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Flow Of Nanofluid With Stenosis And Post Stenotic Dilatation Through An Inclined Artery

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Abstract

The present paper analyzed the flow of an incompressible Nanofluid in an inclined artery with stenosis and post stenotic dilatation. Homotopy Perturbation method is used to get the solution for velocity, pressure drop, wall shear stress of coupled equations. It is observed that Brownian motion number (N_b), Grashof number (G_r) and volumetric flow rate (q) increases resistance to the flow decreases. It is noticed that wall shear stress increases as height of the stenosis (δ_1),thermophoresis parameter (N_t),Grashof number (G_r),local nanoparticle Grashof number (B_r), and inclination (α) increases and decreases with Brownian motion number (N_b).

Keywords: *Stenosis, Dilatation, Nanofluid, Homotopy Perturbation method, Brownian motion number*

1.Introduction :

One of the major health problems causing large number of death is due to diseases in the blood vessels and in the heart like heart attack and stroke. The common reason for these death is stenosis. It is reveals that deposition of fatty substances , cellular waste products on the wall of the artery generates stenosis, but the actual cause of stenosis is not well known. Due to the blockage in the artery the blood flow is restricted and weaken the arterial wall. At the downstream of the stenosis due to the weaken of the arterial wall post dilatation generates. Post stenotic dilatation of the artery are common in multiple stenoses as mentioned by [11]. To prevent arterial diseases detailed knowledge of blood flow in stenosis arteries and post dilatation is required.

Based on this numerous mathematical and experimental models were proposed by various researchers in the past decaded to understand the blood flow characteristics in arteries due to the presence of stenosis.[20,3,6,18].

In recent days ,many researchers focused on study of nanofluid flow in different geometries. Nanofluids is defined as the study of fluid flow in an around nano-sized objects. Choi[2] was the first person who initiated nanofluid technology, where as Buongiorno[1] investigated in detailed about convective transport in nanofluids. Das *et.al* [4] examined in detailed of Pool boiling of nanofluids on horizontal narrow tubes. Many researchers have discussed blood flow analysis in stenosis arteries Nadeem *et.al* [14,15], Mekheimer and El Kot [12].Blood flow in the arteries in the presence of overlapping stenosis was studied by Riahi et al. [17]. Blood flow of nanofluid through an artery with composite stenosis and permeable walls was studied by R. Ellahi *et.al* [5].

Mukesh Kumar Sharma *et.al* [13] have seen the effect of overlapping stenosis and dilatation on Non-Newtonian blood flow in an inclined artery. The effects of post-stenotic dilatations on the flow of coupled stress fluid through stenosed arteries was studied by KM Prasad *et.al* [8]. Priyadarshini *et.al* [16] have analyzed the flow of Herschel-Bulkley fluid through a tapered arterial stenosis with dilatation. The Effect of slip on Herschel- Bulkley Fluid Flow Through An Artery With Stenosis and Post Stenotic Dilatation was studied by Syed Waseem Raja *et.al* [19]. K M Prasad *et.al* [10] examined flow of nanofluid through an inclined tube of non uniform cross section with multiple stenoses.

Motivated from all these studied a mathematical model has developed by assuming blood as nanofluid which flows through an artery having stenosis and post stenotic dilatation with inclination.

2. Mathematical Formulation

In the present study we considered the steady flow of Nano fluid through a circular artery containing multiple abnormal segments with inclination.

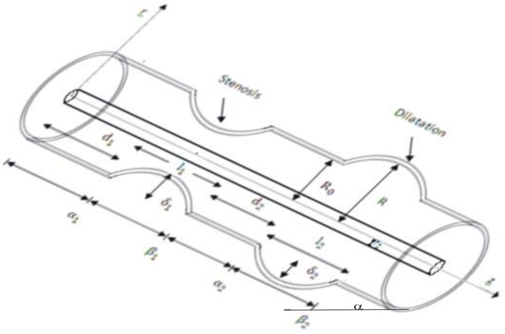

Figure. 1:Geometry of the problem

The geometry of the wall of the tube is taken as ([K Maruti Prasad *et.al* [9])

$$h=\frac{R(z)}{R_o}\begin{cases}1-\frac{\delta_i}{2R_o}\left[1+\cos\frac{2\pi}{L_i}\left(z-\alpha_i-\frac{L_i}{2}\right)\right], & \alpha_i\le z\le\beta_i\\ 1, \ otherwise\end{cases} \quad \text{(i)}$$

The radius of the artery at dilatation , normal artery and the length of the i^{th} abnormal segment is represented by $R,\ R_o, L_i$ respectively. The max. distance of the i^{th} abnormal segment is given by δ_i and it is +ve , –ve for stenosis and aneurysms respectively . The distance from the origin to the start of the i^{th} abnormal segment is denoted by α_i and it is given as

$$\alpha_i=\left[\sum_{j=1}^{i}\left(d_j+L_j\right)\right]-L_i \quad \text{(ii)}$$

The distance from the origin to the end of the i^{th} abnormal segment is given by β_i , i.e

$$\beta_i=\left[\sum_{j=1}^{i}\left(d_j+l_j\right)\right] \quad \text{(iii)}$$

The start of the i^{th} abnormal segment from the end of the $(i-1)^{th}$ distance is given by d_i.
Equation for an incompressible fluid for the balance of mass , momentum, temperature and nanoparticle volume fraction are given as (Maruti Prasad [7]).

$$div\bar{V}=0 \quad (2)$$

$$\rho_f\frac{d\bar{V}}{dt}=-\nabla\bar{p}+\mu\nabla^2\bar{V}+f \quad (3)$$

$$(\rho c)_f\frac{d\bar{T}}{dt}=\begin{bmatrix}k\nabla\bar{T}+\\ (\rho c)_p\left[D_B\nabla\bar{C}.\nabla\bar{T}+\frac{D_{\bar{T}}}{\bar{T}_0}\nabla\bar{T}.\nabla\bar{T}\right]\end{bmatrix} \quad (4)$$

$$\frac{d\bar{C}}{dt}=D_B\nabla^2\bar{C}+\left[\frac{D_{\bar{T}}}{\bar{T}_0}\right]\nabla^2\bar{T} \quad (5)$$

Where ρ_f is the of the fluid, ρ_p is the density of the particle, c is the volumetric volume expansion coefficient, coefficient, $\bar{V}$ is the velocity vector, f is the body force, $\frac{d}{dt}$ represents the material time derivative, $\bar{p}$ is the pressure, $\bar{C}$ is the nanoparticle phenomena, D_B is the Brownian diffusion coefficient and $D_{\bar{T}}$ is the thermophoretic diffusion coefficient.
In component form Eq. (2) and Eq. (5) can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\bar{r}\bar{v}\right)+\frac{\partial\bar{u}}{\partial\bar{z}}=0 \quad (6)$$

$$\rho\left[\bar{v}\frac{\partial\bar{v}}{\partial\bar{r}}+\bar{u}\frac{\partial\bar{v}}{\partial\bar{z}}\right]=-\frac{\partial\bar{P}}{\partial\bar{r}}+ \mu\left[\frac{\partial^2\bar{v}}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial\bar{v}}{\partial\bar{r}}+\frac{\partial^2\bar{v}}{\partial\bar{z}^2}-\frac{\bar{v}}{\bar{r}^2}\right]-\frac{\cos\alpha}{F} \quad (7)$$

$$\rho\left[\bar{v}\frac{\partial \bar{u}}{\partial \bar{r}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{z}}\right]=-\frac{\partial \bar{P}}{\partial \bar{z}}+\mu\left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right] \quad (8)$$

$$+\rho g\alpha\left(\bar{T}-\bar{T}_0\right)+\rho g\alpha\left(\bar{C}-\bar{C}_0\right)+\frac{\sin\alpha}{F}$$

$$\left[\bar{v}\frac{\partial \bar{T}}{\partial \bar{r}}+\bar{u}\frac{\partial \bar{T}}{\partial \bar{z}}\right]=\alpha\left[\frac{\partial^2 \bar{T}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right]$$

$$+\tau\left\{\begin{matrix} D_B\left[\frac{\partial \bar{C}}{\partial \bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}}+\frac{\partial \bar{C}}{\partial \bar{z}}\frac{\partial \bar{T}}{\partial \bar{z}}\right] \\ +\frac{D_{\bar{T}}}{\bar{T}_0}\left[\left(\frac{\partial \bar{T}}{\partial \bar{r}}\right)^2+\left(\frac{\partial \bar{T}}{\partial \bar{z}}\right)^2\right]\end{matrix}\right\} \quad (9)$$

$$\left[\bar{v}\frac{\partial \bar{C}}{\partial \bar{r}}+\bar{u}\frac{\partial \bar{C}}{\partial \bar{z}}\right]=\left[\begin{matrix} D_B\left[\frac{\partial^2 \bar{C}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{C}}{\partial \bar{r}}+\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right] \\ +\frac{D_{\bar{T}}}{\bar{T}_0}\left[\frac{\partial^2 \bar{T}}{\partial \bar{r}^2}+\frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right]\end{matrix}\right] \quad (10)$$

Where $\tau=\frac{(\rho C)_P}{(\rho C)_f}$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid.

The boundary conditions are as follows

$$\frac{\partial \bar{u}}{\partial \bar{r}}=0,\ \frac{\partial \bar{T}}{\partial \bar{r}}=0,\ \frac{\partial \bar{C}}{\partial \bar{r}}=0 \ \ at\ \ \bar{r}=0 \quad (11)$$

$$\bar{u}=0\ ,\ \bar{T}=\bar{T}_0,\ \bar{C}=\bar{C}_0 \ \ at\ \ R(z) \quad (12)$$

Using the following non-dimensional quantities

$$\bar{z}=\frac{z}{B}\ ,\ \bar{d}_1=\frac{d_1}{B},\bar{L}_1=\frac{L_1}{B},\bar{L}_2=\frac{L_2}{B},$$

$$\bar{B}_1=\frac{B_1}{B},\ \bar{v}=\frac{B}{\delta U}v,\ \ \bar{u}=\frac{u}{U}$$

$$\bar{R}(z)=\frac{R(z)}{R_0}\ ,\ \bar{\delta}_i=\frac{\delta_i}{R_0},\bar{P}=\frac{P}{\mu UL/R_0^2},$$

$$\bar{q}=\frac{q}{\pi R_0^2 U},\ R_e=\frac{2\rho c_1 R_o}{\mu},$$

$$N_b=\frac{(\rho C)_P D_{\bar{B}}\bar{C}_o}{(\rho C)_f}\ ,\ N_t=\frac{(\rho C)_P \bar{T}_o}{(\rho C)_f\ \beta}, \quad (13)$$

$$G_r=\frac{g\beta \bar{T}_o R_o^3}{\gamma^2},B_r=\frac{g\beta \bar{C}_o R_o^3}{\gamma^2}$$

Applying the mild stenosis approximation, the Eq.(6) to Eq.(12) reduces to

$$\frac{\partial v}{\partial r}+\frac{v}{r}+\frac{\partial u}{\partial z}=0 \quad (14)$$

$$\frac{\partial P}{\partial r}=-\frac{\cos\alpha}{F} \quad (15)$$

$$\frac{\partial P}{\partial z}-\frac{\sin\alpha}{F}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+G_r\theta_t+B_r\sigma \quad (16)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_t}{\partial r}\right)+N_b\frac{\partial \sigma}{\partial r}\frac{\partial \theta_t}{\partial r}+N_t\left(\frac{\partial \theta_t}{\partial r}\right)^2=0 \quad (17)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \sigma}{\partial r}\right)+\frac{N_t}{N_b}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta_t}{\partial r}\right)\right]=0 \quad (18)$$

Where u is the velocity averaged over section of the tube with Radius $R_0 .\theta_t,\sigma,N_b,N_t,G_r$ and B_r are temperature profile, nanoparticle phenomena, Brownian motion parameter, thermophoresis parameter, local temperature , Grashof number and local nanoparticle Grashof number.

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial r}=0,\ \frac{\partial \theta_t}{\partial r}=0,\ \frac{\partial \sigma}{\partial r}=0 \ \ at\ \ r=0$$

$$u=0\ ,\ \theta_t=0,\ \sigma=0 \ \ at\ \ r=h(z) \quad (19)$$

3. Solution

The solution of the coupled Eq. (17) and Eq. (18) have been solved by using homotopy perturbation method (HPM) as

$$H\left(q_t,\theta_t\right)=\left(1-q_t\right)\left[L\left(\theta_t\right)-L\left(\theta_{10}\right)\right]$$
$$+q_t\left[L\left(\theta_t\right)+N_b\frac{\partial\sigma}{\partial r}\frac{\partial\theta_t}{\partial r}+N_t\left(\frac{\partial\theta_t}{\partial r}\right)^2\right] \quad (20)$$

$$H\left(q_t,\sigma\right)=\left(1-q_t\right)\left[L\left(\sigma\right)-L\left(\sigma_{10}\right)\right]$$
$$+q_t\left[L\left(\sigma\right)+\frac{N_t}{N_b}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta_t}{\partial r}\right)\right\}\right] \quad (21)$$

Where q_t is the embedding parameter which has the range $0\le q_t\le 1$. For our convenience, $L=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)$ is taken as linear operator. The initial guesses θ_{10} and σ_{10} are defined as

$$\theta_{10}\left(r,z\right)=\left(\frac{r^2-h^2}{4}\right),\sigma_{10}\left(r,z\right)=-\left(\frac{r^2-h^2}{4}\right) \quad (22)$$

Define

$$\theta_t\left(r,z\right)=\theta_{to}+q_t\theta_{t1}+q_t^2\theta_{t2}+... \quad (23)$$

$$\sigma\left(r,z\right)=\rho_0+q_t\sigma_1+q_t^2\sigma_2+... \quad (24)$$

The series (23) and (24) are convergent for the most **o**f the cases. The convergent depends on the non linear part of the equation. Adopting same procedure as done by Maruthi Prasad [7], the solution for temperature and nanoparticle phenomena can be written for $q_t=1$ as

$$\theta_t\left(r,z\right)=\left(\frac{r^2-h^2}{64}\right)\left(N_b-N_t\right) \quad (25)$$

$$\sigma\left(r,z\right)=-\left(\frac{r^2-h^2}{4}\right)\frac{N_t}{N_b} \quad (26)$$

Substituting the Eq.(25) and Eq.(26) in the Eq.(16) and applying boundary conditions,
then exact solution for the velocity can be written as

$$u\left(r,z\right)=\left(\frac{r^2-h^2}{4}\right)\left(-\frac{\sin\alpha}{F}+\frac{dP}{dz}\right)$$
$$+B_r\frac{N_t}{N_b}\left(\frac{r^4}{64}-\frac{r^2h^2}{16}+\frac{3h^4}{64}\right) \quad (27)$$
$$-G_r\left(N_b-N_t\right)\left(\frac{r^6}{2304}-\frac{r^2h^4}{256}+\frac{h^6}{288}\right)$$

The dimension less flux q can be calculated as

$$q=\int_0^h 2ru\,dr \quad (28)$$

By substituting the Eq.(27) in Eq. (28) , then the flux is

$$q=\left[\begin{array}{l}-\frac{r^4}{8}\left(-\frac{\sin\alpha}{F}+\frac{dP}{dz}\right)+B_r\frac{N_t}{N_b}h^6\left(0.02083\right)\\-G_r\left(N_b-N_t\right)h^8\left(0.001627\right)\end{array}\right] \quad (29)$$

From Eq. (29), $\frac{dP}{dz}$ can be given as

$$\frac{dP}{dz}=\left[\begin{array}{l}-\frac{8q}{h^4}+\frac{\sin\alpha}{F}+B_r\frac{N_t}{N_b}h^2\left(0.16664\right)\\-G_r\left(N_b-N_t\right)h^4\left(0.0130\right)\end{array}\right] \quad (30)$$

The pressure drop per wave length

$\Delta p=p\left(0\right)-p\left(\lambda\right)$ is

$$\Delta p=-\int_0^1\frac{dp}{dz}dz$$

$$\Delta p=\int_0^1\left[\begin{array}{l}\frac{8q}{h^4}-\frac{\sin\alpha}{F}-B_r\frac{N_t}{N_b}h^2\left(0.16664\right)\\+G_r\left(N_b-N_t\right)h^4\left(0.0130\right)\end{array}\right]dz \quad (31)$$

The resistance to the flow λ is defined as

$$\lambda=\frac{\Delta p}{q}=\frac{1}{q}\int_0^1\left[\begin{array}{l}\frac{8q}{h^4}-\frac{\sin\alpha}{F}-B_r\frac{N_t}{N_b}h^2\left(0.16664\right)\\+G_r\left(N_b-N_t\right)h^4\left(0.0130\right)\end{array}\right]dz \quad (32)$$

The pressure drop in the absence of of stenosis *h=1* is denoted by Δp_n and is
obtained from Eq.(31) as

$$\Delta p_n = \int_0^1 \begin{bmatrix} 8q - \dfrac{\sin\alpha}{F} - B_r \dfrac{N_t}{N_b}\left(0.16664\right) \\ +G_r\left(N_b - N_t\right)\left(0.0130\right) \end{bmatrix} dz \quad (33)$$

The resistance to the flow in the normal artery is denoted by λ_n which is obtained from Eq. (33) as

$$\lambda_n = \frac{\Delta p_n}{q} = \frac{1}{q}\int_0^1 \begin{bmatrix} 8q - \dfrac{\sin\alpha}{F} - B_r \dfrac{N_t}{N_b}\left(0.16664\right) \\ +G_r\left(N_b - N_t\right)\left(0.0130\right) \end{bmatrix} dz \quad (34)$$

The normalized resistance to the flow denoted by

$$\bar{\lambda} = \frac{\lambda}{\lambda_n} \quad (35)$$

The wall shear stress

$$\tau_h = -\frac{h}{2}\frac{dP}{dz} = -\frac{h}{2}\begin{bmatrix} -\dfrac{8q}{h^4} + \dfrac{\sin\alpha}{F} + B_r \dfrac{N_t}{N_b} h^2\left(0.16664\right) \\ -G_r\left(N_b - N_t\right)h^4\left(0.0130\right) \end{bmatrix} \quad (36)$$

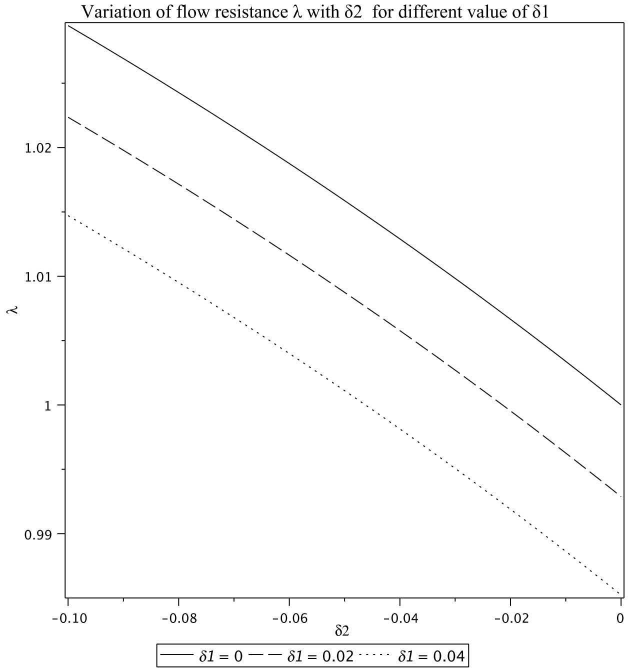

Fig.2,d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:L:=1:F:=0.3:Br:=0.3: Nb:=0.1:Nt:=0.3:Gr:=0.2:qα=0.1$\frac{1}{6}$ π

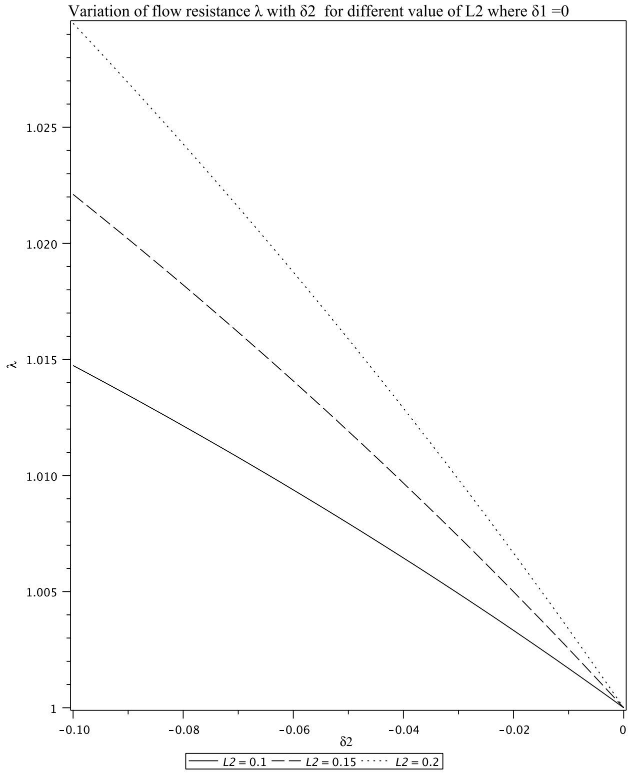

Fig.3, d1:=0.2:d2:=0.2:L1:=0.2:L:=1:F:=0.3:Br:=0.3:Nb:=0.1:Nt:=0.3:Gr:=0.2:q:=0.1:α:=π/6, $\delta_1 = 0$

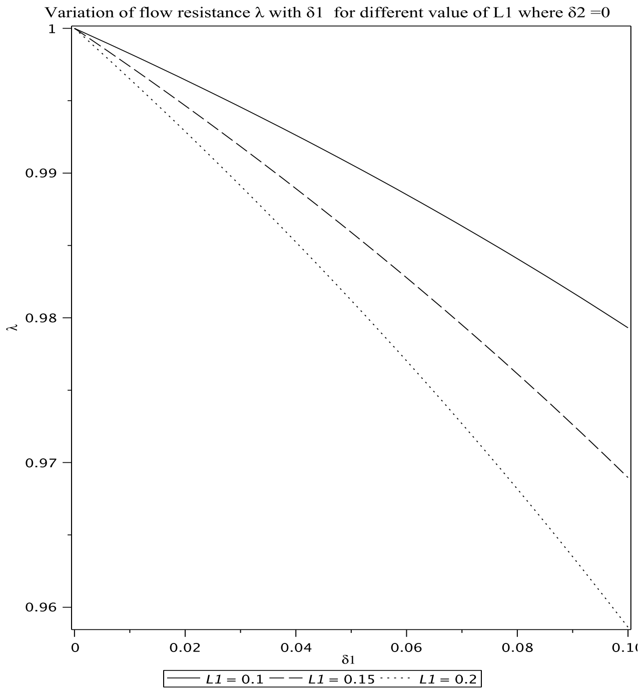

Fig.4, d1:=0.2:d2:=0.2:L2:=0.2:L:=1:F:=0.3:Br:=0.3:Nb:=0.1:Nt:=0.3:Gr:=0.2:q:=0.1:α:=π/6, δ_2 = 0

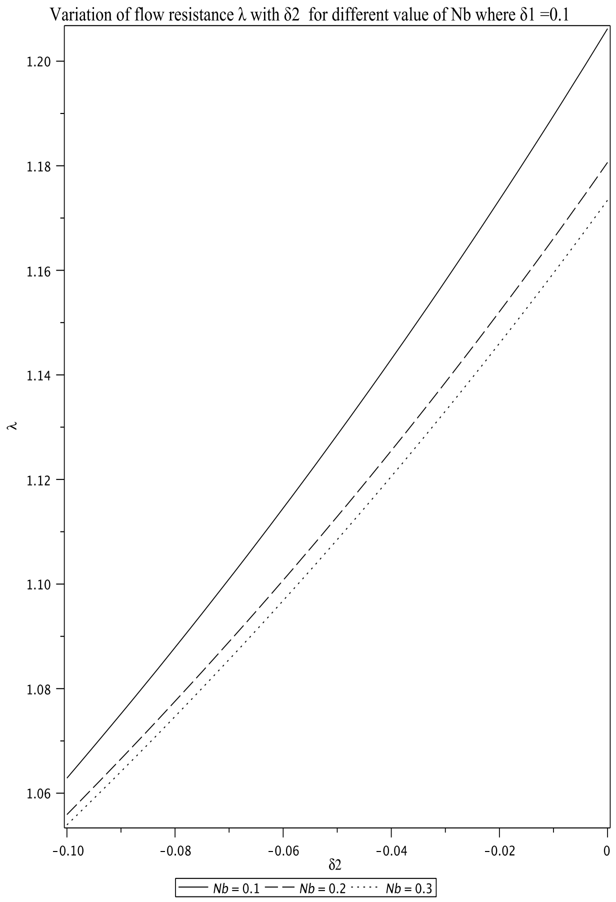

Fig.5, d1=d2:=0.2:L1:=L2:=0.2:F:=0.3: Br:=0.3:Nt:=0.3,Gr:=0.2:q:=0.3:L:=1: α:=π/6,δ_1 = 0.1

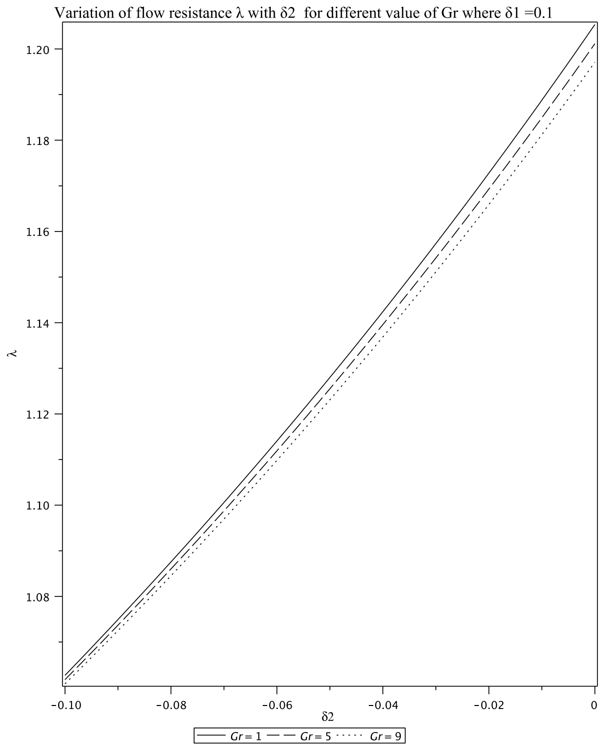

Fig.6, d1:=d2:=0.2:L1:=L2:=0.2:F:=0.3: Br:=0.3:Nb:=0.1:Nt:=0.3:q:=0.3:L:=1: α:=π/6δ_1 = 0.1

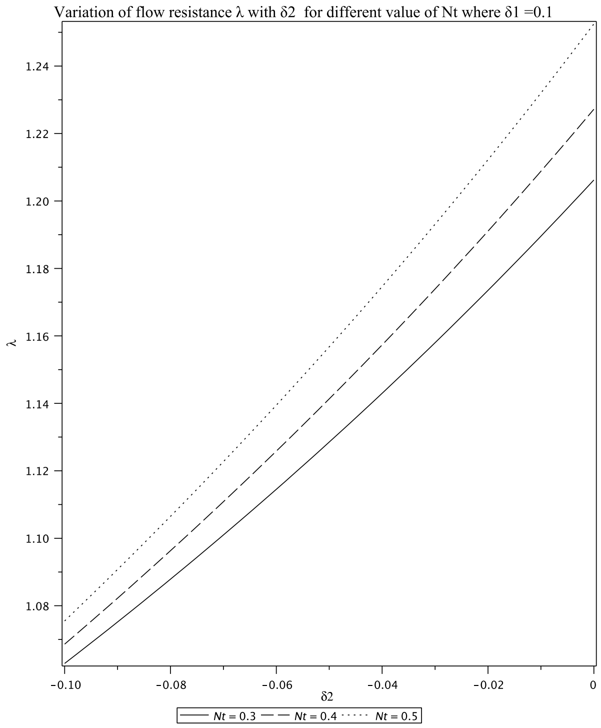

Fig.8, d1:=d2:=0.2:L1:=L2:=0.2:F:=0.3: Br:=0.3:Nb:=0.1:Gr:=0.2:q:=0.3:L:=1: α:=π/6$\boldsymbol{\delta}_1$ = 0.1

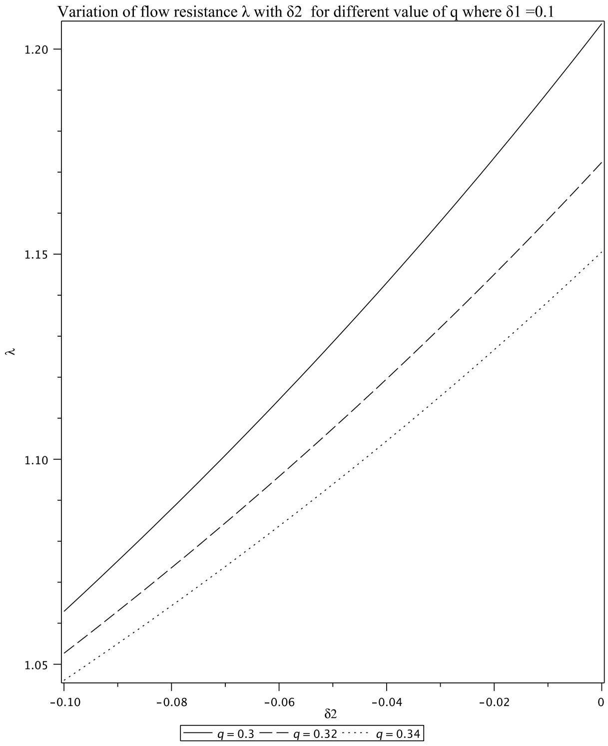

Fig.7, d1:=d2:=0.2:L1:=L2:=0.2:F:=0.3: Gr:=0.2:Nb:=0.1:Nt:=0.3:α:=π/6:L:=1: Br:=0.3: $\delta_1 = 0.1$

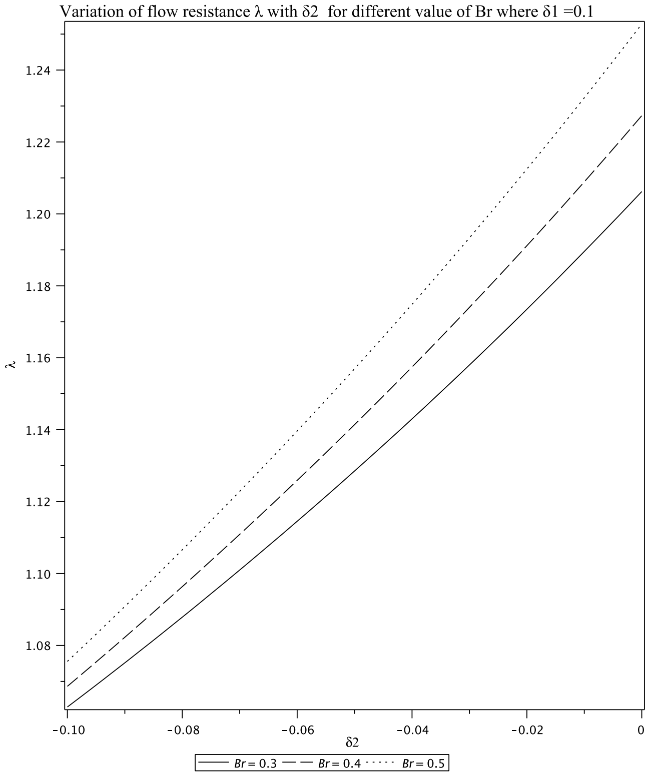

Fig.9, d1:=d2:=0.2:L1:=L2:=0.2:F:=0.3: Gr:=0.2:Nb:=0.1:Nt:=0.3:q:=0.3:L:=1: α:=π/6 $\delta_1 = 0.1$

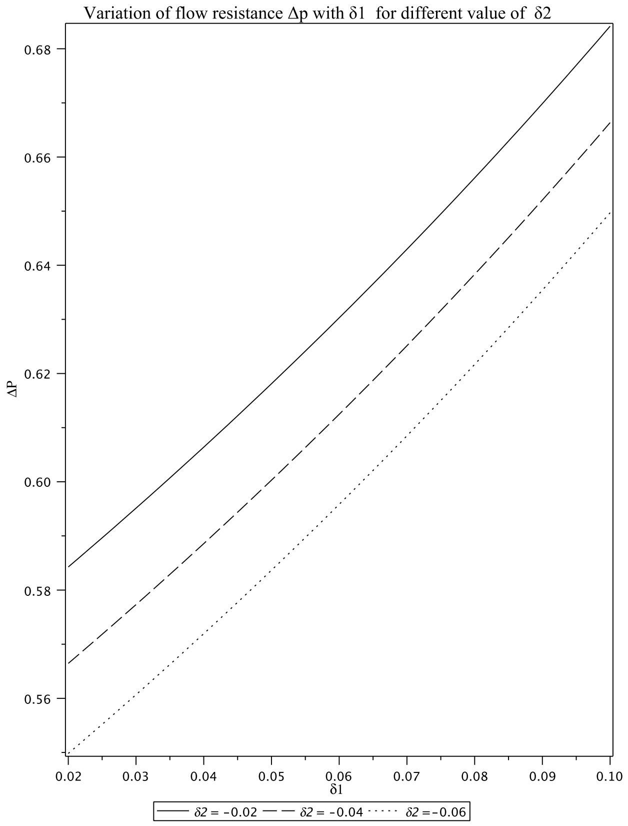
Variation of flow resistance Δp with δ1 for different value of δ2

Fig.10, d1=d2=0.2:L1=L2=0.2:q=0.3:F=0.3:Br=0.3: Gr=0.2:Nb:=0.1:Nt:=0.3:α:=π/6:L:=l

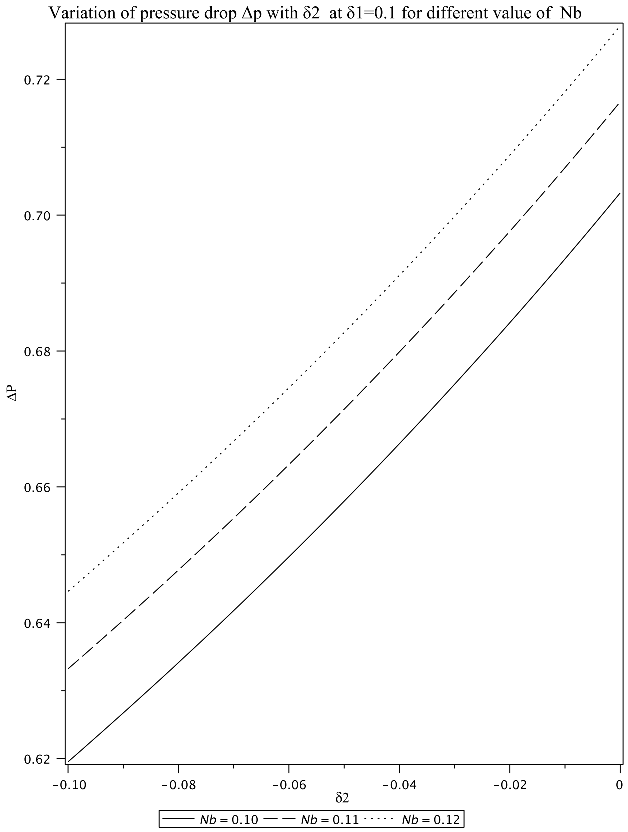
Variation of pressure drop Δp with δ2 at δ1=0.1 for different value of Nb

Fig.11, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:q:=0.3:F:=0.3:Br:=0.3: Gr:=0.2:delta1:=0.1:Nt:=0.3:α:=π/6:L:

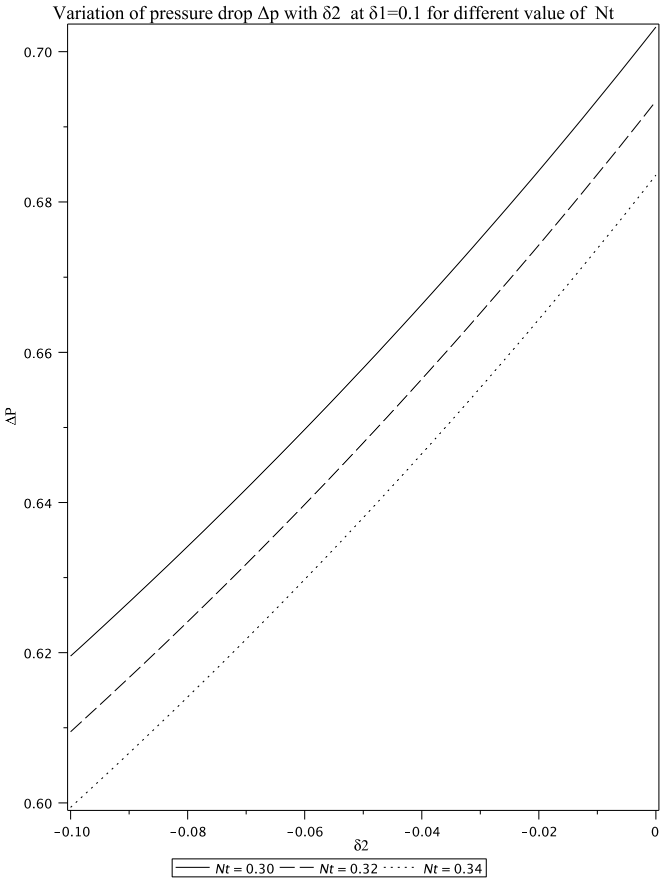
Variation of pressure drop Δp with δ2 at δ1=0.1 for different value of Nt

Fig.12, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:q:=0.3:F:=0.3:Br:=0.3: Gr:=0.2:delta1:=0.1:Nb:=0.1:α:=π/6:L

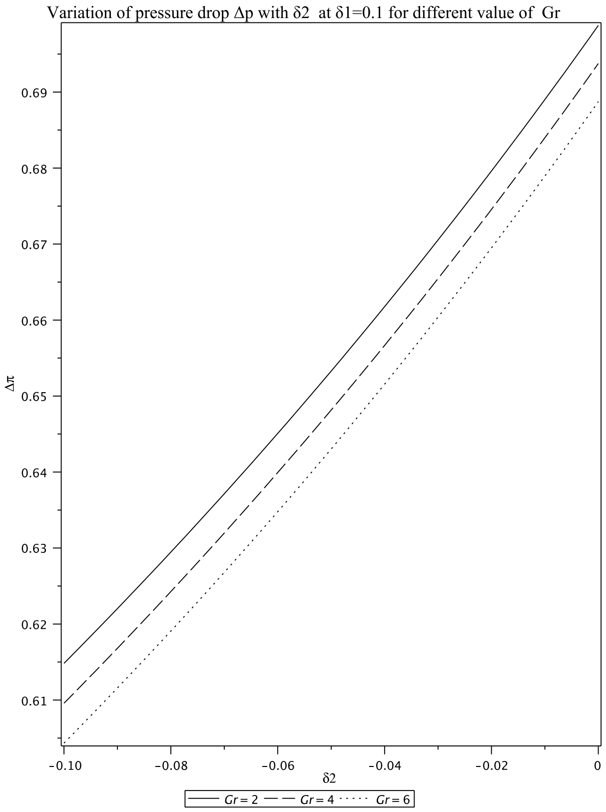
Variation of pressure drop Δp with δ2 at δ1=0.1 for different value of Gr

Fig.13, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:q:=0.3:F:=0.3:Br:=0.3: Nt:=0.3:delta1:=0.1:Nb:=0.1:α:=π/6:L:

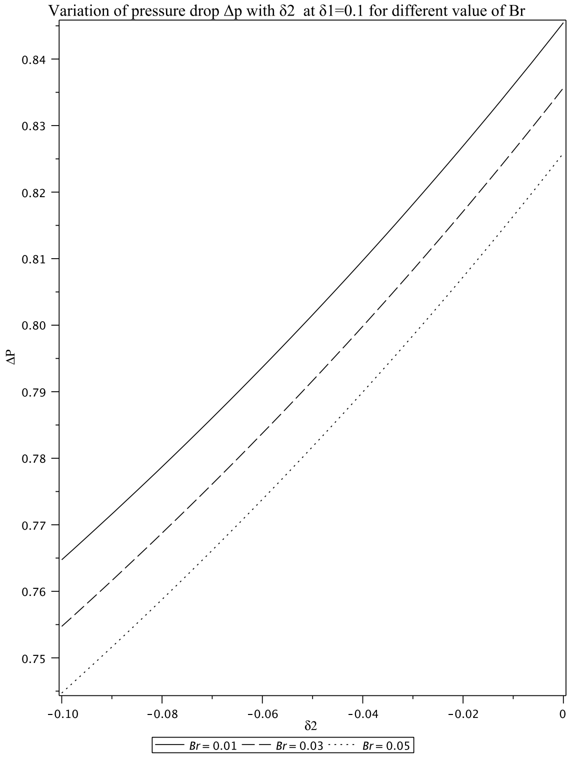

Fig.14, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:q:=0.3:F:=0.3:Gr:=0.2: Nt:=0.3:delta1:=0.1:Nb:=30.1:α:=π/6:L

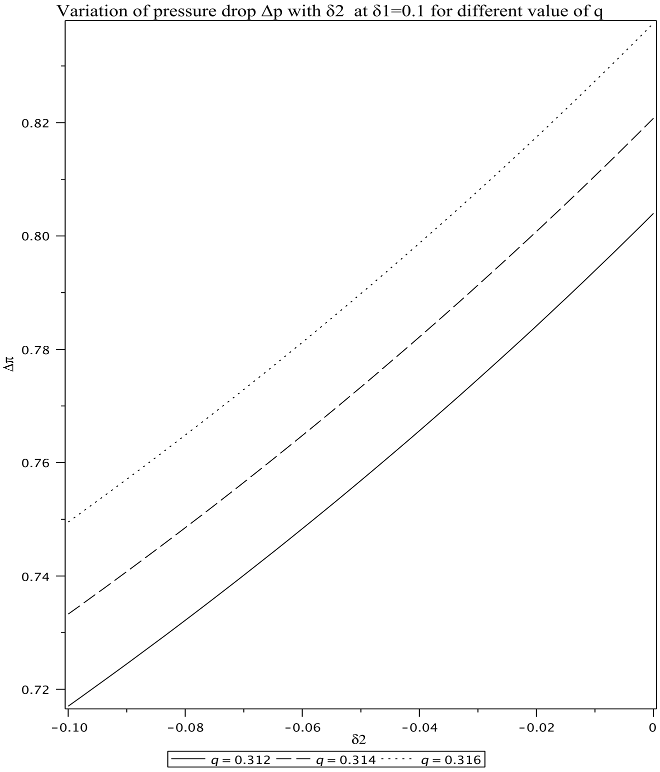

Fig.16, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:alpha:=Pi/6:F:=0.3:Gr:=0.2: Nt:=0.3:delta1:=0.1:Nb:=30.1:Br:=0.

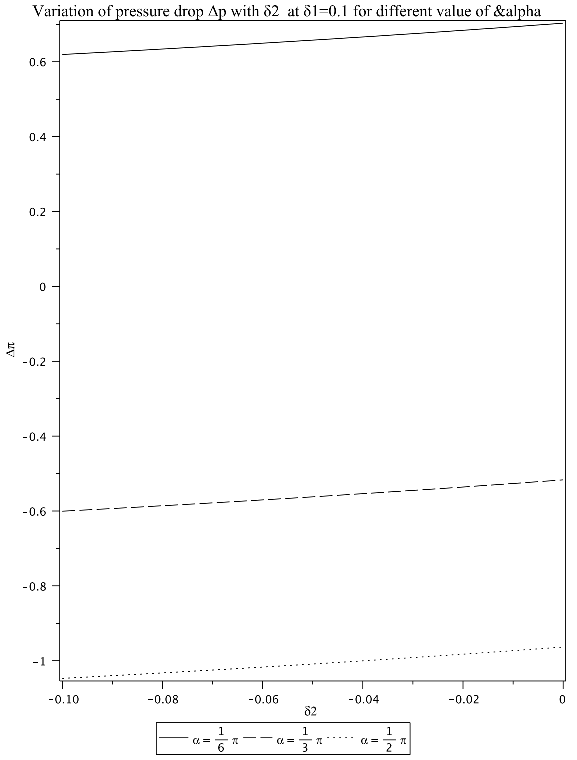

Fig.15, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:q:=0.3:F:=0.3:Gr:=0.2: Nt:=0.3:δ1:=0.1:Nb:=30.1:Br:=0.3:L:

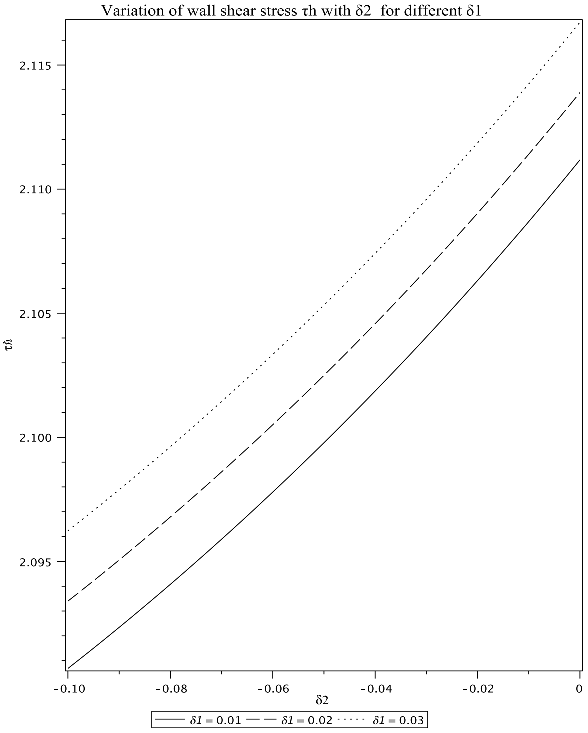

Fig.17, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:alpha:=Pi/6:F:=0.3:Gr:=0.2: Nt:=0.3:q:=0.3:Nb:=30.1:Br:=0.3:

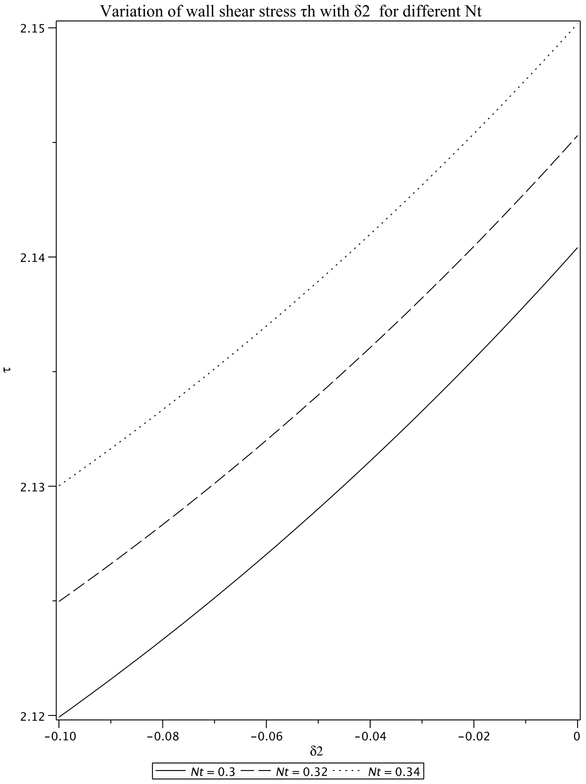

Fig.18, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:alpha:=Pi/6:F:=0.3:Gr:=0.2: Nb:=0.1:q:=0.3:Br:=0.3:L:=1:q:=

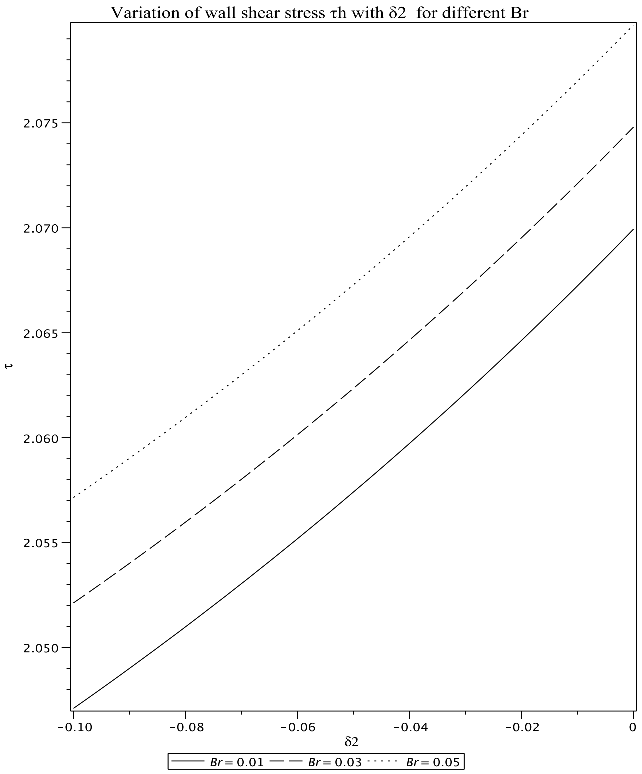

Fig.20, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:alpha:=Pi/6:F:=0.3:Nt:=0.3: Nb:=0.1:q:=0.3:Gr:=0.32:L:=1:q:=

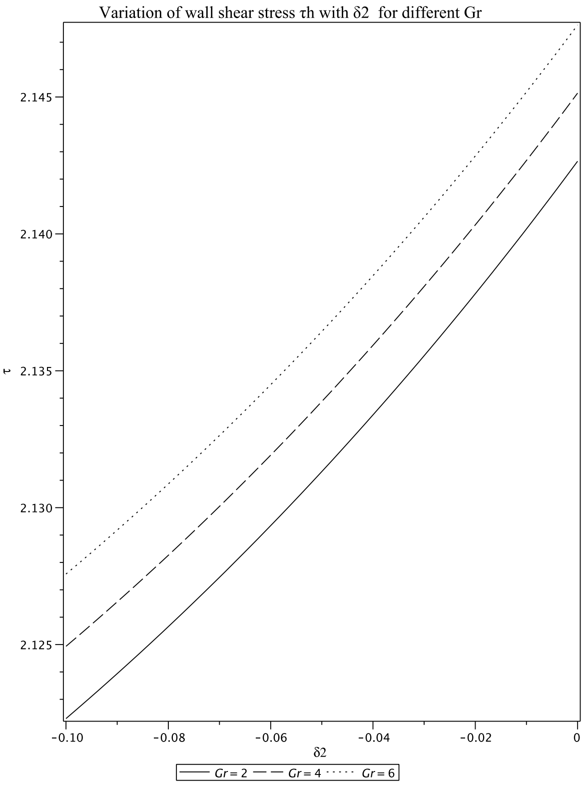

Fig.19, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:alpha:=Pi/6:F:=0.3:Nt:=0.3: Nb:=0.1:q:=0.3:Br:=0.3:L:=1:q:=

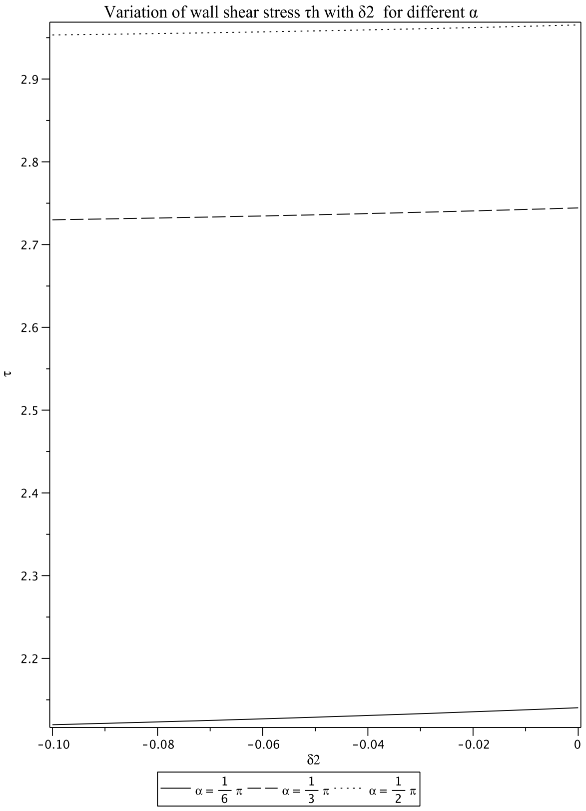

Fig.21, d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:Br:=0.3:F:=0.3:Nt:=0.3: Nb:=0.1:q:=0.3:Gr:=0.32:L:=1:q:=0

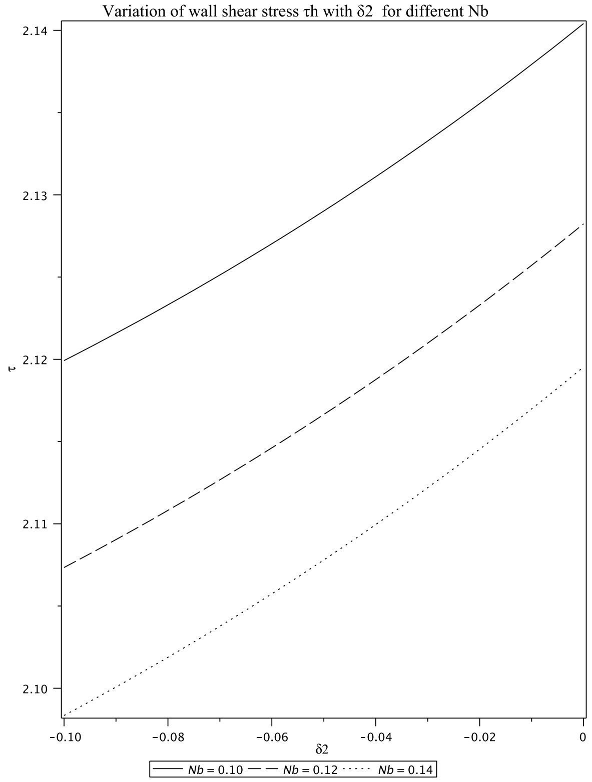

Fig.22 , d1:=0.2:d2:=0.2:L1:=0.2:L2:=0.2:alpha:=Pi/6:F:=0.3:Gr:=0.2: Nt:=0.3:q:=0.3:Br:=0.3:L:=1:q:=

4. Result And Discussison

The analytical solution of pressure drop (Δp), resistance to the flow ($\bar{\lambda}$) and wall shear stress (τ_h) are given in equation (31), Eqn. (35) and Eqn. (36). The effects of various parameters are presented graphically.

It is observed from Fig. 2, as flow resistance decreases with height of the stenosis and increases with the length of post stenotic dilatation (see Fig. 3), further it is observed that as length of the stenosis (δ_1)increases it flow resistance decreases (Fig. 4).

It is also observed from Figs. (5-9) that Brownian motion number (N_b), Grashof number (G_r), volumetric flow rate (q) increases resistance to the flow decreases (Figs. 5-7), where as $\bar{\lambda}$ increases Brownian motion number (N_b),local nanoparticle Grashof number (B_r) increases ,(Figs. 8-9).

The variation of pressure drop (Δp) for different parameters are given in Figs. (10-16). It is seen from these figures as pressure drop (Δp) , height of the dilatation (δ_2) , thermophoresis parameter (N_t), Grashof number (G_r), local nanoparticle Grashof number (B_r), inclination (α) increases pressure drop increases (Figs. 10,12-15), whereas increase in Brownian motion number (N_b) and volumetric flow rate (q) increases the pressure drop (Figs. 11,16).

Figures (17-22) depicts the variations of wall shear stress(τ_h) with variations of different parameters. It is noticed from Figs. (17-21) that wall shear stress increases as height of the stenosis (δ_1),thermophoresis parameter (N_t),Grashof number (G_r),local nanoparticle Grashof number (B_r), and inclination (α) increases and decreases with Brownian motion number (N_b) (Fig.22).

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