$. The resulting graph H has $2^{n+1}(m+1)-2m-1$ vertices and $4m[2^n-1]$ edges. Let u_1 and u_2 be the isolates necessary to super vertex sum label the graph. The algorithm to Super Vertex Sum Labeling the super subdivision of Perfect Binary Tree is as follows:
Get input of the number of level(height)of Binary Tree as 'n' and the value of 'm' for super subdivision.

Define $f:V(H)\rightarrow \{1,2,3,\ldots,[2^{n+1}(m+1)-2m-1]+2\}$ as follows.
Step 1: Choose the vertex with degree 2m as first vertex and label it as $f(v_0)=1$
Step 2: Initialize i = 1, L = 1
Step 3: if ($L\neq n$) go to Step 4; else Step 10
Step 4: Initialize x = 0
Step 5: if ($x\leq 2^L-1$) go to Step 6; else go to Step 9
Step 6: for any unvisited (unlabeled) vertex 'v' at $d(v,v_0) = 2L$ and deg(v) = 3m or $mf(v_i) = 2^L + x$
Step 7:
for j = 1 to m
{for any unvisited (unlabelled) vertex 'v' at $d(v,v_i)=1$, $d(v,v_0) = 2L-1$ and deg(v)=2
f (vij) = i+1+j+ (m+1) (2}^{n+1-2-i)}
Step 8: increment i = i+1 ; x = x+1; go to step 5
Step 9: increment L = L+1; go to Step 3
Step 10: Label the isolates as $f(u_1)=2^{n+1}(m+1)-2m$ and $f(u_1)=2^{n+1}(m+1)-2m+1$
Step 11: Stop

Example 2
Super vertex sum labeling of super-subdivision of perfect binary tree (with n=3 and m=3) is given in figure 2.

5. Conclusion

In this paper, we provided optimal super vertex sum labeling for the super subdivision of perfect binary tree. For further studies, optimal super vertex sum labeling for other family of trees, the optimal super vertex sum number for arbitrary super subdivision of the same graphs can be explored.

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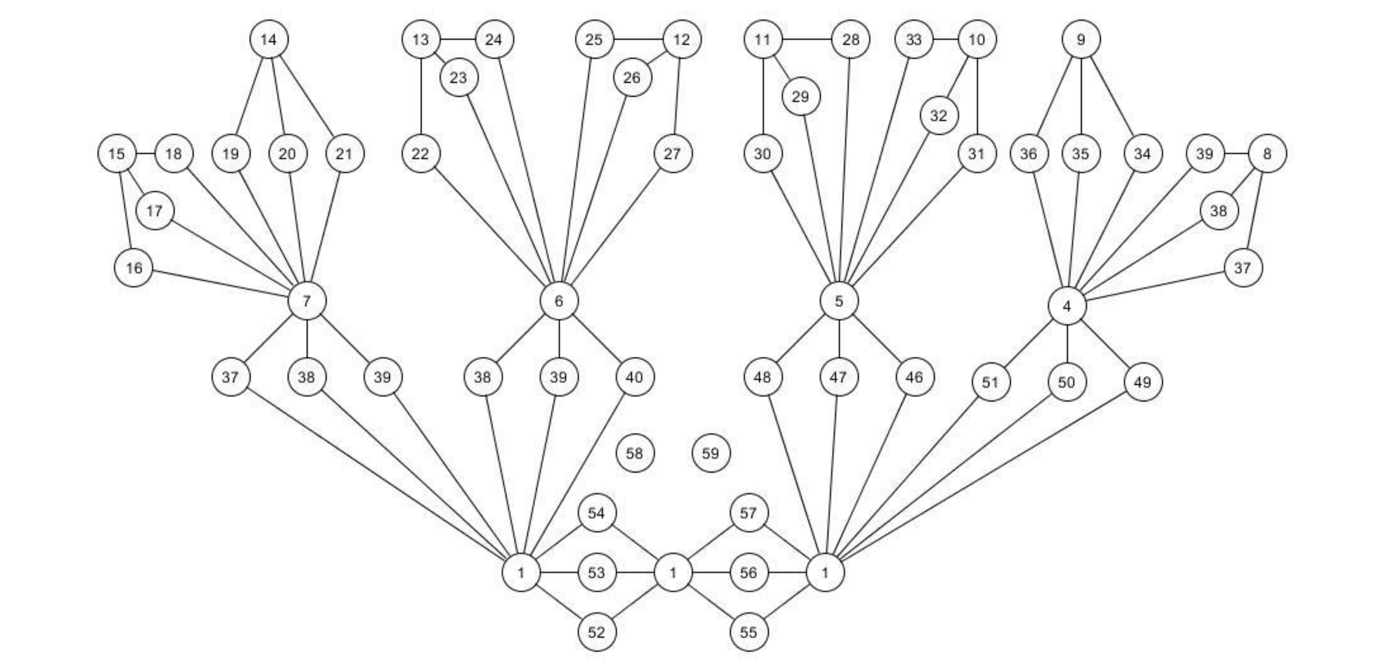

Fig. 2 Super vertex sum labeling of super-subdivision of perfect binary tree (with n=3 and m=3).

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