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Fuzzy economic production lot-size model under imperfect production process with cloudy fuzzy demand rate

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Abstract

The aim of the article is to develop classical economic production lot-size (EPL) model of an item produced in imperfect production process with fixed set up cost and without shortages in fuzzy environment where demand rate of an item is cloudy fuzzy number and production rate is demand dependent. In general, fuzziness of any parameter remains fixed over time but in practice, fuzziness of parameter begins to reduce as time progress because of gathering experience and knowledge. The model is solved in crisp, general fuzzy and cloudy fuzzy environment using Yager's index method and De and Beg's ranking index method and comparison are made for all cases. Here, the average cost function is minimized using dominance based Particle Swarm Optimization (DBPSO) algorithm to find decision for the decision maker (DM). The model is illustrated with some numerical examples and some sensitivity analyses have been done to justify the notion.

Keywords: *EPL, Reliability, De and Beg's ranking index method, cloudy fuzzy number, DBPSO.*

1. Introduction

In the development of economic production lot-size model, usually researchers consider the demand rate as constant in nature. In the real world, it is observed that these quantities will have little changes from the exact values. Thus in practical situations, demand variable should be treated as fuzzy in nature. Recently fuzzy concept is introduced in the production/ inventory problems. At first, Zadeh (1965) introduced the fuzzy set theory. After that, it has been applied by Bellman and Zadeh (1970) in decision making problems. Numerous researches have been done in this area. Researchers like Kaufmann and Gupta(1992), Mandal and Maiti (2002), Maiti et.al (2014), Maiti and Maiti(2006,2007), Bera and Maiti (2012), Mahata and Goswami(2007, 2013), De and Sana(2015) etc. have investigated extensively over this subject. Kau and Hsu(2002) developed a lot-size reorder point inventory model with fuzzy demands. In this study, a cloudy fuzzy inventory model is developed depending upon the learning from past experience. In defuzzification methods, specially on ranking fuzzy numbers, after Yager (1981), some researchers like Ezzati et at. (2012), Deng (2014), Zhang et al. (2014) and others adopted the method for ranking of fuzzy numbers based on centre of gravity. Moreover, De and Beg (2016) and De and Mahata (2016) invented new defuzzification method for triangular dense fuzzy set and triangular cloudy fuzzy set respectively. In this model, fuzziness depends upon time. As the time progress, fuzziness become optimum at the optimum time. This idea is incorporated in cloudy fuzzy environment. ***Till now***, n***one has addressed this type of realistic production inventory model with cloudy fuzzy demand rate***.

In the classical economic production lot-size (EPL) model, the rate of production of single item or multiple items is assumed to be inflexible and predetermined. However, in reality, it is observed that the production is influenced by the demand. When the demand increases, consumption by the customer obviously more and to meet the additional requirement of the customer, the manufactures bound to increase their production. Converse is true for reverse situation. In this connection, several researchers developed EPL models for single/multiple items considering either uniform or variable production rate (depend on time, demand and/or on hand inventory level). Bhunia and Maiti (1997), Balkhi and Benkherouf (1998), Abad (2000), Mandal and Maiti (2000) etc. developed their inventory models considering either uniform or variable production rate. However, manufacturing flexibility has become more important factor in inventory management. Different types of flexibility in manufacturing system have been identified in the

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literature among which volume flexibility is the most important one. Volume flexibility of a manufacturing system is defined as its ability to be operated profitably at different output levels. Cheng (1989) first developed the demand dependent production unit cost in EPQ model; Khouja (1995) introduced volume flexibility and reliability consideration in EPQ model. Shah and shah (2014) developed EPQ model for time declining demand with imperfect production process under inflationary conditions and reliability.

Items are produced using conventional production process with a certain level of reliability. Higher reliability means that the products with acceptable quality are more consistently produced by the process reducing the cost of scraps, rework of substandard products, wasted materials, labor hours etc. A considerable number of research paper have been done on imperfect production by Rosenblatt and Lee(1986), Ben-Daya and Hariga(2000), Goyal et al. (2003), Maiti et al. (2006), Sana et al. (2007), Manna et al. (2014), Pal et al. (2014), etc. Recently, Manna et al. (2016) considered multi-item EPQ model with learning effect on imperfect production over fuzzy random planning horizon. Khara et al. (2017) developed an inventory model under development dependent imperfect production and reliability dependent demand.

Use of soft computing techniques for inventory control problems is a well established phenomenon. Several authors use Genetic Algorithm (GA) in different forms to find marketing decisions for their problems. Pal *et al*. (2009) uses GA to solve an EPQ model with price discounted promotional demand in an imprecise planning horizon. Bera and Maiti (2012) used GA to solve multi-item inventory model incorporating discount. Maiti *et al*.(2009) used GA to solve inventory model with stochastic lead time and price dependent demand incorporating advance payment. Mondal and Maiti (2002), Maiti(2006,2007), Maiti et.al (2014) many other researchers uses GA in inventory control problems. Also, Bhunia and Shaikh (2015) used PSO to solve two-warehouse inventory model for deteriorating item under permissible delay in payment. ***Here, dominance based particle swarm optimization has been developed to solve this fuzzy inventory model.***

Here, fuzzy inventory model under imperfect production process with cloudy fuzzy demand rate is developed where production rate is demand dependent. The model is solved in crisp , general fuzzy and cloudy fuzzy environment using Yager's index method and De and Beg's ranking index method for defuzzification and compare the results obtained in crisp, fuzzy and cloudy fuzzy environment. In this study, objective is to minimize average total cost to obtain the optimum order quantity and the cycle time using dominance based Particle Swarm Optimization (PSO) algorithm to find decision for the decision maker (DM). The model is illustrated with some numerical examples and some sensitivity analyses have been presented.

2. Definitions and Preliminaries

2.1 Normalized General Triangular Fuzzy Number (NGTFN):

A NGTFN $\tilde{A} = (a_1, a_2, a_3)$ (cf. Fig-1) has three parameters a_1, a_2, a_3 where $a_1 < a_2 < a_3$ and is characterized by its continuous the membership function $\mu_{\tilde{A}}(x): X \to [0,1]$, where X is the set and $x \in X$, is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, \quad otherwise \end{cases}$$

(1)

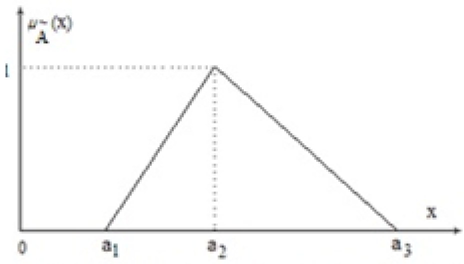

Fig-1: Membership function of a triangular fuzzy number

2.2 α -Cut of a fuzzy number:

A α cut of a fuzzy number $\tilde{A}$ in X is denoted by A_α and is defined as crisp set A_α = $\{x : \mu_{\tilde{A}}(x) \ge \alpha, x \in X\}$ *where* $\alpha \in [0,1]$. Here, A_α is a non-empty bounded closed interval contained in X and it can be denoted by $A_\alpha = [A_L(\alpha), A_R(\alpha)]$ where $A_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ is alled left α -cut and $A_R(\alpha) = a_3 - (a_3 - a_2)\alpha$ is called the right α -cut of $\mu_{\tilde{A}}(x)$ respectively. (2)

2.3 Yager's Ranking Index:

If $A_L(\alpha)$ *and* $A_R(\alpha)$ be the left and right α cuts of a fuzzy number $\tilde{A}$ then the Yager's Ranking index is computed for defuzzification as $I(\tilde{A}) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]\, d\alpha = \frac{1}{4}(a_1 + 2a_2 + a_3)$ (3)

Also, the degree of fuzziness (d_f) is defined by the formula $d_f = \frac{U_b - L_b}{m}$ where U_b and L_b are the upper and lower bounds of the fuzzy numbers respectively and m being their respective mode.

2.**4 Cloudy Normalized Triangular Fuzzy Number (CNTFN) (De and Beg (2016)):**

After infinite time, the normalized triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ becomes a crisp singleton then fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is called the cloudy fuzzy number. This means that both $a_{1,}\ a_3 \to a_2$ *as* $t \to \infty$.

So, the cloudy fuzzy number takes the form $\tilde{A} = (a_2(1 - \frac{\rho}{1+t}),\ a_2,\ a_2(1 + \frac{\sigma}{1+t}))$ for $0 < \rho, \sigma < 1$ (4)

It is to be noted that $\lim_{t\to\infty} (1 - \frac{\rho}{1+t})a_2 = a_2$ *and* $\lim_{t\to\infty} (1 + \frac{\sigma}{1+t})a_2 = a_2$. So, $\tilde{A} \to \{a_2\}$

Its membership function becomes a continuous function of x and t, defined by

$$\mu(x,t) = \begin{cases} \dfrac{x - a_2(1 - \frac{\rho}{1+t})}{\frac{a_2\rho}{1+t}}, & if\ \ a_2(1 - \frac{\rho}{1+t}) \le x \le a_2 \\ \dfrac{a_2(1 + \frac{\sigma}{1+t}) - x}{\frac{a_2\sigma}{1+t}}, & if\ \ a_2 \le x \le a_2(1 + \frac{\sigma}{1+t}) \\ 0, & otherwise \end{cases} \quad (5)$$

The graphical representation of CNTFN is appeared in the Fig-2. Let left and right α -cut of $\mu(x,t)$ from (5) denoted as $L(\alpha,t)$ *and* $R(\alpha,t)$ respectively. According to definition of α -cut defined in subsection 2.2,

$$L(\alpha,t) = a_2(1 - \frac{\rho}{1+t} + \frac{\rho\alpha}{1+t}) \text{ and } R(\alpha,t) = a_2(1 + \frac{\sigma}{1+t} - \frac{\sigma\alpha}{1+t}) \quad (6)$$

2.5 De and Beg's Ranking Index on CNTFN:

Let left and right α -cut off $\mu(x,t)$ from (5) denoted as $L(\alpha,t)$ *and* $R(\alpha,t)$ respectively. Then the defuzzification formula under time extension of Yager's ranking index is given by

$$J(\tilde{A}) = \frac{1}{2T}\int_{\alpha=0}^{1}\int_{t=0}^{T} \{L(\alpha,t) + R(\alpha,t)\}\, d\alpha\ dt \quad (7)$$

Note that α *and* t independent variables. Thus using (5), (6) becomes

$$J(\tilde{A}) = \frac{a_2}{2T}\left[2T + \frac{\sigma - \rho}{2}\log(1+T)\right] \quad (8)$$

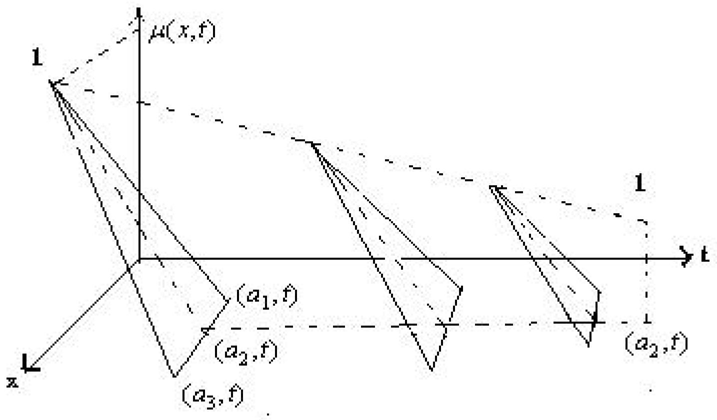

Fig2: Membership function of CNTFN

Obviously, $\lim_{T\to\infty}\dfrac{\log(1+T)}{T}=0$ (Using L'Hopital's rule) and therefore $J(\tilde{A})\to a_2\ as\ T\to\infty$. Note that

$\dfrac{\log(1+T)}{T}$ is taken as cloud index(CI) (9)

In practices, T is measured in days/months.

2.6 Arithmetic Operations on Normalized General Triangular Fuzzy Number (NGTFN):

Let $\tilde{A}=(a_1,a_2,a_3)\ and\ \tilde{B}=(b_1,b_2,b_3)$ are two triangular fuzzy numbers, then for usual arithmetic operations $+, -, \times, \div$ respectively namely addition, subtraction, multiplication and division between $\tilde{A}\ and\ \tilde{B}$ are defined as follows:

(i) $\tilde{A}+\tilde{B}=(a_1+b_1,\ a_2+b_2,\ a_3+b_3)$

(ii) $\tilde{A}-\tilde{B}=(a_1-b_3,\ a_2-b_2,\ a_3-b_1)$

(iii) $\tilde{A}\times\tilde{B}=(a_1b_1, a_2b_2, a_3b_3)$

(iv) $\dfrac{\tilde{A}}{\tilde{B}}=(\dfrac{a_1}{b_3},\dfrac{a_2}{b_2},\dfrac{a_3}{b_1}),\ b_1,b_2,b_3\neq 0$

(v) $k\tilde{A}=(ka_1,ka_2,ka_3)\ if\ k\geq 0$

and $k\tilde{A}=(ka_3,ka_2,ka_1)\ if\ k<0$

3. Dominance based Particle Swarm Optimization technique (DBPSO)

During the last decade, nature inspired intelligence becomes increasingly popular through the development and utilization of intelligent paradigms in advance information systems design. Among the most popular nature inspired approaches, when task is to optimize with in complex decisions of data or information, PSO draws significant attention. Since its introduction a very large number of applications and new ideas have been realized in the context of PSO (Najafi *et al*., 2009; Marinakis and Marinaki, 2010). A PSO normally starts with a set of solutions (called swarm) of the decision making problem under consideration. Individual solutions are called particles and food is analogous to optimal solution. In simple terms, the particles are flown through a multi-dimensional search space, where the position of each particle is adjusted according to its own experience and that of its neighbors. The particle *i* has a position vector $(X_i(t))$, velocity vector $(V_i(t))$, the position at which the best fitness $X_{pbesti}(t)$ encountered by the particle so far and the best position of all particles $X_{gbest}(t)$ in current generation *t*. In generation *(t+1)*, the position and velocity of the particle are changed to $X_i(t+1)$ and $V_i(t+1)$ using following rules:

$$V_i(t+1)=wV_i(t)+\mu_1 r_1(X_{pbest\,i}(t)-X_i(t))+\mu_2 r_2(X_{gbest}(t)-X_i(t)) \quad (10)$$

$$X_i(t+1)=X_i(t)+V_i(t+1) \quad (11)$$

The parameters μ_1 and μ_2 are set to constant values, which are normally taken as 2, r_1 and r_2 are two random values uniformly distributed in [0,1], w (0<w<1) is inertia weight which controls the influence of previous velocity on the new velocity. Here $(X_{pbesti}(t))$ and $(X_{gbest}(t))$ are normally determined by comparison of objectives due to different solutions. So for optimization problem involving crisp objective the algorithm works well. If objective value due to solution X_i dominates objective value due to solution X_j, we say that X_i dominates X_j. Using this dominance property PSO can be used to optimize crisp optimization problem. This form of the algorithm is named as dominance based PSO (DBPSO) and the algorithm takes the following form. In the algorithm V_{max} represent maximum velocity of a particle, $B_{il}(t)$ and $B_{iu}(t)$ represent lower and upper boundary of the i-th variable respectively. check_constraint $(X_i(t))$ function check whether solution $X_i(t)$ satisfies the constraints of the problem or not. It returns 1 if the solution $X_i(t)$ satisfies the constraints of the problem otherwise it returns 0.

3.1 Proposed DBPSO algorithm

1. Initialize μ_1, μ_2, w, N and Maxgen.
2. Set iteration counter $t=0$ and randomly generate initial swarm $P(t)$ of N particles (solutions).
3. Determine objective value of each solution $X_i(t)$ and find $X_{gbest}(t)$ using dominance property.
4. Set initial velocity $V_i(t)$, $\forall X_i(t) \in P(t)$ and set $X_{pbesti}(t)=X_i(t)$, $\forall X_i(t) \in P(t)$.
5. While (t<Maxgen) do
6. For i=1:N do
7. $V_i(t+1) = wV_i(t) + \mu_1 r_1 (X_{pbest\,i}(t) - X_i(t)) + \mu_2 r_2 (X_{gbest}(t) - X_i(t))$
8. If $(V_i(t+1)>V_{max})$ then set $V_i(t+1)=V_{max}$.
9. If $(V_i(t+1)<-V_{max})$ then set $V_i(t+1)=-V_{max}$
10. $X_i(t+1)=X_i(t)+V_i(t+1)$
11. If $(X_i(t+1)>B_{iu}(t))$ then set $X_i(t+1)=B_{iu}(t)$.
12. If $(X_i(t+1)<B_{il}(t))$ then set $X_i(t+1)=B_{il}(t)$.
13. If check_constraint $(X_i(t+1))=0$
14. Set $X_i(t+1)=X_i(t)$, $V_i(t+1)=V_i(t)$
15. Else
16. If $X_i(t+1)$ dominates $X_{pbesti}(t)$ then set $X_{pbesti}(t+1)=X_i(t+1)$.
17. If $X_i(t+1)$ dominates $X_{gbest}(t)$ then set $X_{gbest}(t+1)=X_i(t+1)$.
18. End If.
19. End For.
20. Set $t=t+1$.
21. End While.
22. Output: $X_{gbest}(t)$.
23. End Algorithm

3.2 Implementation of DBPSO

(a) Representation of solutions: A n-dimensional real vector $X_i=(x_{i1}, x_{i2},\ldots\ldots,x_{in})$, is used to represent i-th solution, where $x_{i1}, x_{i2},\ldots\ldots, x_{in}$ represent n decision variables of the decision making problem under consideration.

(b) Initialization: N such solutions $X_i=(x_{i1}, x_{i2},\ldots\ldots,x_{in})$, i=1,2,….,N, are randomly generated by random number generator within the boundaries for each variable $[B_{jl}, B_{ju}]$, j=1,2,……,n. Initialize (P(0)) sub function is used for this purpose.

(c) Dominance property: For crisp maximization problem, a solution X_i dominates a solution X_j if objective value of X_i is greater than that of X_j.

(d) Implementation: With the above function and values the algorithm is implemented using C-programming language. Different parametric values of the algorithm used to solve the model are as follows (Engelbrech, 2005), $\mu_1 = 1.49618$, $\mu_2 = 1.49618$, $w = 0.7298$.

4. Notations and Assumptions

The following notations and assumptions are adopted to develop the proposed inventory model.

4.1 Notations

k	Production rate per cycle.
d	Demand rate per cycle (d<k).
r	Production process reliability.
q(t)	Instantaneous inventory level
Q	Maximum inventory level(decision variable)
T	Cycle length (decision variable).
t_1	Production period (decision variable)
c	Production cost per unit.
c_3	Setup cost per cycle.
h	Inventory carrying cost per unit quantity per unit time.
Z	Average total inventory cost.
Q^*	Optimum value of Q.
T^*	Optimum value of T.
Z^*	Optimum value of Z.
t_1^*	Optimum value of t_1.

4.2. Assumptions

(i) Replenishment occurs instantaneously on placing of order quantity so lead time is zero.
(ii) The inventory is developed for single item in an imperfect production process.
(iii) Shortages are not allowed.
(iv) The time horizon of the inventory system is infinite.
(vi) The production rate k is demand dependent and is of the form $k=a + b\,d$ (12)
where a and b are positive constants.

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(vii) At the beginning of inventory system, ambiguity of demand rate is high because the decision maker (DM) has no any definite information how many people are accepting the product and how much will be demand rate. As the time progress, DM will begin to get more information about the expected demand over the process of inventory and learn whether it is below or over expected. It is generally observed that when new product comes into the market, people will take much more time (no matter what offers /discounts have been declared or what's the quality of product) to adopt/accept the item. Gradually, the uncertain region (cloud) getting thinner to DM's mind. In this respect, demand rate is assumed to be cloudy fuzzy (§ 2.4).

5. Model development and analysis

The process reliability r means that amongst the items produced in a production run, only r percent are acceptable that can be used to meet the customer's demand. Initially, the production process starts with zero inventories with production rate k and demand rate d. During the interval $[0, t_1]$, inventory level gradually built up at a rate $rk - d$ and reaches at its maximum level Q at the end of production process. The inventory level gradually depleted during the period $[t_1, T]$ due to customer's demand and ultimately becomes at zero at t=T. The graphical representation of this model is shown in Fig-2. The instantaneous state of $q(t)$ describing the differential equations in the interval $[0,T]$ of that item is given by

$$\frac{dq(t)}{dt} = rk - d, \quad 0 \le t \le t_1$$
$$= -d, \qquad t_1 \le t \le T \qquad where\ \ rk - d > 0 \quad (13)$$

with boundary condition $q(0) = 0,\ q(t_1) = Q,\ q(T) = 0$ (14)

The solution of the differential equation (13) using the boundary condition (14) is given by

$$q(t) = \begin{cases} (rk-d)t, & 0 \le t \le t_1 \\ d(T-t), & t_1 \le t \le T \end{cases} \quad (15)$$

The length of each cycle is $T = \dfrac{Q}{rk-d} + \dfrac{Q}{d} = \dfrac{Qrk}{d(rk-d)}$

(16)

Total holding cost for each cycle is given by $hH_1(Q,r,k)$, where (17)

$$H_1(Q,r,k) = \int_0^T q(t)\,dt = \int_0^{t_1} (rk-d)t\,dt + \int_{t_1}^T d(T-t)dt = \frac{Q^2 rk}{2d(rk-d)}$$

Total production cost per cycle is $cP_c(Q,r,k)$, where (18)

$$P_c(Q,r,k) = \int_0^{t_1} k dt = kt_1 = k\frac{Q}{rk-d} \quad where\ Q = (rk-d)t_1$$

Total cost=Production cost + Set up cost + Holding cost

$= cP_c(Q,r,k) + c_3 + hH_1(Q,r,k)$

$= \dfrac{ckQ}{rk-d} + c_3 + \dfrac{hQ^2 rk}{2d(rk-d)}$

Therefore, the total average cost is $Z = \left[\dfrac{ckQ}{rk-d} + c_3 + \dfrac{hQ^2 rk}{2d(rk-d)}\right]/T$

$$= \frac{cd}{r} + \frac{c_3}{T} + \frac{hT(rk-d)d}{2rk}$$
$$= \frac{cd}{r} + \frac{c_3}{T} + \frac{hdT(ar + (br-1)d)}{2(a+bd)r}$$

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Hence, our problem is given by Minimize Z= $\dfrac{cd}{r}+\dfrac{c_3}{T}+\dfrac{hdT(ar+(br-1)d)}{2(a+bd)r}$

$$subject\ to\ \ d(T-t_1)=(rk-d)t_1\ \ i.e.\ r\,k\,t_1=d\,T,\ Q=d(T-t_1) \tag{19}$$

Now, the problem is reduced to minimize the average cost Z and to find the optimum value of Q and T for which $Z(Q, T)$ is minimum and the corresponding value of t_1. The average cost is minimized by DBPSO.

5.1 Fuzzy mathematical model

Initially, when production process starts, demand rate of an item is ambiguous. Naturally, demand rate is assumed to be general fuzzy over the cycle length. Then fuzzy demand rate $\tilde{d}$ as follows $\tilde{d}=(d_1,\, d_2,\, d_3)$ for NGTFN.

Therefore the problem (19) becomes fuzzy problem, is given by

$$Minimize\ \tilde{Z}=\frac{c\tilde{d}}{r}+\frac{c_3}{T}+\frac{h\tilde{d}T(ar+(br-1)\tilde{d})}{2(a+b\tilde{d})r}$$

$$\text{subject to } r\,\tilde{k}\,t_1=\tilde{d}\,T,\ \tilde{Q}=\tilde{d}(T-t_1) \tag{20}$$

Now, using (1), the membership function of the fuzzy objective, fuzzy order quantity and fuzzy production rate under NGTFN are given by

$$\mu_1(Z)=\begin{cases}\dfrac{Z-Z_1}{Z_2-Z_1}, & Z_1\le Z\le Z_2\\ \dfrac{Z_3-Z}{Z_3-Z_2}, & Z_2\le Z\le Z_3\\ 0, & otherwise\end{cases}\qquad where\begin{cases}Z_1=\dfrac{c\,d_1}{r}+\dfrac{c_3}{T}+\dfrac{h\,d_1T\left\{ar+(br-1)d_1\right\}}{2r(a+bd_3)}\\ Z_2=\dfrac{c\,d_2}{r}+\dfrac{c_3}{T}+\dfrac{h\,d_2T\left\{ar+(br-1)d_2\right\}}{2r(a+bd_2)}\\ Z_3=\dfrac{c\,d_3}{r}+\dfrac{c_3}{T}+\dfrac{h\,d_3T\left\{ar+(br-1)d_3\right\}}{2r(a+bd_1)}\end{cases}$$

(21)

$$\mu_2(Q)=\begin{cases}\dfrac{Q-Q_1}{Q_2-Q_1}, & Q_1\le Q\le Q_2\\ \dfrac{Q_3-Q}{Q_3-Q_2}, & Q_2\le Q\le Q_3\\ 0, & otherwise\end{cases}\qquad where\begin{cases}Q_1=d_1\,(T-t_1)\\ Q_2=d_2\,(T-t_1)\\ Q_3=d_3\,(T-t_1)\end{cases}$$

(22)

$$\mu_3(k)=\begin{cases}\dfrac{k-k_1}{k_2-k_1}, & k_1\le k\le k_2\\ \dfrac{k_3-k}{k_3-k_2}, & k_2\le k\le k_3\\ 0, & otherwise\end{cases}\qquad where\begin{cases}r\,k_1\,t_1=d_1\,T\\ r\,k_2\,t_1=d_2\,T\\ r\,k_3\,t_1=d_3\,T\end{cases} \tag{23}$$

The index value of the fuzzy objective, fuzzy order quantity and fuzzy production rate are respectively obtained using (2) and (3) as

$$\begin{cases} I(\tilde{Z}) = \dfrac{1}{4}(Z_1 + 2Z_2 + Z_3) \\ \quad = \dfrac{c(d_1+2d_2+d_3)}{4r} + \dfrac{c_3}{T} + \dfrac{hT}{8r}\left[\dfrac{d_1\{ar+(br-1)d_1\}}{a+bd_3} + \dfrac{2d_2\{ar+(br-1)d_2\}}{a+bd_2} + \dfrac{d_3\{ar+(br-1)d_3\}}{a+bd_1}\right] \\ I(\tilde{Q}) = \dfrac{1}{4}(Q_1+2Q_2+Q_3) = \dfrac{(T-t_1)}{4}(d_1+2d_2+d_3) \\ I(\tilde{k}) = \dfrac{1}{4}(k_1+2k_2+k_3) = \dfrac{T}{4rt_1}(d_1+2d_2+d_3)\ [using\ (21),(22)\ and\ (23)] \end{cases} \tag{24}$$

5.1.1 Particular cases

Subcase-4.1.1.1: If $d_1, d_2, d_3 \to d$ then $I(\tilde{Z}) \to \dfrac{cd}{r} + \dfrac{c_3}{T} + \dfrac{hdT(ar+(br-1)d)}{2(a+bd)r}$

$$I(\tilde{Q}) \to d(T-t_1)$$

$$\text{and } I(\tilde{k}) \to \frac{dT}{rt_1}$$

This is a classical EPQ model with process reliability ***r*.**

Subcase-4.1.1.2 If $r \to 1, b \to 0$ then $I(\tilde{Z}) \to cd + \dfrac{c_3}{T} + \dfrac{hdT}{2a}(a-d)$

$$I(\tilde{Q}) \to d(T-t_1)$$

$$I(\tilde{k}) \to \frac{dT}{t_1}$$

Also, this is classical EPQ model with production rate ***a*.**

5. 2 Cloudy fuzzy mathematical model

Initially, when production process starts, demand rate of an item is ambiguous. As the time progress, hesitancy of demand rate tends to certain demand rate over the cycle length. Then fuzzy demand rate $\tilde{d}$ becomes cloudy fuzzy following the equation (4)

Now, using (5), the membership function of the fuzzy objective, fuzzy order quantity and fuzzy production rate under CNTFN are given by

$$\chi_1(Z,T) = \begin{cases} \dfrac{Z - Z_{11}}{Z_{12} - Z_{11}}, & Z_{11} \le Z \le Z_{12} \\ \dfrac{Z_{13} - Z}{Z_{13} - Z_{12}}, & Z_{12} \le Z \le Z_{13} \\ 0, & otherwise \end{cases} \quad where \begin{cases} Z_{11} = \dfrac{c(1-\frac{\rho}{1+T})d}{r} + \dfrac{c_3}{T} + \dfrac{hTd(1-\frac{\rho}{1+T})}{2r}\left[\dfrac{ar+(br-1)d(1-\frac{\rho}{1+T})}{a+bd(1+\frac{\sigma}{1+T})}\right] \\ Z_{12} = \dfrac{cd}{r} + \dfrac{c_3}{T} + \dfrac{hdT\{ar+(br-1)d\}}{2r(a+bd)} \\ Z_{13} = \dfrac{c(1+\frac{\sigma}{1+T})d}{r} + \dfrac{c_3}{T} + \dfrac{hTd(1-\frac{\rho}{1+T})}{2r}\left[\dfrac{ar+(br-1)d(1+\frac{\sigma}{1+T})}{a+bd(1-\frac{\rho}{1+T})}\right] \end{cases}$$

(25)

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$$\chi_2(Q,T)=\begin{cases}\dfrac{Q-Q_{11}}{Q_{12}-Q_{11}},\ Q_{11}\le Q\le Q_{12}\\ \dfrac{Q_{13}-Q}{Q_{13}-Q_{12}},\ Q_{12}\le Q\le Q_{13}\\ 0,\quad otherwise\end{cases}\quad where\ \begin{cases}Q_{11}=d(1-\dfrac{\rho}{1+T})(T-t_1)\\ Q_{12}=d(T-t_1)\\ Q_{13}=d(1+\dfrac{\sigma}{1+T})(T-t_1)\end{cases}$$

(26)

$$\chi_3(k,T)=\begin{cases}\dfrac{k-k_{11}}{k_{12}-k_{11}},\ k_{11}\le k\le k_{12}\\ \dfrac{k_{13}-k}{k_{13}-k_{12}},\ k_{12}\le k\le k_{13}\\ 0,\quad otherwise\end{cases}\quad where\ \begin{cases}k_{11}=d(1-\dfrac{\rho}{1+T})\dfrac{T}{r\,t_1}\\ k_{12}=\dfrac{d\,T}{r\,t_1}\\ k_{13}=d(1+\dfrac{\sigma}{1+T})\dfrac{T}{r\,t_1}\end{cases}\qquad (27)$$

Using (7) the index value of the fuzzy objective, fuzzy order quantity and fuzzy production rate are respectively are given by

$$J(\tilde{Z})=\frac{1}{4\tau}\int_{T=0}^{\tau}(Z_{11}+2Z_{12}+Z_{13})\,dT$$

$$=\frac{1}{4\tau}\int_0^{\tau}\left[\frac{c\,d}{r}(4+\frac{\sigma-\rho}{1+T})+\frac{4c_3}{T}\right]dT$$

$$+\frac{1}{4\tau}\int_0^{\tau}\frac{h\,d\,T}{2r}\left[(1-\frac{\rho}{1+T})\frac{a\,r+(b\,r-1)\,d(1-\frac{\rho}{1+T})}{a+b\,d(1+\frac{\sigma}{1+T})}+2\frac{a\,r+(b\,r-1)\,d}{a+b\,d}+(1+\frac{\sigma}{1+T})\frac{a\,r+(b\,r-1)\,d(1+\frac{\sigma}{1+T})}{a+b\,d(1-\frac{\rho}{1+T})}\right]dT$$

[Using (25)]

$$=I_1+\frac{h\,d}{8\tau\,r}(I_2+I_3+I_4)$$

(28)

The expression of $I_1, I_2, I_3\ and\ I_4$ are given in Appendix-1

$$J(\tilde{Q})=\frac{1}{\tau}\int_0^{\tau}\frac{1}{4}(Q_{11}+2Q_{12}+Q_{13})\,dT\ =\frac{d}{4\tau}\int_0^{\tau}\left(4+\frac{\sigma-\rho}{1+T}\right)(T-t_1)\,dT\quad \text{[Using (26)]}$$

$$=\frac{d}{4\tau}\left[2\tau^2-4\tau\,t_1+(\sigma-\rho)(\tau-(1+t_1)\ln|1+\tau|)\right]\qquad (29)$$

$$J(\tilde{k})=\frac{1}{\tau}\int_0^{\tau}\frac{1}{4}(k_{11}+2k_{12}+k_{13})\,dT\ =\frac{1}{4\tau}\int_0^{\tau}\frac{d\,T}{r\,t_1}\left\{4+\frac{(\sigma-\rho)}{1+T}\right\}dT\quad \text{[Using (27)]}$$

$$=\frac{d}{4\tau\,r\,t_1}\left[2\tau^2+(\sigma-\rho)(\tau-\ln|1+\tau|)\right]\qquad (30)$$

5.2.1Stability analysis and particular cases

(i) If $\rho,\sigma\to 0\ then\ p\to q\ and\ u\to v$ Also, $I_2\to\dfrac{p}{2u}\tau^2,\ I_4\to\dfrac{p}{2u}\tau^2,\ I_3=\dfrac{p}{u}\tau^2$

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So, $J(\tilde{Z}) \to \frac{cd}{r} + \frac{c_3}{\tau}\ln\left|\frac{\tau}{\varepsilon}\right| + \frac{hd}{4r\tau}\frac{p\tau^2}{u}$, $J(\tilde{Q}) \to d(\frac{\tau}{2} - t_1)$, $J(\tilde{k}) \to \frac{d}{2rt_1}\tau$

(ii) If $\rho, \sigma \to 0$ then the model reduces to (i). The above expressions deduced in (i) are in the form of classical EPQ model. Thus we choose ε in such a way that above expressions reduced to classical EPQ model.

Hence, $\frac{cd}{r} + \frac{c_3}{\tau}\ln\left|\frac{\tau}{\varepsilon}\right| + \frac{hd}{4r\tau}\frac{p\tau^2}{u} \cong \frac{cd}{r} + \frac{c_3}{T} + \frac{hdT}{2}\frac{p}{ur}$. Comparing we have $\frac{1}{T} = \frac{1}{\tau}\ln\left|\frac{\tau}{\varepsilon}\right|$, $T = \frac{\tau}{2}$

From these, we get $\varepsilon = \frac{2T}{e^2}$. Also, if $\tau = 2$ *then* $T = 1.$ *Hence,* $\varepsilon \to 2e^{-2} << 1$

Since $2 < e \Rightarrow \frac{2T}{e^2} < \frac{T}{2} \Rightarrow \varepsilon < \frac{T}{2}$

6. Numerical Illustration

The following values of inventory parameters are used to calculate the minimum values of average cost function (Z^*) along with the optimum inventory level (Q^*), optimum production period (t_1^*) and optimum cycle length (T^*)
a=100, *b*=1.22, c_3=$300 , *h*=$ 1.5 per unit, *c*= $ 3 per unit, *r*=.8, *d*= 500 units for the crisp model; for fuzzy model demand rate <d_1, d_2, d_3>=<460, 500, 600> units keeping other inventory parameters are same as taken in crisp model and that for the cloudy fuzzy model, $\sigma = 0.16$, $\rho = 0.13, \varepsilon = 0.6$. Optimum results are obtained via dominance based particle swarm optimization and presented in Table-1.

It is noted that for computation of degree of fuzziness, apply formula $d_f = \frac{U_b - L_b}{m}$ where U_b, L_b respectively are the upper and lower bounds of fuzzy components and m is the Mode which is obtained using the formula Mode(m)=3 $\times$ Median-2 $\times$ Mean. For fuzzy demand rate <460, 500, 600>, Median=500, Mean=520, U_b=600, L_b=460, m=460

Table-1: Optimum values of EPL model by DBPSO

Model	t_1*(months)	T^*(months)	Q^* units	Z^*($)	$d_f = \frac{U_b - L_b}{m}$	CI= $\frac{\log(1+T)}{T}$
Crisp	1.5	1.704	102.00	2127.56		
Fuzzy	1.9	2.58	346.30	2164.49	0.304	
Cloudy Fuzzy	1.85	2.22	183.03	2115.33		0.227

From the above results, it has been observed that minimum cost is obtained in cloudy fuzzy model and the value of optimum cost Rs. 2115.33 after the completion 2.22 months. In cloudy fuzzy environment degree of fuzziness is less than the general triangular number as the hesitancy of fuzzy gradually decreases due to the taking experience over time.

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6.1 Sensitivity Analysis of Cloudy Fuzzy Model

Table-2: Sensitivity analysis for cloudy fuzzy model

Parameters	% change	Average cost (z^*)	$\frac{(z'-z)}{z}\times 100\%$
d	-15%	1833.44	-13.32
	-10%	1927.49	-8.88
	-5%	2021.45	-4.44
	5%	2209.13	4.43
	10%	2302.87	8.86
	15%	2396.55	13.29
a	-15%	2099.51	-0.75
	-10%	2104.86	-0.49
	-5%	2110.13	-0.25
	5%	2120.45	0.24
	10%	2125.51	0.48
	15%	2130.48	0.69
b	-15%	2006.4	-5.15
	-10%	2046.12	-3.27
	-5%	2082.28	-1.56
	5%	2145.66	1.43
	10%	2173.58	2.75
	15%	2199.39	3.97
c_3	-15%	2108.56	-0.32
	-10%	2110.82	-0.21
	-5%	2113.07	-0.11
	5%	2122.09	0.32
	10%	2128.87	0.64
	15%	2135.63	0.96
c	-15%	1833.27	-13.37
	-10%	1927.29	-8.9
	-5%	2021.31	-4.44
	5%	2209.35	4.44
	10%	2303.37	8.89
	15%	2397.38	13.33
h	-15%	2100.38	-0.71
	-10%	2105.36	-0.47
	-5%	2110.35	-0.23
	5%	2120.31	0.23
	10%	2125.28	0.47
	15%	2130.28	0.71

Using the above numerical illustration, the effect of under or over estimation of various parameters on average cost is studied. Here using $\Delta z = \frac{(z' - z)}{z} \times 100\%$ as a measure of sensitivity where z is the true value and z' is the estimated value. The sensitivity analysis is shown by increasing or decreasing the parameters by 5%, 10% and 15% , taking one at a time and keeping the others as true values. The results are presented in Table-2.

It is seen form the Table-3 that the parameters *d* and *c* are highly sensitive. For the changes of demand at -15% , average inventory cost reduces to -13.32% and for 15%, the average inventory cost increases at +13.29% respectively. Also the same results observed for the changes of unit production cost. These phenomena agree with reality. But for the changes of *a*, *b*, c_3, *h* from -15% to +15%, there are moderately variations on the average cost. This sensitivity table reveals that the observations done on inventory model are more realistic and more practicable.

6.2 Effect of changing cycle time

Comparing the results obtained in crisp, general fuzzy and cloudy fuzzy environment, it has been observed from the graphical illustration (**Fig-3**) that cloudy fuzzy model predicts the minimum cost 2068.57 ($) and the minimum cost is obtained at cycle time 4 months which is shown in **Fig-4**. In Fig-4, the curve shown U shape pattern under the cloudy fuzzy model. So the curve is convex. So, it is interesting to note that cloudy fuzzy model is more reliable.

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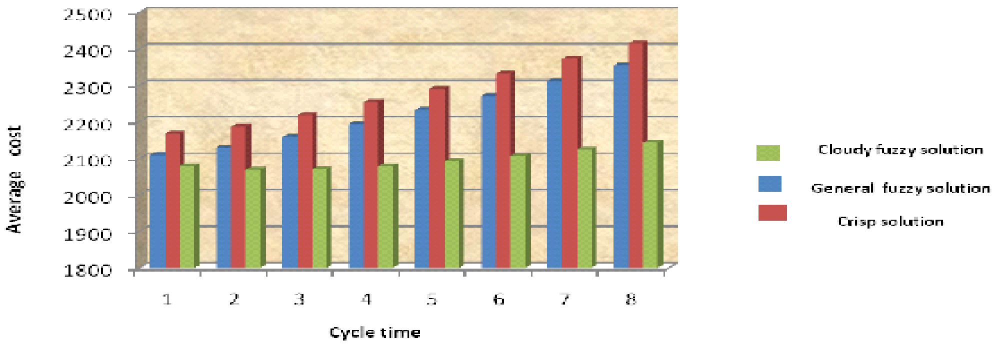

Fig-3: Average cost vs cycle time

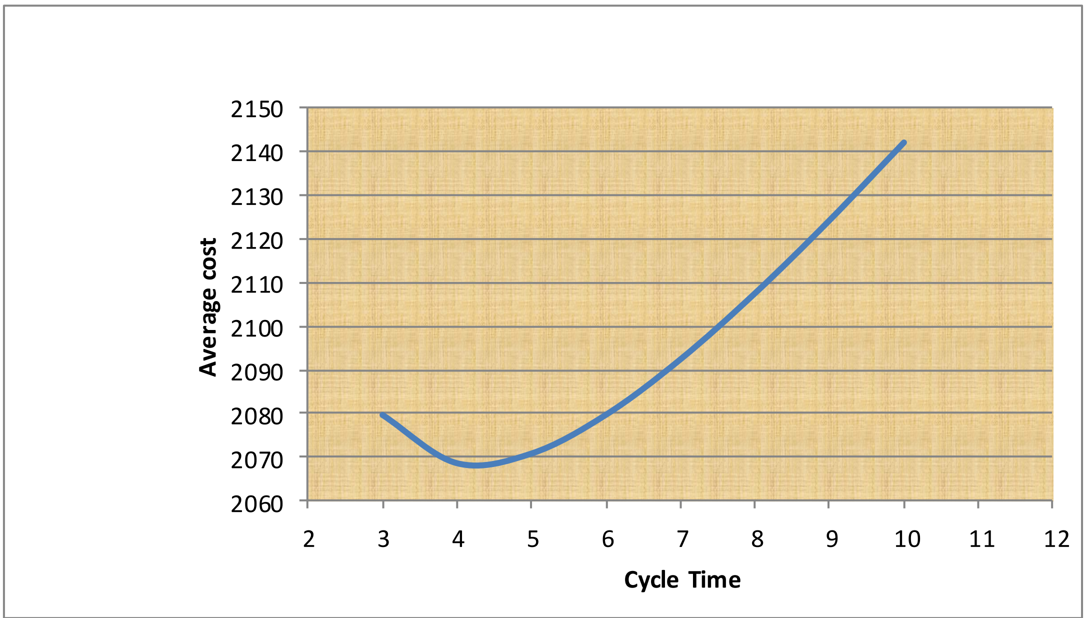

Fig-4: Average cost vs cycle time for cloudy fuzzy model

6.3 Effect of changing reliability

Relibility is the most important factor in manufacturing system as reliability defined to be capability of manufacturing units without breakdown of the system. It has been observed from the graphical illustration (**Fig-5**) that as the reliability increases, average cost gradually decreases as because increase of reliabilty resulted in increase of production rate. So, cost of finished good consistently decreases.

Also, the performance level as measured by reliability can significantly improved the manufacturing system. Since the present is minimization problem, so averege cost decreases with the increase of reliability.

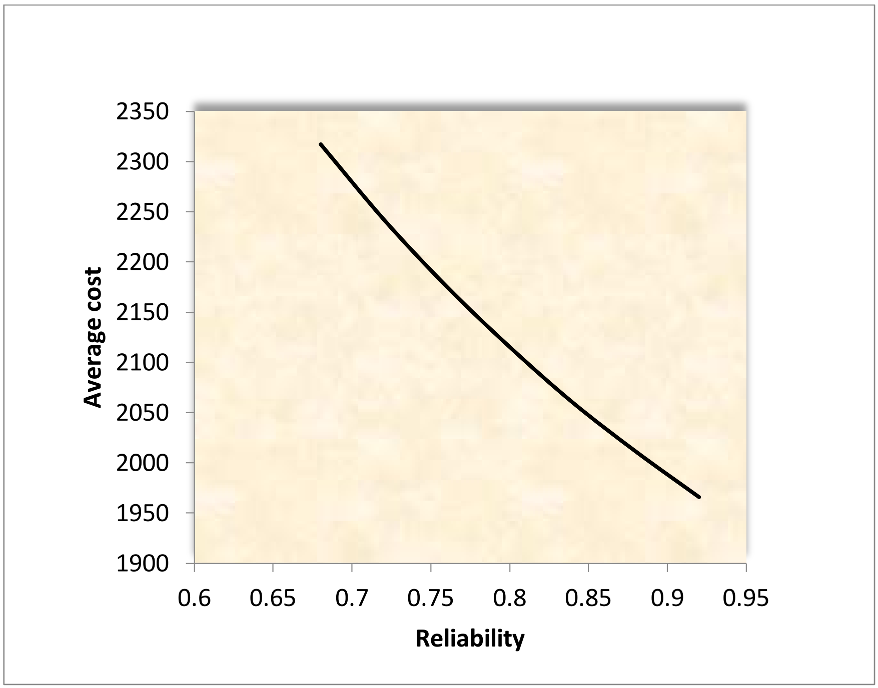

Fig-5:Average cost vs reliability for cloudy fuzzy model

6.4 Comparison of average cost under different cycle time

It has been observed that difference of average inventory cost of crisp model as well as general fuzzy model with respect to cloudy fuzzy model for different value of cycle time are shown in Table-3.

From this Table-3, it is seen that cloudy fuzzy model giving the minimum average inventory cost at time 4 months which is the better choice of inventory practinioner as well as decision maker.

Table-3: Average cost under diferent model

Crisp model				Genral fuzzy model			Cloudy fuzzy model		
Cycle time T	t_1^*	Q^*	Z^*	t_1^*	Q^*	Z^*	t_1^*	Q^*	Z^*
3	2.64	179.58	2109.86	2.68	164.80	2167.25	1.35	74.68	2079.64
4	3.52	239.46	2129.58	3.62	195.70	2187.59	1.80	99.52	**2068.57**
5	4.41	299.29	2159.47	4.55	231.75	2217.92	2.21	149.04	2070.79
6	5.20	359.15	2194.36	5.51	252.35	2253.35	2.69	154.26	2079.68
7	6.11	419.01	2232.11	6.45	283.25	2291.45	3.13	189.16	2092.40
8	7.04	478.87	2271.65	7.37	323.42	2331.43	3.59	203.02	2107.53
9	7.92	538.73	2312.33	8.34	339.90	2372.59	4.04	228.91	2124.26
10	8.81	598.59	2353.95	9.20	394.49	2414.60	4.53	238.13	2142.13

7. Conclusion and future research

In this paper, fuzzy inventory model under imperfect production process with cloudy fuzzy demand rate is developed where production rate is demand dependent. The model is solved in crisp , general fuzzy and cloudy fuzzy environment using Yager's index method and De and Beg's ranking index method using new defuzzification method and the results obtained in crisp, fuzzy and cloudy fuzzy environment are compared. ***For the first time, this type of inventory model has been successfully solved by DBPSO in cloudy fuzzy environment.*** Further extension of this model can be done considering some realistic situation such as multi-item, quantity discount, price and reliability dependent, learning effect etc. Moreover, in future, this model can be formulated with random planning horizon, fuzzy planning horizon in stochastic, fuzzy stochastic environments.

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Reference

[1] Abad, P.L. (2000). Optimal lot-size for a perishable good under conditions of finite production and partial backordering and lost sale. *Computers and Industrial Engineering*, 38, 457-465.

[2] Balkhi, Z.T. & Benkherouf, L. (1998). A production lot size inventory model for deteriorating and arbitrary production demand rates. *European Journal of Operational Resaerch*, 31, 52-60.

[3] Bellman, R.E. & Zadeh, L.A.(1970).Decision making in a fuzzy environment, *Management Science*, 17, B141-B164.

[4] Bera, U. &Maiti, A.K. (2012). A soft computing approach to multi-item fuzzy EOQ model incorporating discount, *International Journal of Information and Decision Sciences*, 4(4), 313-328.

[5] Ben-Daya, M. & Hariga, M.(2000). Economic lot scheduling problem with imperfect production processes, *Journal of Operational Research Society*, 51, 875-881.

[6] Bhunia, A. K. & Maiti, M. (1997). Deterministic inventory models for variable production. *Journal of Operational Research Society*,48, 221-224.

[7] Bhunia, A.K. & Shaikh, A.A. (2015). An application of PSO in a two warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies, *Applied Mathematics and Computation*, 256, 831-850.

[8] Cheng, T.C.E.(1989).An economic production quantity model with flexibility and reliability consideration, *European Journal Of Operational Research*, 39,174-179.

[9] De, S.K & Sana, S.S. (2015). An EOQ model with backlogging, *International Journal of Management Science & Engineering Management*, 1-12.

[10] De, S.K & Beg, I. (2016). Triangular dense fuzzy Neutrosophic sets, Neutrosophic Sets System, 1(2), 1-12.

[11] De, S.K & Mahata, G.C. (2016). Decision of fuzzy inventory with fuzzy backorder model under cloudy fuzzy demand rate, *International J. Appl. Comput. Math.*, DOI 10.1007/s40819-016-0258-4.

[12] Deng, H. (2014). Comparing and ranking fuzzy numbers using ideal solutions, *Applied Mathematical Modelling*, 38, 1638-1646.

[13] Engelbreach, A.P.(2005). Fundamentals of computational swarm intelligence, Wiley.

[14] Ezzati, R., Allahviranloo, T., Khezerloo,S. & Khezerloo,M. (2012). An approach for ranking of fuzzy numbers, *Expert System Application*, 39, 690-695.

[15] Goyal, S.K., Hung, C.K. & Chen, K.C. (2003). A Simple integrated production policy of an imperfect item for vendor and buyer, *Production Planning and Control*, 14, 596-602.

[16] Kao, C. and Hsu, W. K. (2002), Lot Size-Reorder Point Inventory Model with Fuzzy Demands,*Computers & Mathematics with Applications*, 43, 1291-1302.

[17] Kauffman, A. & Gupta, M.M.(1992). Introduction to fuzzy arithmetic, theory and applications, Van Nostrand Reinhold, New York.

[18] Khara, B., Dey, J.K. & Mondal, S.K. (2017). An inventory model under development cost dependent imperfect production and reliability dependent demand, *Journal of Management Analytics*,1-18.

[19] Khouja, M.(1995). The economic production lot size model under volume flexibility, *Computers and Operations Research*, 22, 515-525.

[20] Mahata, G & Goswami, A.(2007). An EOQ model for deteriorating items under trade credit financing in the fuzzy sense, *Production Planning & Control*, 681-692.

[21] Mahata, G & Goswami, A.(2013). Fuzzy inventory models for items with imperfect quality and shortage backordering under crisp and fuzzy decisions, *Computers and Industrial Engineering*, 64, 190-199

[22] Maiti, A. K., Maiti, M.K. & Maiti, M. (2009), Inventory model with stochastic lead time and price dependent demand incorporating advance payment, *Applied Mathematical Modelling*, 33, 2433-2443.

[23] Maiti. A.K., Maiti, M.K. and Maiti, M. (2014), An EOQ model of an item with imprecise seasonal time via genetic algorithm, *International Journal of Operational Research*, 19(3), 358-384.

[24] Maiti, M.K. and Maiti, M. (2006). Fuzzy inventory model with two warehouses under

possibility constraints, *Fuzzy sets and fuzzy systems*, 157, 52-73.

*[25]*Maiti, M.K. and Maiti, M. (2007). Two storage inventory model in a mixed environment, *Fuzzy*

[26] *Optimization and Decision Making*, 6, 391-426.

[27] Maiti, A.K., Bhunia, A.K. & Maiti, M.(2006). An application of genetic algorithm in bi-level production strategy for multi-items under flexibility and reliability considerations, *Tamsui Oxford Journal of Management Sciences*, 22(1), 01-22

[28] Mandal, M & Maiti, M. (2000). Inventory of damageable items with variable replenishment and stock-dependent demand, *Asia Pacific Journal of Operational Research*, 17, 41-54.

[29] Mandal, S.& Maiti, M. (2002). Multi-item fuzzy EOQ models, using Genetic Algorithm, *Computers & Industrial Engineering*, 44, 105-117.

[30] Manna, A.K., Dey, J.K. & Mondal, S.K. (2014). Three layers supply chain in an imperfect production inventory model with two facilities under fuzzy rough environment, *Journal of Uncertainty Analysis and Applications*, 2-17.

[31] Manna, A.K., Das, B., Dey, J.K. & Mondal, S.K. (2016). Multi-item EPQ model with learning effect on imperfect production over fuzzy random planning horizon, *Journal of Management Analytics*,1-31.

[32] Marinakis, Y., Marinaki, M.(2010), A hybrid genetic-particle swarm optimization algorithm for the Vehicle routing problem. *Experts Syst. Appl*., 37, 1446-1455.

[33] Najafi, A.A., Niakib, S.T.A., Shahsavara, M. (2009), A parameter-tuned genetic algorithm for the resource investment problem with discounted cash flows and generalized precedence relations. *Computers and Operations Research*, 36, 2994-3001.

[34] Pal, S., Maiti, M. K. and Maiti, M. (2009), An EPQ model with price discounted promotional demand in an impreciseplanning horizon via Genetic Algorithm, *Computers & Industrial Engineering*, 57, 181-187.

[35] Pal, B., Sana, S.S., & Chaudhuri, K.S.(2014). Joint pricing and ordering policy for two echelon imperfect production inventory model with two cycles, *International Journal of Production Economics*, 155, 229-238.

[36] Rosenblatt, M.J. & Lee, H.L.(1986). Economic production cycles with imperfect production process, *IIE Transactions*, 18, 48-55.

[37] Sana, S.S.,Goyal, S.K. & Chaudhuri, K.(2007). An imperfect production process in a volume flexible inventory model, *International Journal of Production Economics*, 105, 548-559.

[38] Shah, N.H & Shah, B.J. (2014). EPQ model for time declining demand with imperfect production process under inflationary conditions and reliability, *International Journal of Operational Research*, 11(3), 91-99.

[39] Yager, R.R. (1981). A procedure for ordering fuzzy subsets of the unit interval, *Information Science*, 24, 143-161.

[40] Zadeh, L.A. (1965). Fuzzy sets, Information Control, 8, 338-356.

[41] Zhang,F., Ignatius, J., Lim, C.P. & Zhao, Y. (2014). A new method for ranking fuzzy numbers and its application to group decision making, *Applied Mathematical Modelling*, 38, 1563-1582.

Appendix-1: The expression of I_1, I_2, I_3 *and* I_4 are given below.

$$I_1 = \frac{1}{4\tau}\int_0^{\tau}\left\{\frac{cd}{r}(4+\frac{\sigma-\rho}{1+T})+\frac{4c_3}{T}\right\}dT \quad = \frac{cd}{r}(1+\frac{\sigma-\rho}{4\tau}\ln|1+\tau|)+\frac{c_3}{\tau}\ln\left|\frac{\tau}{\varepsilon}\right|$$

$$I_2 = \int_0^{\tau} T(1-\frac{\rho}{1+T})\frac{\left\{ar+(br-1)d(1-\frac{\rho}{1+T})\right\}}{a+bd(1+\frac{\sigma}{1+T})}dT$$

$$= \int_0^{\tau} T(1-\frac{\rho}{1+T})\frac{T(ar+(br-1)d)+ar+(br-1)d(1-\rho)}{T(a+bd)+a+bd(1+\sigma)}dT$$

$$= \int_0^{\tau} T(1-\frac{\rho}{1+T})\frac{pT+q}{Tu+v}dT$$

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$$[\ p = a\,r + (b\,r-1)\,d\ ,\ q = a\,r + (b\,r-1)\,d\,(1-\rho),\ u = a + b\,d\ ,\ v = a + b\,d(1+\sigma)\,]$$

$$= p\int_0^\tau \frac{T^2}{T\,u+v}\,dT + q\int_0^\tau \frac{T}{T\,u+v}\,dT - \rho\,p\int_0^\tau \frac{T^2}{(T\,u+v)(1+T)}\,dT - \rho\,q\int_0^\tau \frac{T}{(T\,u+v)(1+T)}\,dT$$

$$= I_{21} + I_{22} - I_{23} - I_{24}$$

$$where\ I_{21} = p\int_0^\tau \frac{T^2}{T\,u+v}\,dT = \frac{p}{u^3}\left[\frac{u^2\,\tau^2}{2} - u\,v\,\tau + v^2\ln\left|\frac{v+u\,\tau}{v}\right|\right]$$

$$I_{22} = q\int_0^\tau \frac{T}{T\,u+v}\,dT = \frac{q}{u}\left[\tau - \frac{v}{u}\ln\left|\frac{v+\tau\,u}{v}\right|\right]$$

$$I_{23} = \rho\,p\int_0^\tau \frac{T^2}{(T\,u+v)(1+T)}\,dT = \frac{\rho\,p}{u-v}\left[\tau - \ln|1+\tau| - \frac{v\,\tau}{u} + \frac{v^2}{u^2}\ln\left|\frac{v+\tau\,u}{v}\right|\right]$$

$$I_{24} = \rho\,q\int_0^\tau \frac{T}{(T\,u+v)(1+T)}\,dT = \frac{\rho\,q}{v-u}\left[\frac{v}{u}\ln\left|\frac{v+\tau\,u}{v}\right| - \ln|1+\tau|\right]$$

$$I_3 = 2\int_0^\tau \frac{a\,r + (b\,r-1)\,d}{a+b\,d}\,dT = \frac{p}{u}\tau^2$$

$$I_4 = \int_0^\tau T(1+\frac{\sigma}{1+T})\frac{\left\{a\,r + (b\,r-1)\,d\,(1+\frac{\sigma}{1+T})\right\}}{a+b\,d\,(1-\frac{\rho}{1+T})}\,dT$$

$$= \int_0^\tau T(1+\frac{\sigma}{1+T})\frac{T(a\,r+(b\,r-1)d) + a\,r + (b\,r-1)\,d\,(1+\sigma)}{T(a+b\,d) + a + b\,d\,(1-\rho)}\,dT$$

$$= \int_0^\tau T(1+\frac{\sigma}{1+T})\frac{pT+y}{T\,u+s}\,dT$$

$$[\ p = a\,r + (b\,r-1)\,d\ ,\ y = a\,r + (b\,r-1)\,d\,(1+\sigma),\ u = a + b\,d\ ,\ s = a + b\,d(1-\rho)\,]$$

$$= p\int_0^\tau \frac{T^2}{T\,u+s}\,dT + y\int_0^\tau \frac{T}{T\,u+s}\,dT + \sigma\,p\int_0^\tau \frac{T^2}{(T\,u+s)(1+T)}\,dT + \sigma y\int_0^\tau \frac{T}{(T\,u+s)(1+T)}\,dT$$

$$= I_{41} + I_{42} + I_{43} + I_{44}$$

$$where\ I_{41} = p\int_0^\tau \frac{T^2}{T\,u+s}\,dT = \frac{p}{u^3}\left[\frac{u^2\,\tau^2}{2} - u\,s\,\tau + s^2\ln\left|\frac{s+u\,\tau}{s}\right|\right]$$

$$I_{42} = y\int_0^\tau \frac{T}{T\,u+s}\,dT = \frac{y}{u}\left[\tau - \frac{s}{u}\ln\left|\frac{s+\tau\,u}{s}\right|\right]$$

$$I_{43} = \sigma\,p\int_0^\tau \frac{T^2}{(T\,u+s)(1+T)}\,dT = \frac{\sigma\,p}{u-s}\left[\tau - \ln|1+\tau| - \frac{s\,\tau}{u} + \frac{s^2}{u^2}\ln\left|\frac{s+\tau\,u}{s}\right|\right]$$

$$I_{44} = \sigma\,y\int_0^\tau \frac{T}{(T\,u+s)(1+T)}\,dT = \frac{\sigma\,y}{s-u}\left[\frac{s}{u}\ln\left|\frac{s+\tau\,u}{s}\right| - \ln|1+\tau|\right]$$