Investigation on MHD Micropolar Fluid flow over an exponentially Stretching Surface

S.Anuradha¹ and R.Punithavalli ²

¹Professor& Head, PG & Dept Of Mathematics, Hindusthan College of Arts & Science, Coimbatore-28,Tamilnadu, India
² Research Scholar, PG & Dept Of Mathematics, Hindusthan College of Arts & Science, Coimbatore-28, Tamilnadu, India

Abstract
In this Investigation, MHD micropolar fluid flow through an exponentially stretching sheet with heat and mass transfer has been considered. The steady two dimensional incompressible flows are taken into account. A mathematical governing model has developed for the continuity equation, momentum, temperature and concentration boundary layer equations. The governing partial differential equations are transformed by using similarity transformation and then the resulted set of system of nonlinear ordinary coupled differential equations with fitting boundary conditions are solved numerically by fourth order Runge-kutta method with shooting techniques. To study the flow characteristics, the behavior of various non-dimensional parameters which is involved in this problem on the dimensionless velocity, temperature and concentration are studied numerically with the help of graphs.

Keywords: Micropolar fluid, Stretching sheet, Boundary layer flow, Shooting technique, Runge-kutta method

1.Introduction

velocity at the plate by using the perturbation method Hayat et al. (2011) discussed the soret and Dufour effect on the stagnation point flow of a micropolar fluid towards a stretching sheet. Olanrewaju et al. (2011) examined steady MHD mixed convective stagnation point flow past in a vertical surface with the influence of thermal radiation.

Yacob et al. (2011) investigated Melting heat transfer analysis in boundary layer stagnation-point flow towards a stretching/shrinking sheet in the micropolar fluid by using Runge–KuttaFehlberg method with Shooting Technique. Mahmood et al. (2012) reported the influence of slip velocity on the flow and heat transfer techniques for an electrically conducting micropolar fluid over a permeable stretching surface with variable heat flux, heat generation/absorption, and a transverse magnetic field. Adhikari (2013) discussed stagnation point flow of MHD micropolar fluid on a vertical surface under induced magnetic field with radiative heat flux. Loganathan et al. (2013) investigated the effect of a chemically reacting micropolar fluid on thermophoresis particle deposition through moving porous plate in presence of heat generation/absorption and variable viscosity.

micropolar fluid with the effect of magnetic field, this research extends the work of Anuradha et al (2018) to study the effect of steady MHD boundary layer flow of heat and mass transfer of a micro polar fluid flowing over an exponentially stretching sheet. The main objective of this article is to discuss the characteristics of magnetic field parameters numerically with the help of graphs.

2. Mathematical formulation

MHD micropolar fluid flow through an exponential stretching sheet with heat and mass transfer has been considered. The steady two dimensional incompressible flows are taken into account. Let us consider the Cartesian coordinate axes as (x, y, z) with corresponding velocities (u, v, 0) and the origin has located at the leading edge of the sheet. The x axis is along the stretching sheet and y axis is normal to it in which uniform magnetic field B0 is applied towards the positive direction of y axis. Assume that the stretching velocity is in exponential form with velocity $U_0 = ae^{2x/L}$ with $a>0$ which a is a stretching constant. Under the boundary layer approximation, the governing equations are as follows:

**Continuity Equation:****

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

**Momentum Equation:****

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \left( \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} (\rho u^2 + \frac{\partial}{\partial y} (\rho + \frac{\partial^2 u}{\partial y^2}) (U-u) \right) \quad (2)$$

**Angular momentum Equation:**

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma}{\rho} \frac{\partial^2 N}{\partial y^2} + \frac{k}{\rho} (2N + \frac{\partial u}{\partial y}) \quad (3)$$

**Energy Equation:**

$$\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \left( \frac{\partial}{\partial y} (\rho c_p T) + \frac{\partial}{\partial y} (\rho c_p T \frac{\partial u}{\partial y}) \right) \quad (4)$$

**Concentration Equation:**

$$\frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D_m \frac{\partial^2 c}{\partial y^2} - k(C-C_s) \quad (5)$$

The associated boundary conditions are

$$u = U_0, \ v = 0, \ N = n \frac{\partial v}{\partial x} \text{ at } y = 0$$

$$u = U_0, \ N = 0, \ T = T_e(x), \ C = C_s(x) \text{ at } y \to \infty$$

where $\nu$ is the kinematic viscosity, $\mu$ the dynamic viscosity, $\rho$ the density, $N$ the microrotation, $j$ is the micro inertia per unit mass, $\gamma$ is the spin gradient viscosity, $k$ is the vortex viscosity, $T$ is the temperature, $L$ is the reference length, $D_m$ is the modular diffusivity, the exponential stretching sheet expression for $U_0$, $U_0$, $T_0$ and $C_0$ are defined as

$$U_0 = ae^{2x/L}, \ U_0 = be^{2x/L}, \ T_0 = T_e + ce^{2x/L}, \ C_0 = C_s + de^{2x/L} \quad (7)$$

The continuity equation is satisfied by Cauchy Riemann equations

$$u = \frac{\partial \psi}{\partial x}, \ v = -\frac{\partial \psi}{\partial y} \quad (8)$$

where $\psi(x,y) =$ the stream function.

The following similarity transformation is introduced to get the set of ordinary differential equations in terms of non-dimensional variables

$$\theta = \frac{T-T_e}{T_i-T_e}, \ \eta = \frac{y}{2aL}, \ \psi = \frac{C-C_s}{C_i-C_s} \quad (9)$$

By using equation (9), the nonlinear partial differential equation (2) to (5) becomes

$$f' + \frac{1}{\alpha L} (f' - 2f' + 2) + K \frac{\partial \psi}{\partial \eta} = 0 \quad (10)$$

$$g' + \frac{1}{\alpha L} (g' - 2f' + 2) = \frac{K \psi}{\sqrt{1 - \psi}}(2g' + f') = 0 \quad (11)$$

$$\theta' + Pr(f \theta' - 2f \theta) + Pr Ec Mf' \theta' + Pr \theta = 0 \quad (12)$$

$$\phi' - Sc f \phi' + Sc f \phi' - Sc \phi = 0 \quad (13)$$

The boundary conditions are

$$f(0) = 0, \ f(0) = \varepsilon, \ f \to 1 \text{ as } \eta \to \infty$$

$$M(0) = -nf(0), \ M \to 0 \text{ as } \eta \to \infty$$

$$\theta(0) = 1, \ \theta \to 0 \text{ as } \eta \to \infty$$

$$\phi(0) = 1, \ \phi \to 0 \text{ as } \eta \to \infty$$

From the equations (10) - (13), the non-dimensional parameters are found to analyze the characteristics of velocity, concentration and temperature profiles. Where $f$ = the dimensionless stream function, $\theta$ = the dimensionless temperature, $\Phi$ = the dimensional concentration, $g$ = the angular velocity, $M$ = Magnetic parameter, $Pr$ = Prandtl number, $Sc$ = Schmidt number, $Re$ = non-linear Reynolds number, $K$ = Micropolar parameter, $\gamma$ = non-dimensional chemical reaction parameter, $Ec$ = Eckert number

3. Numerical Solution

The governing equations of this problem had been transformed into set of coupled non-linear boundary layer equations (10) – (13) with the boundary conditions (14) and then solved by using Runge-Kutta method along with the shooting technique. To study the flow characteristics, the behavior of various non-dimensional parameters which is involved in this problem on the dimensionless velocity, temperature and concentration are studied numerically with the help of graphs.

4. Results and Discussions

Numerical computation is carried out to get clear insight of physical problem for MHD boundary layer micropolar fluid flow through exponentially
stretching sheet. The impact of various flow controlling parameters like Magnetic parameter, Prandtl number, Eckert number, Schmidt number and chemical reaction parameter on velocity, temperature and concentration profiles analyzed numerically and graphically.

Figures 1 illustrates the effect of the Magnetic parameter $M$ on velocity profile, temperature profile and concentration profile. Increasing values of Magnetic parameter $M$ decreases the velocity profile which produces the Lorenz force to the flow. Due to Lorenz force, fluid motion and thickness of boundary layer reduced. Increasing values of Magnetic parameter $M$ increase the profiles of temperature and concentration.

Figures 2 demonstrate the influence of Prandtl Number ($Pr$) on the temperature profile and concentration profile. It is observed that smaller values of Prandtl number larger thermal diffusivity. Concentration profile decreases with an increasing value of Prandtl number ($Pr$).

The effect of Schmidt number ($Sc$) on velocity profile and concentration profile are presented in figure 3. An enhancement in Schmidt number reduce both profiles which impact that diffusional effects dominate the viscous force and hence causes the slower movement of fluid. Figures 4 portrayed the behavior of the non-dimensional
parameter Eckert number (Ec) on velocity profile, temperature profile and concentration profile respectively. Rising values of Eckert number (Ec) enhance both velocity profile and temperature profile but it decreases the concentration profile. The viscous dissipation produces the heat due to the presence of Eckert number in energy equation. Figure 5 depict the effect of chemical reaction parameter (γ) on concentration profile. Larger values of chemical reaction parameter (γ) decrease the concentration profile.

5. Conclusion

In this present investigation, MHD micropolar fluid flow through an exponentially stretching sheet with heat and mass transfer has been examined. Mathematical model formulated for the fluid flow. The resulting governing boundary layer non-linear equations are solved numerically by shooting method along with fourth order Runge-Kutta method and the impact of various flow controlling parameters like Magnetic parameter, Prandtl number, Eckert number, Schmidt number and chemical reaction parameter on velocity, temperature and concentration profiles analyzed with the help of graphs. The conclusions are as follows:

- Increasing values of Magnetic parameter M decreases the velocity profile which produces the Lorenz force to the flow. Due to Lorenz force, fluid motion and thickness of boundary layer reduced.

- Increasing values of Magnetic parameter M increase the profiles of temperature and concentration.

- It is observed that smaller values of Prandtl number larger thermal diffusivity. Concentration profile decreases with an increasing value of Prandtl number (Pr)

- An enhancement in Schmidt number reduce velocity profile and concentration profile which impact that diffusional effects dominate the viscous force and hence causes the slower movement of fluid.

- Rising values of Eckert number (Ec) enhance both velocity profile and temperature profile but it decreases the concentration profile. The viscous dissipation produces the heat due to the presence of Eckert number in energy equation.

- Larger values of chemical reaction parameter (γ) decrease the concentration profile.

References


[10] Bakr AA.(2011) “Effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a


