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Free Convective Heat Transfer over Truncated Cone with Convective Heating: Numerical Analysis

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Abstract

This paper is focused on the study of heat transfer characteristics of MHD non-Newtonian Tangent Hyperbolic boundary layer flow over a truncated cone with Biot number effect. The governing partial differential equations of the flow field are converted to a system of non-linear coupled non-similarity ordinary differential equations. Employing Shooting technique followed by finite centered difference technique (Keller Box Method), the system is solved numerically. The numerical code is validated with previous studies. The results of flow and heat analysis on velocity and temperature profiles are given diagrammatically. The numerical results point out that, increasing MHD parameter (M) serves to decelerate the flow, but increased temperature values and increase in the Biot number (Bi) is observed to enhance velocity and temperature.

Keywords: *Lorentz force, Convective boundary condition, Keller-box numerical method, Casson Viscoplastic model.*

1. Introduction

Magnetic field effects on a conducting fluid received good attention from researchers. The phenomenon of Magnetohydrodynamic is important with regard to tremendous applications like electrical power generation, solar system technology, space vehicles, missiles, propulsion devices for aircraft, nuclear plants, astrophysical flows, and many others. Although ample studies were generated for the boundary layer flow in the presence of thermal radiation, the fluid thermal conductivity in such cases is treated as a constant. These include petroleum drilling muds [1], biological gels [2] and polymer processing [3]. Amanulla et al. [4] presented extensive numerical solutions for Magneto-hydrodynamic and non-Newtonian fluid flow over vertical cone with the presence of slip effects, using finite difference method. Cheng [5] investigated heat transfer of power law nanofluid over a truncated cone embedded in a porous medium. Elbashbeshy et al. [6] analyzed radiation effects non-Newtonian fluid flow past a vertical truncated cone. Subba Rao et al. [7] discussed the Newtonian heating on hydrodynamic fluid flow over a circular cylinder. Reddy and Pradeepa [8] studied viscous disspation and sorret effects on natural convection flow over a truncated cone in the presence of biot number. Several authors [9-11] investigated heat transfer of the non-Newtonian fluid of various physical problems.

The present work, we examine theoretically and computationally the steady-state transport phenomena in magneto-hydrodynamic non-Newtonian fluid flow from a truncated cone with biot number effects. Magnetic fields have been found to profoundly influence heat transfer and velocity characteristics in curved body flows. Relevant examples include Bég et al. [12] (for cylindrical geometries), Alkasasbeh et al. [13] who addressed radiative effects, who considered porous medium drag effects and Kasim et al. [14] who used a viscoelastic model.

2. MATHEMATICAL VISCOPLASTIC FLOW MODEL

Major Consider the steady, viscous, two-dimensional, incompressible, free convection flow from a non-isothermal vertical permeable cone embedded in a Casson non-Newtonian fluid. Figure 1(a) depicts the flow model and physical coordinate system.

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right) e_{ij}, \pi \geq \pi_c \\ 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right) e_{ij}, \pi < \pi_c \end{cases} \quad (1)$$

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In which and is the (i,j)th component of deformation rate, denotes the product of the product base deformation rate with itself, shows a critical value of this product based on the non-Newtonian model, the plastic dynamic viscosity of non-Newtonian fluid and the yield stress of fluid.

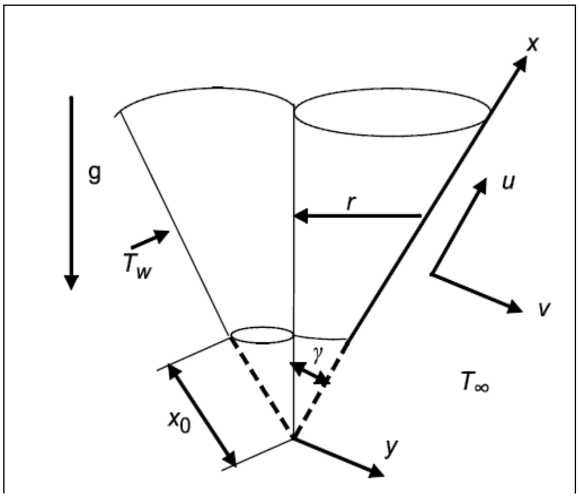

Fig. 1. Schematic diagram of the problem

The equations for mass continuity, momentum and energy, can be written as follows:

$$\frac{\partial(ur)}{\partial x}+\frac{\partial(vr)}{\partial y}=0 \qquad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu(1-n)\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

The boundary conditions for the considered flow with conductive heating:

At $y=0, u=0,\ v=0,\ -k\dfrac{\partial T}{\partial y}=h_w(T_w-T)$

As $y\to\infty,\ u\to 0,\ v\to 0,\ T\to T_\infty$ (5)

The stream function ψ is defined by $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, and therefore, the continuity equation is automatically satisfied. In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced:

$$\eta=C_1 y x^{1/4}, C_1=\frac{g\beta(T_w-T_\infty)}{4\nu^2},$$

$$f(\xi,\eta)=\frac{\psi}{4\nu C_1 x^{3/4}}, \xi=\left(\frac{x}{L}\right)^{1/2}$$

$$\theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \phi(\xi,\eta)=\frac{C-C_\infty}{C_w-C_\infty} \qquad (6)$$

$$Gr=\frac{g\beta(T_w-T_\infty)L^3}{\nu^2}$$

The transformed boundary layer equations for momentum and energy emerge as:

$$(1-n)f'''+3ff''-2f'^2+nWef''f''' +\theta-Mf'=2\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \qquad (7)$$

$$\frac{\theta''}{\Pr}+3f\theta'=2\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \qquad (8)$$

Along the transformed boundary conditions (4) are

At $\eta=0, f=0, f'=1, \theta'=-\gamma(1-\theta(0))$

As $\eta\to\infty,\ f'\to 0,\ \theta\to 0$ (9)

The skin-friction coefficient (cone surface shear stress function) and Nusselt number (heat transfer rate) can be defined using the transformations described above with the following expressions:

$$C_f=4\nu\mu C_1^3 x^{1/4}\left((1-n)f''(\xi,0)+\frac{n}{2}Wef''(\xi,0)\right) \qquad (10)$$

$$Nu=-k\Delta T C_1 x^{-1/4}\theta'(\xi,0) \qquad (11)$$

3. COMPUTATIONAL FINITE DIFFERENCE SOLUTIONS

In this study, the efficient implicit (finite difference) Keller-Box method has been employed to solve the general flow model defined by equations (6) – (7) with boundary conditions (8). This method was originally developed for low speed aerodynamic boundary layers by Keller [15]. This method has been used extensively and effectively for over three decades in a large spectrum of nonlinear fluid mechanics problems. These include laminar transport phenomena [16] and viscoplastic boundary layer flows [17,18].

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4. Results and Discussion

The Comprehensive solutions have been obtained with Keller Box Method and are presented in Figs. 2a-b to 7a-b. The numerical problem comprises three dependent thermo-fluid dynamic variables and five multi-physical control parameters, We, M, Pr and Bi. The influence of stream wise space variable ξ is also investigated. For accuracy, we compared our results with Yih [19] and Reddy and Pradeepa [8] and we observed that a very good agreement with previous results as shown in Table1.

Figure 2a-2b displays the impact of viscoelastic flow parameter (We) on velocity, and temperature profiles severally. Velocity profile and boundary layer thickness decrease for the higher values of viscoelastic flow parameter (We) as shown in figure 2a. However, via coupling of the energy eqn. (7) and momentum equation (free convection), the effect of viscoelastic parameter is indirectly transmitted to the temperature field. Since the Weissenberg number is also present in the wall boundary condition, the acceleration effect is only confined to the region close to the cone surface. Overall however the dominant influence of We, is near the wall and is found to be assistive to momentum development. Only a very small decrease in temperature is observed with a large enhancement in Weissenberg number, as shown in Fig. 2b.

The influence of power law index (n), on the velocity (f') and temperature (θ) in the presence of the MHD $(M = 1.0)$ is displayed in **Figs. 3a-b.** It is observed that the velocity profile is significantly decrease with increasing rheological power law index (n). Momentum diffusion is reduced and hydrodynamic boundary layer thickness is decreased at the cone surface. Conversely, temperature is consistently reduced with increasing values of power law index.

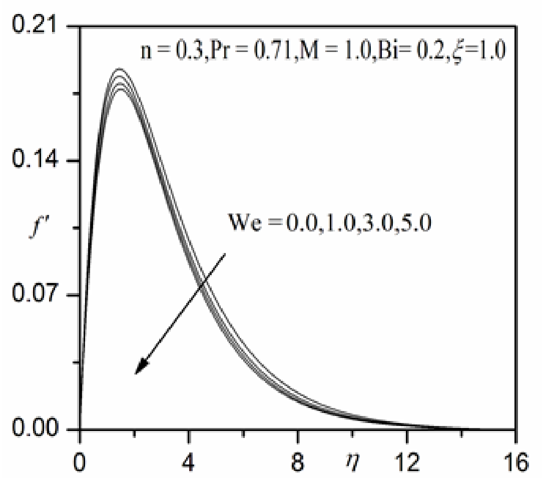

Fig. 2a. Effect of Viscoelastic fluid parameter *We* over velocity profile

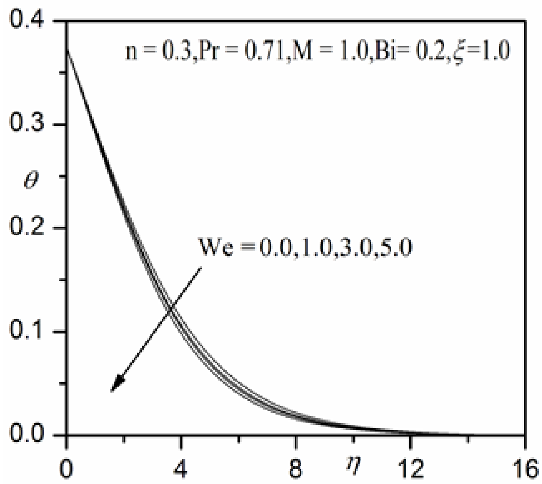

Fig. 2b. Effect of Viscoelastic fluid parameter *We* over Temperature profile

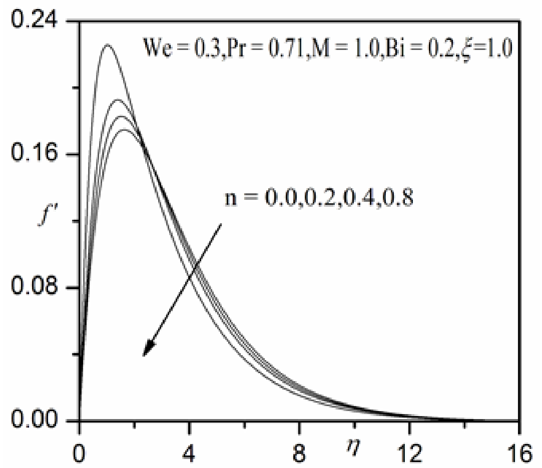

Fig. 3a. Effect of Power law index over Velocity profile

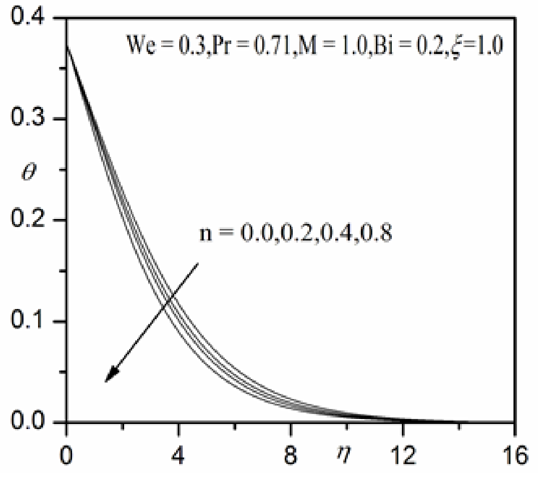

Fig. 3b. Effect of Power law index over Temperature profile

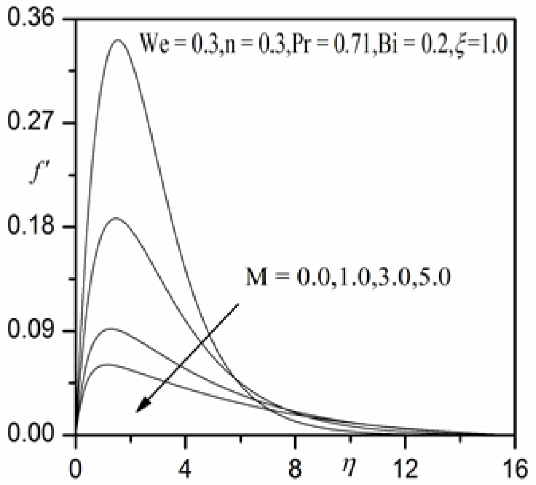

Fig. 4a. Effect of magnetic field parameter over velocity profile

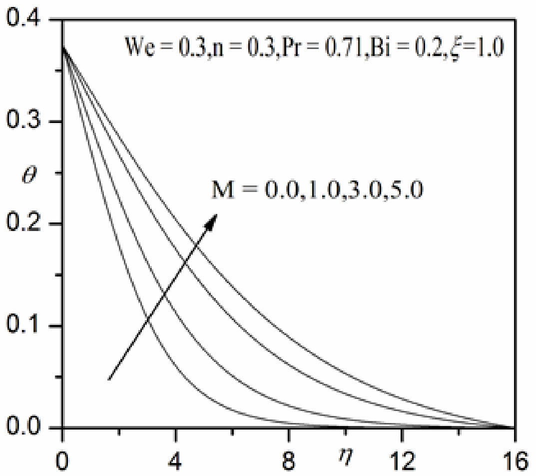

Fig. 4b. Effect of magnetic field parameter over Temperature profile

In figure 4a, it's determined that the velocity profile decreases with the rise of the magnetic parameter values, as a results of the presence of a magnetic field in an electrically conducting fluid introduces a force referred to as the Lorentz force, that acts against the flow if the magnetic field is applied in the normal direction, as within the gift study. This resistive force slows down the fluid velocity. For the temperature profiles will increase by increasing the magnetic parameter component as display in figure 4b. These results concur with other investigations of magnetic non-Newtonian heat transfer including Kasim *et al*. [14] and Megahed [20].

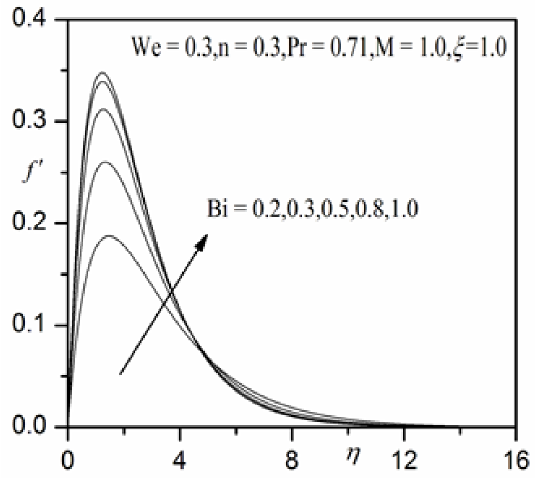

Fig. 5a. Influence of Biot number parameter over Velocity profile

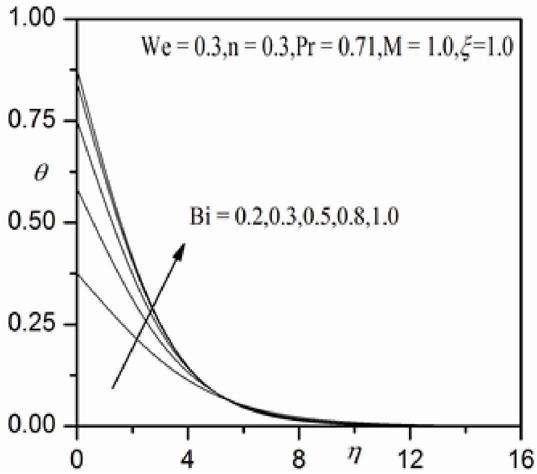

Fig. 5b. Influence of Biot number parameter over temperature profile

Figs. 5a-b illustrate the variation of velocity and temperature with transverse coordinate (η), for different values of Biot number (*Bi*). Biot number is imposed in the augmented wall boundary condition in eqn. (8). With increasing Biot Number, more heat is transmitted to the fluid and this energizes the boundary layer. This also leads to a general

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acceleration as observed in fig. 5a and also to a more pronounced restoration in temperatures in fig. 5b, in particular near the wall. The effect of Biot number is progressively reduced with further distance from the wall (cone surface) into the boundary layer and vanishes some distance before the free stream. A similar response has been observed by Rao *et al.* [7].

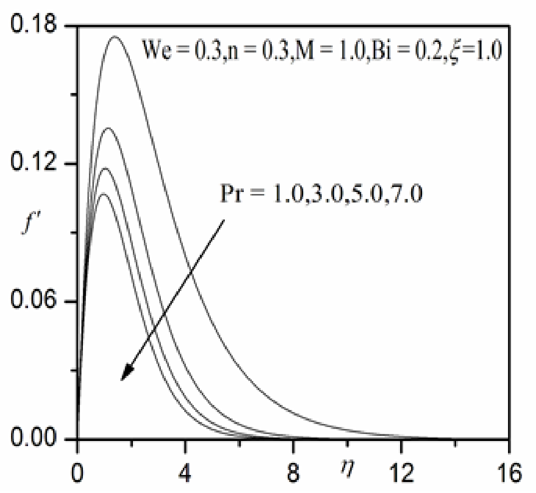

Fig. 6a. Influence of Prandtl number over Velocity profile

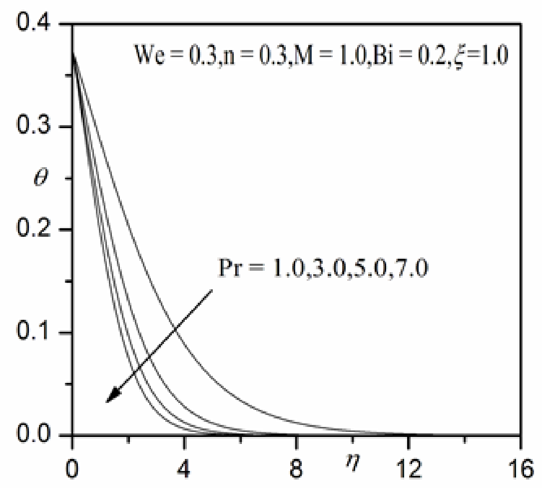

Fig. 6b. Influence of Prandtl number over temperature profile

Figure 6a-6b shows the variation of the velocity and temperature profile for numerous values of Prandtl number (Pr). For *Pr equal to unity* both the momentum and thermal diffusion rates are the same, as are the momentum and thermal boundary layer thicknesses. An increment in *Pr* from 1.0 to 7.0, which corresponds to increasing momentum diffusivity and decreasing thermal diffusivity, results in a tangible reduction in velocity magnitudes throughout the boundary layer. For $Pr < 1$, thermal diffusivity surpasses momentum diffusivity i.e. heat will diffuse at a faster rate than momentum. In this manner for lower *Pr* fluids (e.g. $Pr = 0.71$ which physically relate to liquid metals), the flow will be accelerates whereas for greater Pr fluids (e.g. $Pr = 1$ for low weight molecular polymers [10]). The result shows that an increases of Prandtl number leads to a decreasing the thermal boundary layer thickness. it will be strongly decelerated, as observed in Fig.6b. For $Pr < 1$, the momentum boundary layer thickness is lesser than thermal boundary layer thickness.

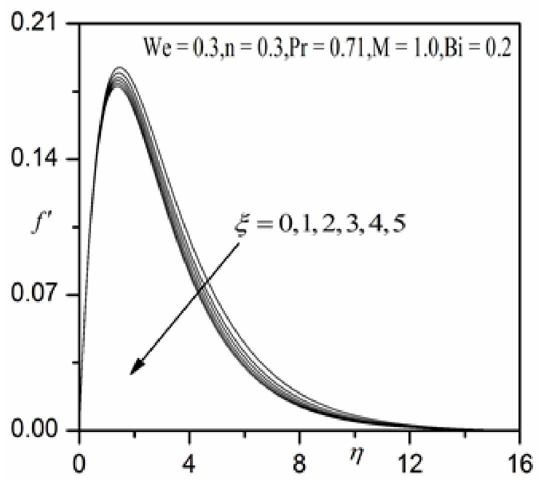

Fig. 7a. Influence of ξ over Velocity profiles

Figs. 7a-b present the distributions for velocity and temperature fields with stream wise coordinate ξ, for the viscoelastic. Increasing ξ values correspond to progression around the periphery of the cone, from the leading edge ($\xi=0$). As ξ *increases, there is a weak deceleration in the flow (fig. 7a),* which is strongest nearer the cone surface and decays with distance into the free stream. Conversely there is a weak elevation in temperatures (fig. 7b) with increasing stream wise coordinate.

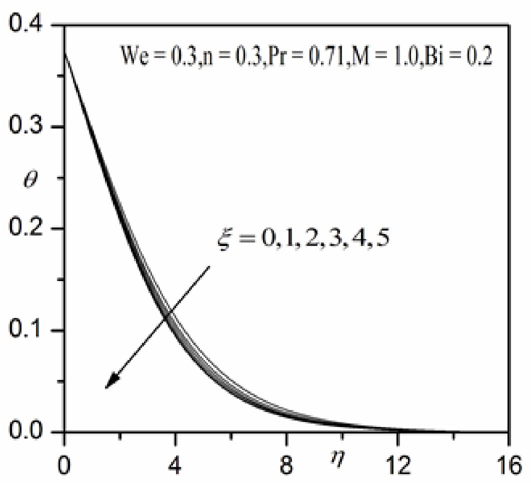

Fig. 7b. Influence of ξ over Temperature profile

Table 2 presents the influence of magnetic body force parameter (*M*), the power law index (*n*), along with a variation in the Weissenberg number (*We*). It is observed that the increasing *M* and power law index, skin friction is reduced. And increasing *We*, accelerates the Skin friction.

Table 2 Numerical values of skin friction coefficient for different values of M, We, n

n	M	We=0.0	We=0.5	We=1.0
0.0	0.0	0.4333	0.4333	0.4333
0.1	0.0	0.4183	0.4193	0.4201
0.2	0.0	0.4015	0.4029	0.4035
0.3	0.5	0.2966	0.2964	0.2963
0.3	1.0	0.2479	0.2478	0.2476
0.3	1.5	0.2167	0.2167	0.2166

6. Conclusions

In this paper, we presented a mathematical model for steady, MHD, two-dimensional incompressible tangent hyperbolic flow through an isothermal truncated cone with biot number effect. Very good correlation between the present computations and the trends of other previous studies has been identified. The most important results in summary are:

1. Enhancing the effect of the magnetic field parameter has the impact of decreasing the fluid flow and increasing temperature.
2. Raising the Weissenberg number (We) and power law index parameter (n), reduced the velocity near the cone surface and also decrease temperature throughout the boundary.
3. Increasing Biot number, increases velocity and temperature for all values of radial coordinate i.e. throughout the boundary layer regime.

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