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Elastic Effect of Two Different Sized Spherical Particles by Particle – Particle Contact Using Discrete Element Method

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Abstract
The objective of this contribution is to exist a numerical simulation technique to model the collision of particles in a plane using object-oriented procedures. This approach is based on the second law of Newton for the translational motion of each particle in the granular material. This comprises a possession track of all forces acting on each particle at every time-step. The back-ground version of DEM and time integration algorithm are established and executed into C++ code. A simple test is regarding the application of time- integration algorithm by particle-particle interaction for which analytical expression exist. In this paper elastic effect due to particle – particle contact on different sized particles are investigated.

Keywords: DEM simulation, Granular materials, Elastic effect, damping effect;

1. Introduction

A granular material is a conglomeration of many discrete solid particles which is characterized by a loss of energy due to dissipative collisions whenever the particles interact. They can be considered as the fourth state of matter which is very different from solids, liquids or gas. Granular materials are very simple. If the particles are non-cohesive, then the forces between them are basically only repulsive so that the shape of the material is determined by external boundaries and gravity. Practically many solid particles which we make use of in the kitchen are granular particles like sugar, rice, coffee, cereals etc. Walking outside we step on the soil which is again a particulate and hence falls under the category of granular matter. The unusual and unique character displayed by granular material systems have led to a resurgence of interest within several scientific and engineering disciplines ranging from physics, soil mechanics and chemical engineering (Jaeger and Nagel [1992]; Behringer [1995]; Bideau and Hansen [1993]; Jaeger et al. [1994, 1996a]). Much of the engineering literature has been dedicated to understanding how to deal with these materials. Prominent contributions in the literature include Coulomb [1773], who proposed the ideas of static friction; Faraday [1831], who discovered the convective instability in a vibrated container filled with powder, and Reynolds [1885], who introduced the motion of dilatancy, which implies that a compacted granular material must expand in order for it to undergo shear. Processes involving particulate or granular flows are prevalent throughout the pharmaceutical, chemical, energy, food handling, mineral processing, powder metallurgy, and mining industries. In addition, numerous phenomena found in nature involve such material flows.

3. Geometry of Kinematic contact of spherical particles.

Consider two particles i and j be in contact with position vectors x_i and x_j with center of gravity lying at O_i and O_j having linear velocities v_i and v_j .

Figure1. Inter particle Contact between two identical spheres *i* and *j*

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The contact point C_{ij} is defined to be at the center of the overlap area position vector x_{cij}.The vector x_{ij} of the relative position point from the center to gravity of particle *i to* that of particle *j* is defined as $x_{ij} = x_i - x_j$ The depth of overlap is h_{ij}. Unit vector in the normal direction of the contact surface through the center of the overlap area is denoted by n_{ij}. It extends from the contact point to the inside of the particle *i* as $n_{ij} = -n_{ji}$

Since the particle shape is assumed to be spherical, for sphere of any dimension the contact parameters can be written as follows:

$$h_{ij} = \begin{cases} R_i + R_j - |x_{ij}|, |x_{ij}| < R_i + R_j \\ 0, |x_{ij}| \geq R_i + R_j \end{cases}$$

Where R_i is the radius of the particle. The relative velocity of the contact point is defined as

$$v_{ij} = v_{cij} - v_{cji}$$

4. Inter particle contact force.

The contact force between particles can be expressed as the sum of normal and tangential components;

$$F_{ij} = F_{n,ij} + F_{t,ij}$$

The normal component of contact force between particles can be expressed as the sum of elastic force and viscous force.

$$F_{n,ij} = F_{n,ij,elastic} + F_{n,ij,viscous}$$

Normal elastic force is based on the linear Hooke's law of a spring with a spring stiffness constant $k_{n.ij}$ and is given by the expression,

$$F_{n,ij,elastic} = K_{n,ij} h_{ij} n_{ij}$$

Normal viscous force is dissipated during real collisions between particles. The linear dependency of force on the relative velocity of the particles at the contact point with a constant normal dissipation coefficient γ_n and is expressed as

$$F_{n,ij,viscous} = -\gamma_n m_{ij} v_{n,ij}$$

5. Computer Implementation

The key computational tasks of DEM at each time step of contact particle can be summarized as follows:

- Finding of contacts between a particle *i* and *j*.
- Calculation of contact forces from relative displacement between particles
- Summary of contact forces to determine the total unbalanced force
- Computation of acceleration from force
- Velocity and displacement by integrating the acceleration
- Updating the position of particles

6. Result and Discussion

6.1. Elastic Impact of different sized Particle – Particle contact using Discrete Element Method

This test simulates the different sized particles collide with each other, and thus particle-particle contact force are formed. Here test confirms the particle-particle interaction of unlike particle with normal stiffness parameter is 3×10^5 units. The impact of different sized particle, impact of particles with same radius and different mass and particles with different radius and same mass are examined. And in each case effect of force with respect to change in diameter and also time, kinetic energy and displacement with respect to time are calculated. When these particles collide with each other, and thus particle-particle contact force are formed.

6.1.1. Effect of Elastic Force with respect to displacement

6.1.1.1. Effect of force on different sized particles

Considers two different sized particles with different radius and mass. The incoming velocity magnitude of first particle was set at 1 m/s and the initial position is 5m

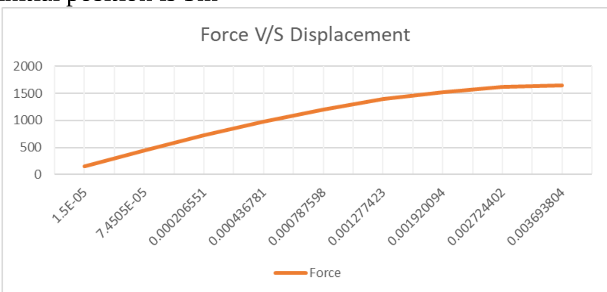

Figure2. Effect of Elastic force on particle with change in Displacement

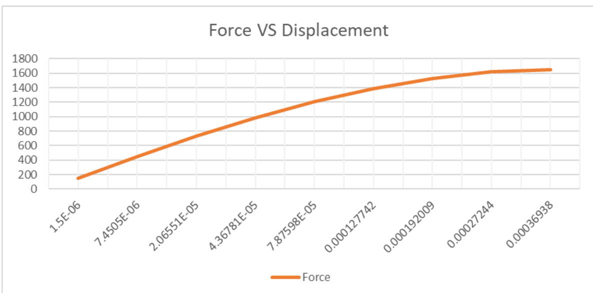

Figure3. Effect of Elastic force on particle with change in Displacement

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If the particles have different radius and mass but these two particles attain same elastic force with respect to displacement. In both case particles attain same effect of elastic force with respect to displacement. Thus change in radius and mass does not change the elastic force on a particle. From the result we can say that elastic force with respect to displacement has no effect on collision with larger particle collide the smaller particle or smaller particle collide the larger particle in both case we get the same elastic force.

6.1.1.2. Elastic effect of Particles with same radius and different mass, same mass and different radius

Case1: Particles with same radius but different mass

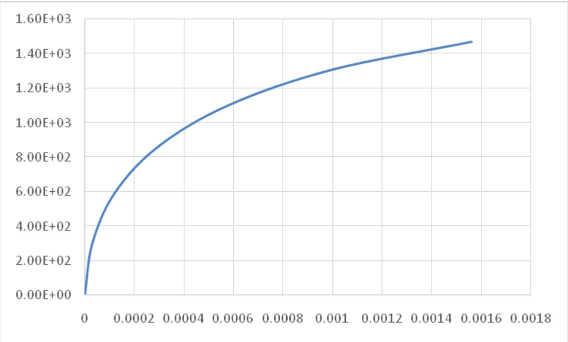

Figure4. Effect of Elastic force on particle with change in Displacement

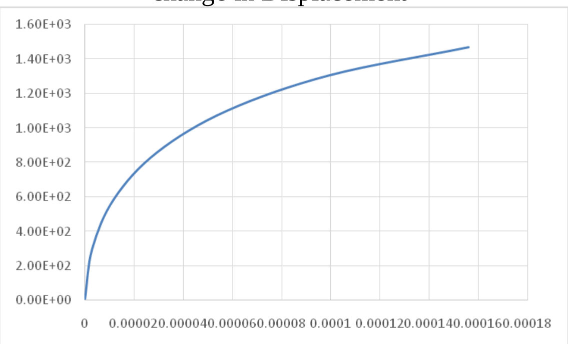

Figure5. Effect of Elastic force on particle with change in Displacement

From the result we can say that elastic force with respect to change in displacement on particles with different radius and same mass, by considering first with particle larger radius particle collide with smaller radius particle and second with smaller radius particle collide with larger radius particle in both case we get same elastic force. Here in both case we get same elastic force with respect to displacement.

Case2: Particles with same mass but different radius

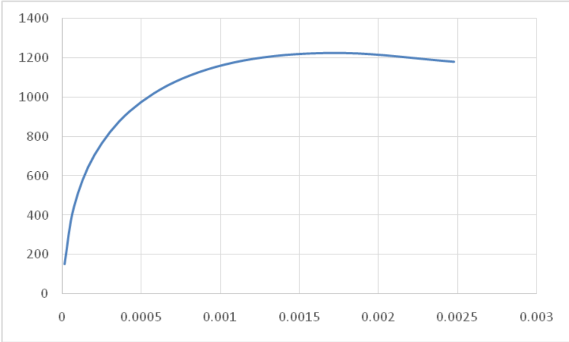

Figure6. Effect of Elastic force on particle with change in Displacement

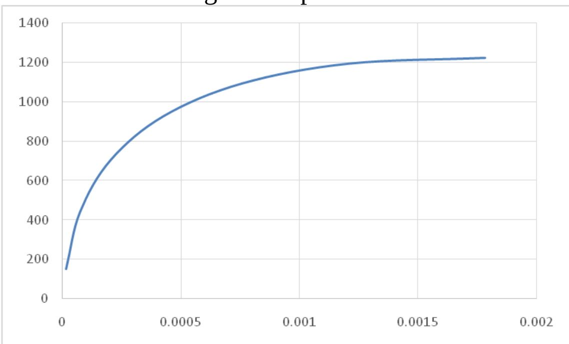

Figure7. Effect of Elastic force on particle with change in Displacement

Particles with same radius and different mass and particles with different mass and same radius in both case elastic forces with respect to displacement are same. But mass vary then there is a change in displacement but we get same elastic force for particles. From the result we can say that elastic force with respect to displacement has no effect on collision with larger mass particle collide the smaller mass particle or smaller mass particle collide the larger mass particle. In both case we get the same elastic force. In both cases displacement change but the force didn't change with respect to displacement. In both case displacement are not same but we get same force.

From the two cases we can say that change in radius of the colliding particle does not affect the displacement with respect to the elastic force on a particle. But change in mass on a particle affect the change in displacement with respect to the elastic force on a particle.

6.1.2. Elastic Effect of Force with respect to Time by different sized particle- particle contact

6.1.2.1. Effect of Force on different sized particles

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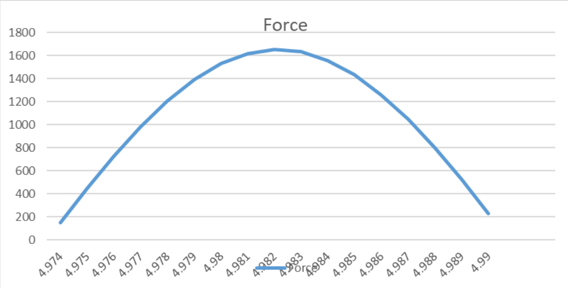

Figure8. Elastic normal impact of particle with change in time

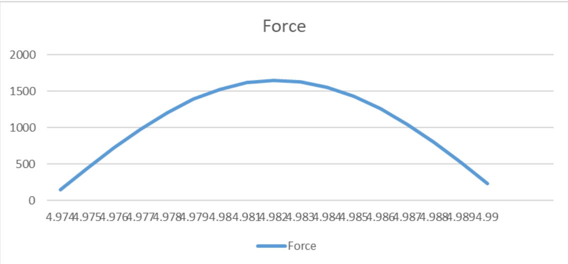

Figure9. Elastic normal impact of particle with change in time

When we consider first particle as smaller and second particle as larger, when these particles collides then the force with respect to time on both the particles are same. From the result we can say that elastic force with respect to change in time has no effect on collision with larger particle collide the smaller particle or smaller particle collide the larger particle in both case we get the same elastic force. Elastic force with respect to time step in both case are same.

6.1.2.2. Elastic effect of Particles with same radius and different mass, different radius and same mass

Case1: Particle with same radius but different mass

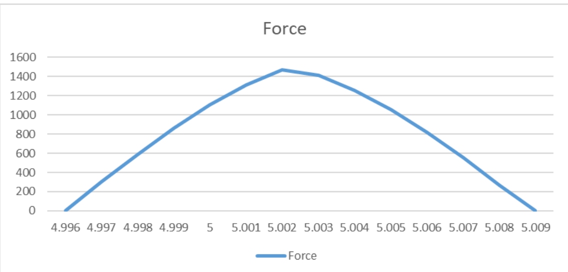

Figure10. Elastic normal impact of particle with change in time

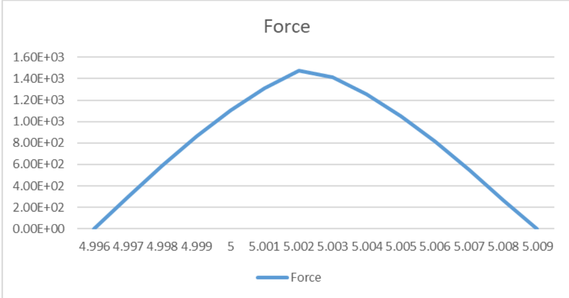

Figure11. Elastic normal impact of particle with change in time

Case 2: Particle with same mass but different radius

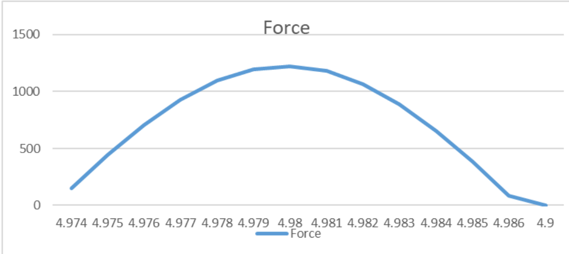

Figure12. Elastic normal impact of particle with change in time

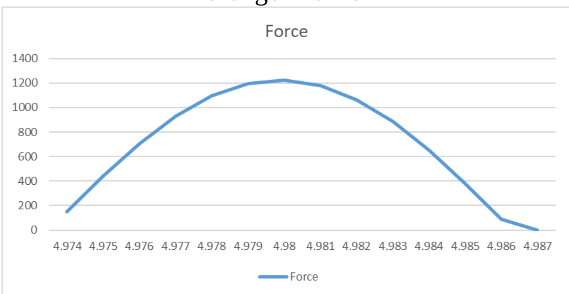

Figure13. Elastic normal impact of particle with change in time

From the result we can say that elastic force with respect to change in time on particles with same radius and different mass, by considering first particle with larger mass and then first particle with smaller mass in both case we get same elastic force. Similarly particles with different radius and same mass, by considering first particle with larger radius and then first particle with smaller radius in both case we get same elastic force. Thus we can say that force has no effect on collision with larger particle collide the smaller particle or smaller particle collide the larger particle. In both case we get the same elastic force.

6.1.3. Kinetic Effect of different sized particles with respect to Time by particle – particle contact

6.1.3.1. Effect of Elastic Force on different sized particles

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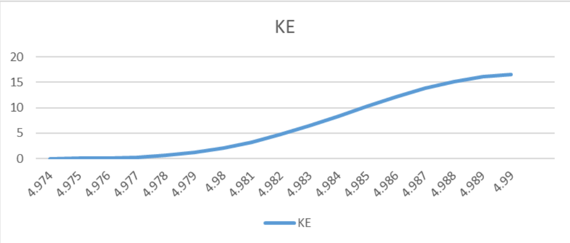

Figure14. Elastic normal effect of particle with change of kinetic energy with respect to time

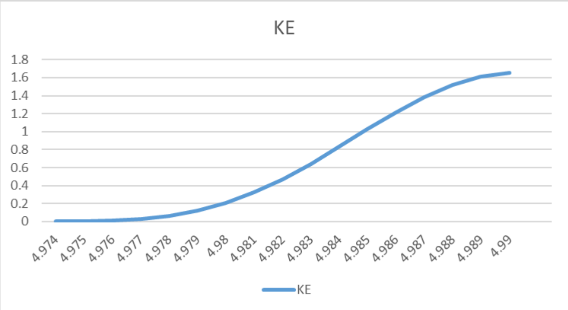

Figure15. Elastic normal effect of particle with change of kinetic energy with respect to time

When two particles with different radius and mass collide then, *first consider larger particle collide the smaller* then first particle attains a decrease in velocity and then remains constant. So kinetic energy decreases to a constant that is there is no collision. In second particle there is an increase in velocity and then remains constant. So kinetic energy also increases and then remains constant. If we consider *first smaller particle* collide with the larger particle then velocity decreases and then increases, so kinetic energy decreases and then increases. But in second particle there is a small increase in velocity. So kinetic energy also increases.

6.1.3.2. Elastic effect of Particles with same radius and different mass, different radius and same mass

Case 1: ***Consider a particle with same radius and different in mass:***

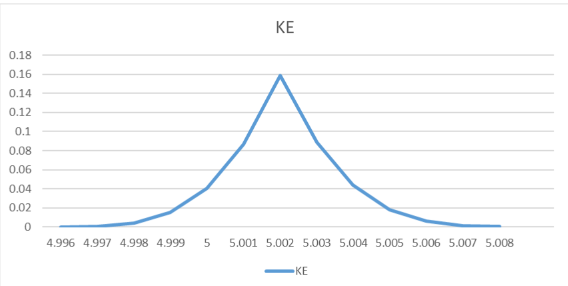

Figure16. Elastic normal effect of particle with same radius and differ in mass

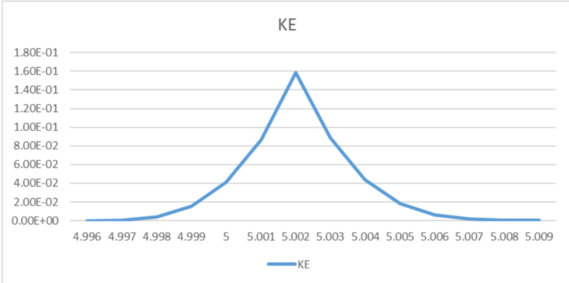

Figure17. Elastic normal effect of particle with same radius and change in mass.

From the result we can see that mass has great effect on kinetic energy of a particle. The velocity occurred by the particle with large mass collide the smaller mass particle attain more velocity compared to the particle with small mass collide the larger particle. So particle with larger mass particle hits the smaller occurs more kinetic energy compared with smaller mass particle hits the larger mass particle.

Consider particles with different radius and same mass

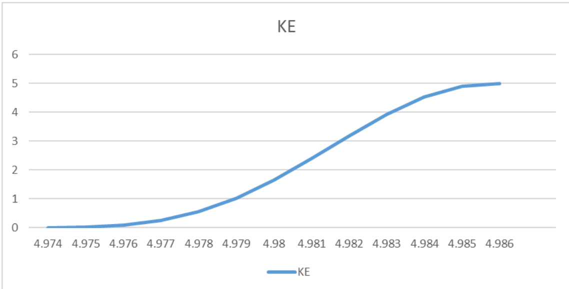

Figure18. Elastic normal effect of particle with same mass and differ in radius

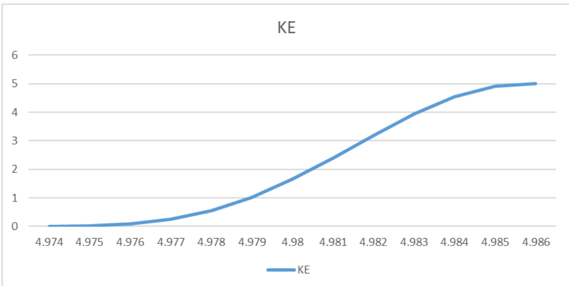

Figure19. Elastic normal effect of particle with same mass and differ in radius

If we consider particles with different radius and same mass. If first particle with larger radius collide with smaller radius particle and smaller radius particle collide with larger radius particle, in both case we get same velocity so the kinetic energy in both the case are same. If there is no collision then kinetic energy remains constant. From the result we can say that the change in mass cause change in

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kinetic energy. And if change in radius has no effect on kinetic energy of a spherical particles.

6.1.4. Displacement of different sized particles with respect to Time by particle – particle contact

6.1.4.1. Effect of Force on different sized particles

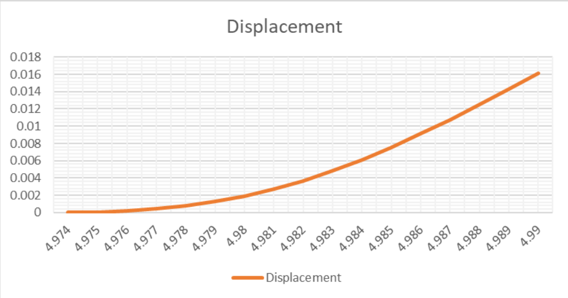

Figure20. Elastic normal effect of particle with change in displacement with respect to time

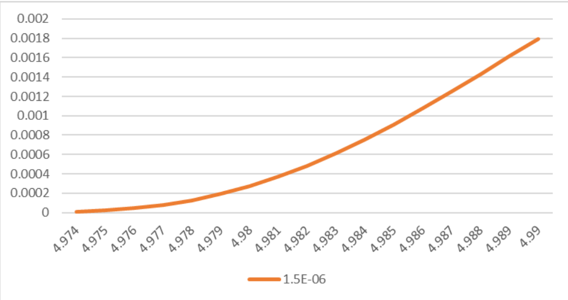

Figure21. Elastic normal effect of particle with change in displacement with respect to time

The first particle having larger radius and mass moving with initial velocity and collide the second particle, there is an increase in displacement with respect to time. But first particle with smaller radius and mass moving and colliding the particle with larger mass and radius then a slight increase in displacement. Change in time in the x-direction and displacement in the y-directions. From the graph we can see displacement changes continuously with time. When larger particle collide with smaller particle then displacement of larger particle is more compared with smaller particle collide with larger particle.

4.2.4.2. Elastic effect of Particles with same radius and different mass, different radius and same mass

Consider particles with same radius and different in mass:

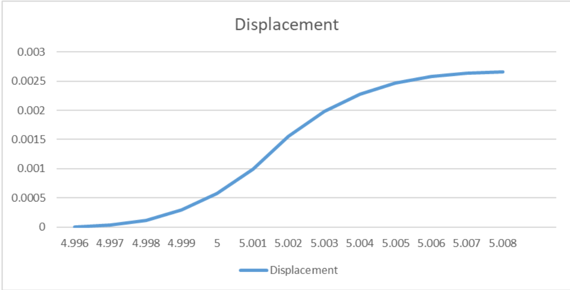

Figure22. Elastic normal effect of particle with same radius and larger in mass

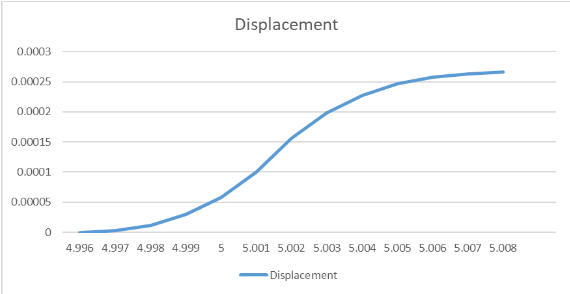

Figure23. Elastic normal effect of particle with same radius and smaller in mass.

When two particles with same radius and difference in mass, if we consider the first particle with larger mass and second particle with smaller mass. When larger mass particle collide the smaller attain a slight increase in displacement compared with small mass particle collide with larger mass particle. Thus we can say that change in mass of the particle change the displacement of the particle.

Consider particles with different radius and same mass

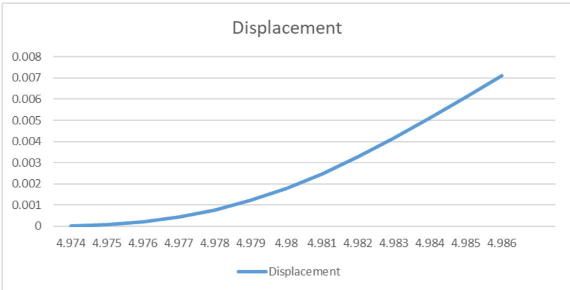

Figure24. Elastic normal effect of particle with same mass and differ in radius

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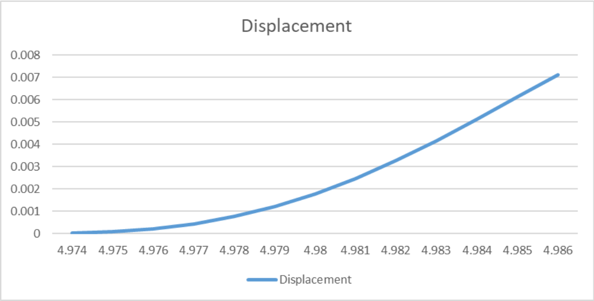

Figure25. Elastic normal effect of particle with same mass and differ in radius

If we consider particles with same mass and different radius. First case in which first particle with larger radius collide with second particle with smaller radius, second case which first particle with smaller radius collide with second particle with larger radius. By comparing we can see that in both cases the particle attain same displacement with respect to time.

7. Conclusion

The result obtained in the present investigation may be generally described as follows:

- The described discrete element model composed of visco-elastic spherical particles is implemented into the developed C++ code. This code open for new elements and interaction models may be considered as the first step in the development of an advanced simulation tool for granular and other inhomogeneous materials and is proposed for modelling more complex geotechnical problems.
- The analytical solutions for the impact of two spheres have been examined and derived. In particle-particle collision the particle rebounds to the original position. The normal force reaches a peak during contact.
- Elastic force with respect to different sized particle with respect to time and displacement are analyzed.
- Elastic effect on kinetic energy on a particle with different time step is analyzed.
- Elastic effect on displacement on a particle with different time step is analyzed.
- Elastic effect of different sized particle with mass change and radius change are analyzed.

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