A New Approach to Solve Fuzzy Travelling Salesman Problem

P.Rajarajeswari[1] and D.Maheswari[2]

[1]Assistant Professor of Mathematics, Chikkanna Government Arts College, Tirupur, Tamilnadu, India

[2]Assistant Professor of Mathematics, Sri Krishna Arts and Science College, Coimbatore, Tamilnadu, India.

Abstract

A fuzzy number can be used to solve many real-life problems like Travelling Salesman, Assignment Problems and so on. In this paper, a new method is proposed for solving Travelling Salesman problems using Transitive Fuzzy Numbers. We use Haar Hungarian algorithm approach to solve a Fuzzy Travelling Salesman Problem (FTSP) and Numerical examples are given to validate the proposed algorithm.

Keywords: *Fuzzy numbers, transitive trapezoidal fuzzy numbers, Haar ranking, Hungarian method.*

1. Introduction

Traveling salesman problem is a classical problem in combinatorial optimization. The objective of the problem is to find the shortest route of salesman starting from a given city, visiting all other cities only once and finally come to the same city where he started. There are different approaches for solving traveling salesman problems. Linear programming method, heuristic methods like cutting plan algorithms and branch and bound method, Markov chain, simulated annealing and so on.

In real life situation, it may not be possible to get the cost or time as a certain quantity. To overcome this Zadeh [1] introduce fuzzy set concepts to deal with imprecision and vagueness. Since then significant advantages have been made in developing numerous methodologies and their applications to various decision problems. If the cost or time or distance is not crisp values, then it becomes a fuzzy TSP. The Fuzzy TSP has been solved for LR-fuzzy parameters by Amit Kumar and Anil Gupta [2]. In this paper, a new algorithm which is similar to the classical assignment method is introduced to solve fuzzy TSP. First, Fuzzy Hungarian method is applied and then modifications are done to satisfy the route conditions by considering element-wise subtraction and Yager's [3] ranking method of fuzzy numbers. Dhanasekar et al., [4] solved fuzzy TSP by using the Hungarian algorithm with element-wise subtraction of fuzzy numbers. Since fuzzy TSP is an NP-hard problem, more numbers of algorithms are still proposed to solve the fuzzy TSP. In this paper, the ordering of fuzzy numbers based on Haar wavelet is used [5], to order the fuzzy numbers. The advantage of using this ranking technique is that it converts a given fuzzy number into average and detailed coefficients using downsampling. The uniqueness of the Haar ranking method [5] is that the fuzzification from the defuzzified value is very easy to obtain through upsampling. In this paper, the Hungarian method using Haar tuples [6] is proposed for solving traveling salesman problem with fuzzy parameters.

2. Preliminaries

Definition 2.1

A Fuzzy set can be obtained by allocating all the elements in the universe of discourse to value lies in [0, 1] by using a membership function.

Definition 2.2

A Fuzzy number Ā with membership function $\mu_{\bar{A}}(x)$ satisfies piece wise continuity, convexity and normality.

Definition 2.3

A fuzzy number $\tilde{A}$ = (a,b,c,d) with membership function of the form is called a trapezoidal fuzzy number.

$$\mu_{\bar{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \dfrac{d-x}{d-c} & c \le x \le d \\ 0 & otherwise \end{cases}$$

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Definition 2.4:

The Fuzzy Operations of fuzzy numbers are defined as
Fuzzy Addition:
$(a_1,b_1,c_1,d_1) + (a_2,b_2,c_2,d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
Fuzzy Subtraction:
$(a_1,b_1,c_1,d_1)-(a_2,b_2,c_2,d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

Definition 2.5:

If $a^{(1)} \sim a^{(4)}$, then the trapezoidal fuzzy number A = ($a^{(1)},a^{(2)},a^{(3)},a^{(4)}$) is called transitive trapezoidal fuzzy number. It is denoted by A = ($a^{(1)}$, $a^{(4)}$), Where $a^{(1)}$ is core (A) , $a^{(4)}$ is left width and right width of c. The parametric form of a transitive trapezoidal fuzzy numbers is represented by
$A = [\, a^{(1)} - a^{(2)}(1\text{-}r) , a^{(1)}+a^{(3)}(1\text{-}r) , a^{(1)} - a^{(1)}(1\text{-}r) \,]$

Definition 2.6:

Consider a Trapezoidal fuzzy number $\tilde{A}$ = (a,b,c,d) the Haar tuple of $\tilde{A}$ can be calculated as $H(\tilde{A})$ = (α, β, γ, δ) where α = (a+b+c+d)/4, β = ((a+b)−(c+d))/4, γ = (a− b)/2, δ = (c−d)/2.
Here a is the average coefficient and the remaining coefficients are all called as detailed coefficients of the given trapezoidal fuzzy number .

Definition 2.7:

Element wise Addition:
$(\alpha_1, \beta_1, \gamma_1, \delta_1) + (\alpha_2, \beta_2, \gamma_2, \delta_2) = (\alpha_1+\alpha_2, \beta_1+\beta_2, \gamma_1 + \gamma_2, \delta_1+\delta_2)$
Element wise Subtraction:
$(\alpha_1, \beta_1,c_1,d_1)-(\alpha_2, \beta_2,c_2,d_2) = (\alpha_1 - \alpha_2, \beta_1 - \beta_2, \gamma_1 - \gamma_2, \delta_1 - \delta_2)$

Fuzzy Travelling Salesman Problem.

Suppose a person has to visit n cities. He starts from a particular city, visits each city once and then returns to the starting point. The fuzzy travelling costs from i^{th} city to j^{th} city is given by $\check{C}_{ij}$. The chosen fuzzy travelling salesman problem may be formulated by
Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$
Subject to
$\sum_{i=1}^{n} x_{ij} = 1$

$\sum_{j=1}^{n} x_{ij} = 1$

3. Proposed Algorithm

Consider the matrix form of the fuzzy travelling salesman problem.
Step: 1 Transform the given fuzzy parameters into Haar tuples.
Step: 2 Fuzzy optimum assignment is obtained by using the Hungarian algorithm.
Step: 3 Inspecting the obtained solution to see whether the solution satisfied route conditions or not. If it is ,then it is the optimal solution of fuzzy TSP. If not, take the next best solution.

3.1 Numerical Example

Consider the following fuzzy TSP discussed in [7]

City → ↓	A	B	C	D
A	∞	(4,3,3,4)	(1,4,2,1)	(5,13,1,5)
B	(2,6,3,2)	∞	(6,4,2,6)	(2,3,7,2)
C	(5,13,1,5)	(5,6,7,5)	∞	(1,4,2,1)
D	(6,12,1,6)	(1,4,2,1)	(2,6,3,2)	∞

Convert the given fuzzy numbers into Haar tuples. The TSP with Haar tuples as its elements is given as follows.

City	A	B	C	D
A	∞	(3.5,0,0.5 ,-0.5)	(2, 0.5, -1.5,0.5)	(6, 3 ,-4,-2)
B	(3.25,0.75, -2,0.5)	∞	(4.5,0.5,1, -2)	(3.5,-1 ,-.5,2.5)
C	(6,3,-4,-2)	(5.75,-.25, -0.5,1)	∞	(2,0.5, -1.5, 0.5)
D	(6.25,2.75 ,-3,-2.5)	(2 ,0.5, -1.5,0.5)	(3.25,1.5 ,-2,0.5)	∞

Apply Hungarian algorithm to get the table,

City	A	B	C	D
A	∞	(1.5,0.5,2, -1)	**(0,0,0,0)**	(4, 2.5, -2.5,-2.5)
B	**(0,0,0,0)**	∞	(1.25,-0.25,3, -2.5)	(0.25, -1.75, 1.5,2)
C	(4,2.5, -2.5, -2.5)	(3.75, 0.75, 1,0.5)	∞	**(0,0,0,0)**
D	(4.25,2.25, -1.5,-3)	**(0,0,0,0)**	(1.25,1, -0.5,0)	∞

The optimal assignment schedule is A→C→D→B→A. Here the route condition is satisfied.

Haar optimal cost is
(2,0.5,-1.5,0.5)+(2,0.5,-1.5,0.5)+(2,0.5,-1.5,0.5)+
(3.25,0.75,-2,0.5) = (9.25,2.25,-6.5,2)
and the corresponding fuzzy optimal cost after defuzzification is given by (5,18,9,5) = Rs.9.28

5. Conclusion

An Algorithm using Haar tuples is proposed for solving transitive fuzzy traveling salesman problem. Since this algorithm is similar to the Hungarian algorithm it is effective and easy to compute. This proposed method is effective than the existing methods because the results were obtained in Haar parameters and further it can be transformed into parameters. The results match with the existing technique and also satisfy the regular TSP conditions.

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