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# Heat transfer effect on an oscillatory flow of Jeffrey fluid through a porous medium in a tube

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**Abstract**

In this paper, we examined the impact of heat transfer on oscillatory flow of Jeffrey fluid through a permeable medium in a circular tube. The expressions for the temperature field and velocity field are obtained analytically. It is discovered that the velocity field increases with increasing or , while it decreases with increasing or . Additionally It is observed that, the temperature field decreases with increasing .

***Keywords:*** *Heat transfer, oscillatory flow, Jeffrey fluid and porous medium*

## 1. Introduction

The examination of oscillatory flow of a viscous fluid in cylindrical tubes has received the consideration of numerous specialists as they assume a critical job in understanding the imperative physiological issue, specifically the blood flow in arteriosclerotic blood vessel. Womersley [17] have looked into the oscillating flow of thin walled elastic tube. Detailed estimations of the oscillating velocity profiles were made by Linford and Ryan [10], Unsteady and oscillatory flow of viscous fluids in locally constricted, rigid, axisymmetric tubes at low Reynolds number has been thought about by Ramachandra Rao and Devanathan [14], Hall [9] and Schneck and Ostrach [15]. Haldar [8] have thought about the oscillatory flow of a blood through an artery with a mild constriction. few different specialists, Misra and Singh [11], Ogulu and Alabraba [12], Tay and Ogulu [16] and Elshahed [7], to make reference to however a couple, have in one way or the other modeled and studied the flow of blood through a rigid tube under the influence of pulsatile pressure gradient.

Lalithajyothi et al. [1] contemplated the pulsatile flow of a jeffrey fluid in a circular tube having internal porous lining.Vajravelu et al. [2] examined the unsteady flow of two immiscible conducting fluids between two permeable beds. Spurred by the above examinations, pulsatile flow of a Jeffrey fluid between permeable beds is investigated. The influence of melting heat transfer and thermal radiation on MHD stagnation point flow of an electrically conducting Jeffrey fluid over a stretching sheet with partial surface slip is performed by Das et al.[3]. Sreenadh et al. [4] explored free convective flow of a Jeffrey fluid in a vertical deformable porous stratum. They observed that the skin friction gets reduced when the porous material is a deformable one and seen that the impact of increasing Jeffrey parameter is to increase the skin friction in the deformable porous stratum. Nallapu and Radhakrishnamacharya [5] studied Jeffrey fluid flow in the presence of magnetic field through porous medium in tubes of small diameters. The impact of slip and heat transfer on the peristatic transport of Jeffrey fluid in a vertical asymmetric channel in porous medium is talked about by Lakshminarayana et al. [6]. Farooq et al.[13] are steadied MHD flow of a Jeffery fluid with Newtonian heating .Kavith et al[18] are analyzed per static transport of a Jeffery fluid in consistent with Newtonian fluid in an inclined channel. Ali et al.[19] are talked about logical solution for oscillatory flow in a channel for Jeffery fluid .

In perspective of these, we inspected the effect of heat transfer on oscillatory flow of Jeffrey fluid through a porous medium in a circular tube. The expressions for temperature field and the

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velocity field are obtained analytically. The effects of different developing parameters on the velocity field and temperature field studied in detail with the help of graphs.

## 2. Mathematical formulation

We consider an oscillatory flow of a Jeffrey fluid through a porous medium in a heated uniform cylindrical tube of constant radius $R$. The wall of the tube is maintained at a temperature $T_w$. We choose the cylindrical coordinates $(r,\theta,z)$ such that $r=0$ is the axis of symmetry. The flow is considered as axially symmetric and fully developed. The geometry of the flow is shown in Fig. 1.

The constitute equation of $S$ for Jeffrey fluid is

$$S=\frac{\mu}{1+\lambda_1}\left(\dot{\gamma}+\lambda_2\ddot{\gamma}\right) \quad (2.1)$$

The equations governing the flow are given by

$$\rho\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+\frac{\partial S_{rz}}{\partial r}-\frac{\mu}{k}w+\rho g\beta\left(T-T_w\right) \quad (2.2)$$

$$\rho c_p\frac{\partial T}{\partial t}=k_0\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \quad (2.3)$$

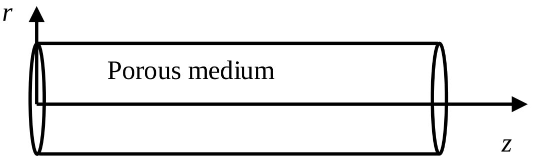


**Fig. 1** The Physical Model

The appropriate boundary conditions are

$$w=0,\quad T=T_w \quad \text{at} \quad r=R$$

$$\frac{\partial w}{\partial r}=0,\quad T=T_\infty \quad \text{at} \quad r=0 \quad (2.4)$$

Introducing the following non-dimensional variables

$$\bar{r}=\frac{r}{R},\ \bar{z}=\frac{z}{R},\ \bar{t}=\frac{w_0 t}{R},\ \alpha^2=\frac{\rho R^2}{\mu},\ \lambda=\frac{R}{w_0},\ \bar{w}=\frac{w}{w_0},$$

$$\bar{p}=\frac{p-p_w}{\mu},\ \theta=\frac{T-T_\infty}{T_w-T_\infty}\ \Pr=\frac{\mu c_p}{k_0},\ \mathrm{Re}=\frac{\rho w_0 R}{\mu},$$

into the Eqs. (2.2) - (2.4), we get (after dropping bars)

$$\mathrm{Re}\frac{\partial w}{\partial t}=-\lambda\frac{dp}{dz}+\frac{1}{1+\lambda_1}\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right]-\frac{1}{Da}w+\frac{Gr}{\mathrm{Re}}\theta \quad (2.5)$$

$$\Pr\mathrm{Re}\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r} \quad (2.6)$$

where $\Pr$ is the Prandtl number, $Da=\frac{k}{R^2}$ is the Darcy number and $\mathrm{Re}$ is the Reynolds number.
The corresponding non-dimensional boundary conditions are

$$w=0,\quad T=1 \quad \text{at} \quad r=1$$

$$\frac{\partial w}{\partial r}=0,\quad T=0 \quad \text{at} \quad r=0 \quad (2.7)$$

### 3. Solution

It is fairly unanimous that, the pumping action of the heart results in a pulsatile blood flow so that we ca represent the pressure gradient (pressure in the left ventricle) as

$$-\frac{dp}{dz}=p_0 e^{iwt} \quad (3.1)$$

and flow variables expresses as

$$\theta(y,t)=\theta_0(r)e^{iwt} \quad (3.2)$$

$$w(y,t)=w_0(r)e^{iwt} \quad (3.3)$$

Substituting Eqs. (3.1) - (3.2) into Eqs. (2.5) and (2.6) and solving the resultant equations subject to the boundary conditions in (2.7), we obtain

$$\theta_0=\frac{I_0(\Omega r)}{I_0(\Omega h)} \quad (3.4)$$

$$w_0=\frac{Gr}{\mathrm{Re}}\frac{(1+\lambda_1)}{\left(\beta_1^2+\Omega^2\right)}\left[\frac{I_0(\beta_1 r)}{I_0(\beta_1 h)}-\frac{I_0(\Omega r)}{I_0(\Omega h)}\right]+\frac{\lambda p_0}{\beta_1^2}(1+\lambda_1)\left[1-\frac{I_0(\beta_1 r)}{I_0(\beta_1 h)}\right] \quad (3.5)$$

here $\Omega^2=i\omega\Pr\mathrm{Re}$ $\quad\beta_1^2=\left(\frac{1}{Da}+i\omega\mathrm{Re}\right)(1+\lambda_1)$

. In Eqs. (3.4) and (3.5), $I_0(x)$ is the modified Bessel function of first kind of order zero
Hence the temperature distribution and the axial velocity are given by

$$\theta=\frac{I_0(\Omega r)}{I_0(\Omega h)}e^{i\omega t} \quad (3.6)$$

$$w=\left(\frac{Gr}{\mathrm{Re}}\frac{(1+\lambda_1)}{\left(\beta_1^2+\Omega^2\right)}\left[\frac{I_0(\beta_1 r)}{I_0(\beta_1 h)}-\frac{I_0(\Omega r)}{I_0(\Omega h)}\right]+\frac{\lambda p_0}{\beta_1^2}(1+\lambda_1)\left[1-\frac{I_0(\beta_1 r)}{I_0(\beta_1 h)}\right]\right)e^{i\omega t} \quad (3.7)$$

## 4. Discussion of the Results

Fig. 2 shows the effects of material parameter $\lambda_1$ on $w$ for $Da=0.1$, $p=1$, $\omega=10$, $\lambda=0.5, Pr=2, Gr=1, \mathrm{Re}=1$ and $t=0.1$. It is observed that, the axial velocity $w$ increases at the axis of tube with increasing material parameter $\lambda_1$.

Effects of Darcy number $Da$ on $w$ for $\lambda_1=0.3$, $p=1$, $\omega=10$, $\lambda=0.5$, $Pr=2$, $Gr=1, \mathrm{Re}=1$ and $t=0.1$ is shown in Fig. 3. It is found that the axial velocity $w$ increases with an increase in Darcy number $Da$.

Fig. 4 depicts the effects of Prandtl number $Pr$ on $w$ for $\lambda_1=0.3$, $p=1$, $\omega=10$, $\lambda=0.5, Da=0.1, Gr=1, \mathrm{Re}=1$ and $t=0.1$. It is noted that, the axial velocity $w$ decreases on increasing Prandtl number $Pr$.

Effects of Grashof number $Gr$ on $w$ for $\lambda_1=0.3$, $p=1$, $\omega=10$, $\lambda=0.5$, $Pr=2, Da=0.1, \mathrm{Re}=1$ and $t=0.1$ is presented in Fig. 5. It is observed that, the axial velocity $w$ increases with increasing Grashof number $Gr$.

Fig. 6 illustrates the effects of Reynolds number $\mathrm{Re}$ on $w$ for $\lambda_1=0.3$, $p=1$, $\omega=10$, $\lambda=0.5, Pr=2, Da=0.1, \mathrm{Re}=1$ and $t=0.1$. It is found that, the axial velocity $w$ decreases with an increase in Reynolds number $\mathrm{Re}$.

Effects of $\lambda$ on $w$ for $\lambda_1=0.3$, $Da=0.1$, $Pr=2$, $Gr=1$, $p=1$, $\omega=10$, $\mathrm{Re}=1$ and $t=0.1$ is shown in Fig. 7. It is observed that, the axial velocity increases on increasing $\lambda$.

Fig. 8 shows the effects of Prandtl number $Pr$ on $\theta$ for $\omega=10, \mathrm{Re}=1$ and $t=0.1$. It is found that, the temperature $\theta$ decreases with increasing Prandtl number $Pr$.

## 5. Conclusions

We studied the effect of heat transfer on oscillatory flow of Jeffrey fluid through a porous medium in a circular tube. The expressions for the velocity field and temperature field are obtained analytically. It is found that the velocity field increases with increasing $\lambda_1, Da, Gr$ or $\lambda$, while it decreases with increasing $\mathrm{Pr}$ or $\mathrm{Re}$. Also It is observed that, the temperature field decreases with increasing $\mathrm{Pr}$.

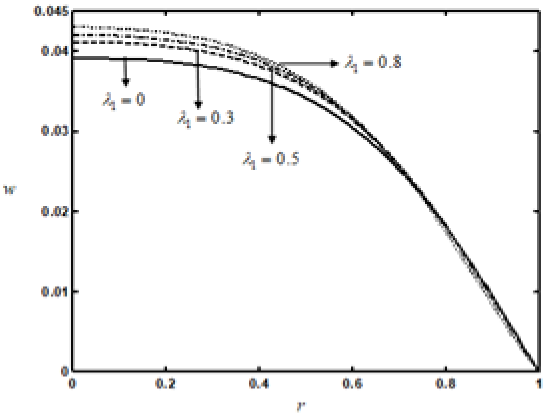


Fig. 2. Effects of material parameter $\lambda_1$ on $w$ for $Da=0.1$, $p=1$, $\omega=10$, $\lambda=0.5, Pr=2, Gr=1, \mathrm{Re}=1$ and $t=0.1$.

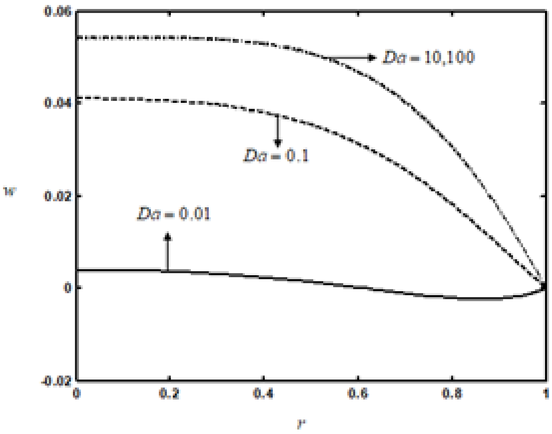


Fig. 3. Effects of Darcy number $Da$ on $w$ for $\lambda_1=0.3$, $p=1$, $\omega=10$, $\lambda=0.5, Pr=2, Gr=1, \mathrm{Re}=1$ and $t=0.1$.

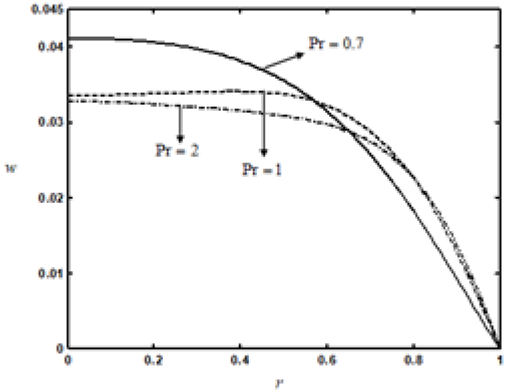


**Fig. 4.** Effects of Prandtl number $Pr$ on $w$ for $\lambda_1 = 0.3$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, $Da = 0.1$, $Gr = 1$, $\mathrm{Re} = 1$ and $t = 0.1$.

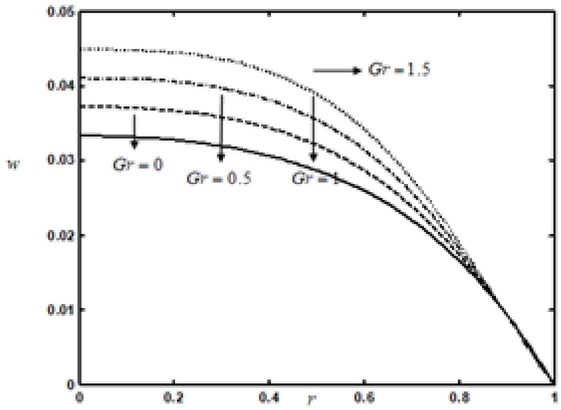


**Fig. 5.** Effects of Grashof number $Gr$ on $w$ for $\lambda_1 = 0.3$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, $Pr = 2$, $Da = 0.1$, $\mathrm{Re} = 1$ and $t = 0.1$.

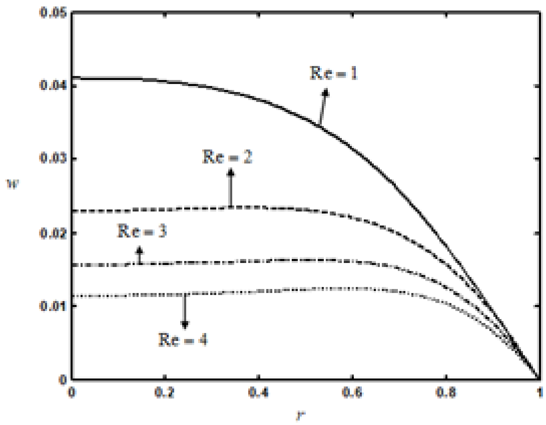


**Fig. 6.** Effects of Reynolds number Re on $w$ for $\lambda_1 = 0.3$, $p = 1$, $\omega = 10$, $\lambda = 0.5$, $Pr = 2$, $Da = 0.1$, $\mathrm{Re} = 1$ and $t = 0.1$.

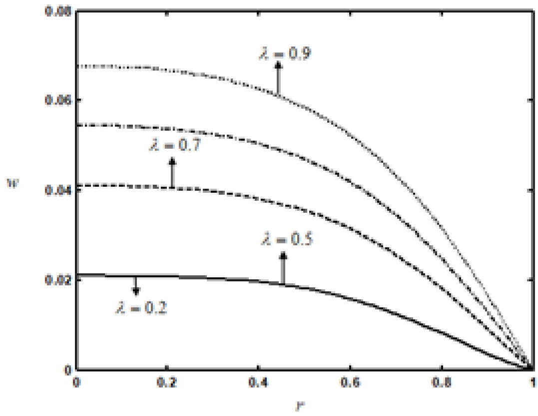


**Fig. 7.** Effects of $\lambda$ on $w$ for $\lambda_1 = 0.3$, $Da = 0.1$, $Pr = 2$, $Gr = 1$, $p = 1$, $\omega = 10$, $\mathrm{Re} = 1$ and $t = 0.1$.

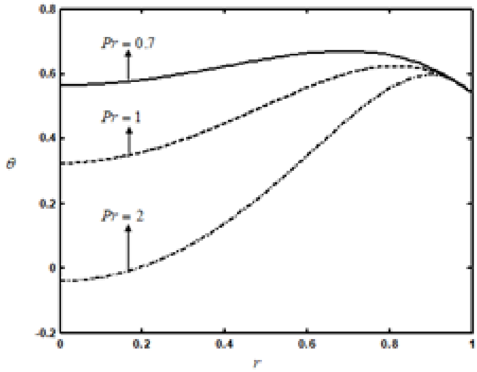


**Fig. 8.** Effects of Prandtl number $Pr$ on $\theta$ for $\omega = 10$, $\mathrm{Re} = 1$ and $t = 0.1$.

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