

Extremally Disconnected Space in Soft Bitopological Spaces

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Abstract

The present paper, We give characterizations of extremally disconnected spaces in soft bitopological spaces by utilizing (1,2)*- soft b- open set. Also we discussed the properties of (1,2)*- soft b- submaximal spaces.

Keywords: (1,2)*- soft b- dense, (1,2)*- soft b- submaximal, (1,2)*-soft b- extremally disconnected.

1. Introduction

Soft set theory was first introduced by Molodtsov [7] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. Shabir and Naz [8] initiated the study of the soft topological spaces. In 1963, J.C. Kelly [5], first initiated the concept of bitopological spaces. After then many authors studied some of basic concepts and properties of bitopological space. In 1996, Andrijevic [1] introduced a new class of open sets in a topological space called b- open sets.

Recently, in 2011, Shabir and Naz [8] initiated the study of the soft topological spaces. They defined soft topology as a collection of soft sets over X. Metin Akdag and Alkan Ozkan [6] are defined soft b- open sets and soft b- continuous map studied their properties. In the year 2014, Basavaraj M. Ittanagi [2] initiated the concept of soft bitopological spaces which are defined over an initial universe with a fixed set of parameters.

In the present paper, we introduce the extremally disconnected spaces and submaximal spaces in soft bitopological spaces by utilizing (1,2)*- soft b-open set. we give the characterizations of these spaces.

2. Preliminaries

Throughout this paper, X is an initial universe, E is the set of parameters, P(X) is the power set of X and A is a nonempty subset of E.

Definition 2.1 :[8] A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{ (x, f_A(x)) : x \in E \}$, where $f_A : E \rightarrow P(X)$ such that $f_A(x) = \phi$ if $x \notin A$. Here f_A is called approximate function of the soft set F_A . The set of all soft sets over X will be denoted by $SS_E(X)$.

Definition 2.2:[8] For two soft sets F_A, F_B over a common universe X, we say that F_A is a soft subset of F_B if

1. $A \subseteq B$ and
2. For all $e \in A$, $F(e)$ and $F(e)$ are identical approximations. We write $F_A \subseteq F_B$. F_A is said to be a soft super set of F_B if F_B is a soft subset of F_A . We denote it by $F_A \supseteq F_B$.

Definition 2.4:[8] Two soft sets F_A and G_B over the common universe X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A .

Definition 2.4:[7] Let $F_A, F_B \in SS_E(X)$. Then soft union of F_A and G_B , denoted by $F_A \cup G_B$, defined by $F_{A \cup B} = F_C$ where $C = A \cup B$, and for

all $e \in C$.

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B \\ G(e) & \text{if } e \in B \setminus A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

Definition 2.5:[8] The soft intersection H_C of two soft sets F_A and G_B over a common universe X , denoted by $F_A \tilde{\cap} G_B$, is defined as $C = A \cap B$ and $H(e) = F(e) \cap G(e)$, for all $e \in C$.

Definition 2.6:[8] A soft set F_E over X is said to be a null soft set or empty soft set denoted by φ if for all $e \in E$, $F(e) = \varphi$. It means that there is no element in X related to the parameter $e \in E$.

Definition 2.7:[8] The difference H_E of two soft set F_E and G_E over X , denoted by $F_E \tilde{\setminus} G_E$, is defined as

$$H_E = \{H(e) = F(e) \setminus G(e) \text{ for all } e \in E\}$$

Definition 2.8:[8] The complement of a soft set F_E , denoted by F_E^C ; $F^C : E \rightarrow P(X)$ is mapping given by $F^C(e) = X - F(e)$, $\forall e \in E$ and F^C is called the soft complement function of F .

Definition 2.9:[8] A soft set F_E over X is said to be an absolute soft set denoted by \tilde{X} or $F_{\tilde{E}}$ if for all $e \in E$, $F(e) = X$. Clearly $\tilde{X}^C = \varphi$ and $\varphi^C = \tilde{X}$.

Definition 2.10:[4] Let $F_E \in SS_E(X)$. We say that $x_e = (e, \{x\})$ is a soft point of F_E if $e \in E$ and $x \in F(e)$.

Definition 2.11:[4] The soft point x_e said to be belonging to the soft set F_E , denoted by $x_e \in F_E$.

Definition 2.12:[3] Let $F_A \in S(X)$. The soft power set of F_A is defined by $\tilde{P}(F_A)$ is defined by $\tilde{P}(F_A) = \{F_{A_i} : F_{A_i} \subseteq F_A \text{ } i \in I \subseteq N\}$ and its cardinality is defined by $|\tilde{P}(F_A)| = 2^{\sum_{x \in E} |f_A(x)|}$ where $|f_A(x)|$ is cardinality of $f_A(x)$.

Example 2.13: [3] Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tilde{X} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$. Then the soft subsets over \tilde{X} are $F_{E_1} = \{(e_1, \{x_1\})\}$, $F_{E_2} = \{(e_1, \{x_2\})\}$, $F_{E_3} = \{(e_1, \{x_1, x_2\})\}$, $F_{E_4} = \{(e_2, \{x_1\})\}$, $F_{E_5} = \{(e_2, \{x_2\})\}$, $F_{E_6} = \{(e_2, \{x_1, x_2\})\}$, $F_{E_7} = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$, $F_{E_8} = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$, $F_{E_9} = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2\})\}$, $F_{E_{10}} = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$, $F_{E_{11}} = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$, $F_{E_{12}} = \{(e_1, \{x_2\}), (e_2, \{x_1, x_2\})\}$, $F_{E_{13}} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\}$, $F_{E_{14}} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\}$, $F_{E_{15}} = \tilde{X}$, $F_{E_{16}} = \varphi$.

Definition 2.14:[8] Let $\tilde{\tau}$ be the collection of soft sets over \tilde{X} , then $\tilde{\tau}$ is said to be a soft topology on X if satisfies the following axioms.

- (i) φ, \tilde{X} belong to $\tilde{\tau}$.
- (ii) the union of any member of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
- (iii) the intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triplet $(\tilde{X}, \tilde{\tau}, E)$ is called soft topological space over X .

Definition 2.15:[10] Let \tilde{X} be a nonempty soft set on the universe X , $\tilde{\tau}_1$ and are $\tilde{\tau}_2$ two different soft topologies on \tilde{X} . Then $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called a soft bitopological space.

Definition 2.16: [10] Let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is be a soft bitopological space and $F_A \in \tilde{X}$. Then F_A is called $\tilde{\tau}_{1,2}$ -open if $F_A = F_B \cup F_C$, where $F_B \in \tilde{\tau}_1$ and $F_C \in \tilde{\tau}_2$. The complement of $\tilde{\tau}_{1,2}$ -open set is called $\tilde{\tau}_{1,2}$ -closed.

Definition 2.17: [10] Let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is be a soft bitopological space and $F_E \in SS_E(X)$. Then

(i) The $\tilde{\tau}_{1,2}$ - closure of F_E , denoted by $\tilde{\tau}_{1,2}$ -cl(F_E), is defined by $\tilde{\tau}_{1,2}$ -cl(F_E) = $\bigcap \{H_E : H_E \supseteq F_E; H_E \text{ is a } \tilde{\tau}_{1,2} \text{- open set}\}$

(ii) The $\tilde{\tau}_{1,2}$ - interior of F_E , denoted by $\tilde{\tau}_{1,2}$ -int(F_E), is defined by $\tilde{\tau}_{1,2}$ -int(F_E) = $\bigcup \{G_E : G_E \subseteq F_E; G_E \text{ is a } \tilde{\tau}_{1,2} \text{- soft open set}\}$

Definition 2.18[9]: Let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a soft bitopological space and $F_A \subseteq \tilde{X}$. Then F_A is called $(1,2)^*$ - soft b-open set (briefly $(1,2)^*$ -sb-open) if $F_A \subseteq \tilde{\tau}_{1,2}$ -int($\tilde{\tau}_{1,2}$ -cl(F_A)) \cup $\tilde{\tau}_{1,2}$ -cl($\tilde{\tau}_{1,2}$ -int(F_A)). The set of all $(1,2)^*$ - soft b-open sets are denoted by $(1,2)^*$ -SbO(\tilde{X}).

Definition 2.19[9]: Let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a soft bitopological space and F_A be a soft set over \tilde{X}

(i) $(1,2)^*$ -soft b-closure (briefly $(1,2)^*$ -sbcl(F_A))

of a set F_A in \tilde{X} defined as the soft intersection of all $(1,2)^*$ - soft b-closed supesets of F_A .

(ii) $(1,2)^*$ -soft b-interior (briefly $(1,2)^*$ -sbint(F_A)) of a set F_A in \tilde{X} defined as the soft union of all $(1,2)^*$ - soft b-open subsets of F_A .

3. $(1,2)^*$ - Soft b-Extremally Disconnected Spaces

In this section, we introduce a new class of soft space called $(1,2)^*$ -soft b- extremally disconnected, and we obtain several characterizations of $(1,2)^*$ -soft b- extremally disconnected spaces by utilizing classes of soft sets.

Definition 3.1: A soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is said to be $(1,2)^*$ -soft extremally disconnected space if $\tilde{\tau}_{1,2}$ - closure of every $\tilde{\tau}_{1,2}$ - open set of \tilde{X} is $\tilde{\tau}_{1,2}$ - open set in \tilde{X} .

Definition 3.2: A soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is said to be $(1,2)^*$ -soft b-extremally disconnected space if $(1,2)^*$ -soft b-closure of every $(1,2)^*$ -soft b-open set of \tilde{X} is $(1,2)^*$ -soft b- open set in \tilde{X} .

Example 3.3: Let $X = \{x, y, z, a\}$,

$E = \{e_1\}$. $\tilde{X} = \{(e_1, \{x, y, z, a\})\}$.

$\tilde{X}, \varphi, G_{E3} = \{(e_1, \{x\})\}$,

$G_{E4} = \{(e_1, \{y\})\}, G_{E5} = \{(e_1,$

$\{z\})\}, G_{E6} = \{(e_1, \{a\})\}$,

$G_{E7} = \{(e_1, \{x, y\})\}, G_{E8} = \{(e_1, \{x,$

$z\})\}, G_{E9} = \{(e_1, \{x, a\})\}$,

$G_{E10} = \{(e_1, \{y, z\})\}, G_{E11} = \{(e_1, \{y,$

$a\})\}, G_{E12} = \{(e_1, \{z, a\})\}$,

$G_{E13} = \{(e_1, \{x, y, z\})\}, G_{E14} = \{(e_1,$

$\{x, y, a\})\}, G_{E15} = \{(e_1, \{y, z, a\})\}$,

$G_{E16} = \{(e_1, \{x, a, z\})\}$. Consider

the soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$,

where $\tilde{\tau}_1 = \{\tilde{X}, \varphi, G_{E6}, G_{E7}, G_{E14}\}$,

$\tilde{\tau}_2 = \{\tilde{X}, \varphi, G_{E3}\}$. Then $\tilde{\tau}_{1,2}$ - open set are

$\{\tilde{X}, \varphi, G_{E3}, G_{E6}, G_{E7}, G_{E14}, G_{E9}\}$, $(1,2)^*$ -soft

b-open sets are $\{\tilde{X}, \varphi, G_{E3}, G_{E6}, G_{E7}, G_{E8}, G_{E9},$

$G_{E12}, G_{E3}, G_{E14}, G_{E16}, G_{E4}\}$. Then $(1,2)^*$ -soft

b-closure of every $(1,2)^*$ -soft b-open set of

\tilde{X} is $(1,2)^*$ -soft b- open set in \tilde{X} . Hence

$(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is $(1,2)^*$ -soft b- extremally

disconnected space.

Remark 3.4: Every $(1,2)^*$ -soft extremally disconnected space is $(1,2)^*$ -soft b- extremally disconnected space but not conversely as shown in the following example.

Example 3.5: Consider the soft bitopological

space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ given in Example 3.3,

where $\tilde{\tau}_1 = \{\tilde{X}, \varphi, G_{E6}, G_{E7}, G_{E14}\}$, $\tilde{\tau}_2 = \{\tilde{X},$

$\varphi, G_{E3}\}$. Then $\tilde{\tau}_{1,2}$ - open set are $\{\tilde{X}, \varphi, G_{E3},$

$G_{E6}, G_{E7}, G_{E14}, G_{E9}\}$, $\tilde{\tau}_{1,2}$ - open set are $\{\tilde{X}, \varphi,$

$G_{E13}, G_{E15}, G_{E5}, G_{E12}, G_{E10}\}$. Here $\tilde{\tau}_{1,2}$ - cl(G_{E6})

$= \tilde{\tau}_{1,2}$ - cl($\{(e_1, \{a\})\}$) = $G_{E12} = \{(e_1, \{z,$

$a\})\}$, which is not a $\tilde{\tau}_{1,2}$ - open set.

Therefore, $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a $(1,2)^*$ -soft b-

extremally disconnected space but not $(1,2)^*$ -soft

extremally disconnected space

Theorem 3.6 : Let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be soft

bitopological space. Then the following statements are equivalent.

- (i) \tilde{X} is $(1,2)^*$ -soft b- extremally disconnected space.
- (ii) $(1,2)^*$ -sbint(F_E) is $(1,2)^*$ -soft b-closed set for each $(1,2)^*$ -soft b-closed set F_E of \tilde{X} .
- (iii) $(1,2)^*$ -sbcl($(1,2)^*$ -sbcl(F_E))^C = $((1,2)^*$ -sbcl(F_E))^C for each $(1,2)^*$ -soft b-closed set F_E of \tilde{X} .
- (iv) $G_E = (1,2)^*$ - sbcl(F_E)^C implies $(1,2)^*$ - sbcl(G_E) = $((1,2)^*$ - sbcl(F_E))^C for each pair of $(1,2)^*$ -soft b-open set F_E and G_E of \tilde{X} .

Proof.(i) \Rightarrow (ii) Let F_E be a $(1,2)^*$ -soft b-closed set of \tilde{X} . Then F_E^C is $(1,2)^*$ -soft b- open. Since \tilde{X} is $(1,2)^*$ -soft b-extremally disconnected space , $(1,2)^*$ - sbcl(F_E^C) is $(1,2)^*$ -soft b- open set. But $(1,2)^*$ - sbcl(F_E^C) = $((1,2)^*$ - sbint(F_E))^C. Therefore $(1,2)^*$ - sbint(F_E) is $(1,2)^*$ -soft b-closed set.

(ii) \Rightarrow (iii) Suppose that F_E is $(1,2)^*$ -soft b-open set F_E of \tilde{X} . Then $(1,2)^*$ -sbcl($(1,2)^*$ -sbcl(F_E))^C = $(1,2)^*$ -sbcl($(1,2)^*$ -sbint(F_E^C)). By assumption $(1,2)^*$ - sbint(F_E^C) is a $(1,2)^*$ -soft b-closed set .So $(1,2)^*$ -sbcl($(1,2)^*$ -sbint(F_E^C)) = $(1,2)^*$ -sbint(F_E^C) = $(1,2)^*$ -sbcl(F_E)^C.

(iii) \Rightarrow (iv) Let F_E and G_E be $(1,2)^*$ -soft b-open set of \tilde{X} . We put $G_E = ((1,2)^*$ -sbcl(F_E))^C. By assumption, $(1,2)^*$ -sbcl(G_E) = $(1,2)^*$ -sbcl($(1,2)^*$ -sbcl(F_E))^C = $((1,2)^*$ -sbcl(F_E))^C.

(iv) \Rightarrow (i) Let F_E be a $(1,2)^*$ -soft b-open set of \tilde{X} . Let $G_E = ((1,2)^*$ -sbcl(F_E))^C. From the assumption, we obtain that $(1,2)^*$ -sbcl(G_E) = $((1,2)^*$ -sbcl(F_E))^C. So $((1,2)^*$ -sbcl(G_E))^C = $(1,2)^*$ -sbcl(F_E). Hence $(1,2)^*$ -sbint(G_E^C) = $(1,2)^*$ -sbcl(F_E) . Thus $(1,2)^*$ -sbcl(F_E) is $(1,2)^*$ -soft b-open set of \tilde{X} . Then \tilde{X} is $(1,2)^*$ -soft b- extremally disconnected space. \square

4. $(1,2)^*$ -Soft b- Submaximal Space

Definition 4.1: A soft subset F_E of a soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called $(1,2)^*$ -soft dense if $\tilde{\tau}_{1,2}$ -cl(F_E) = \tilde{X} .

Definition 4.2: A soft subset F_E of a soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called $(1,2)^*$ -soft b-dense if $(1,2)^*$ -sbcl(F_E) = \tilde{X} .

Proposition 4.3: Every $(1,2)^*$ -soft b-dense set is $(1,2)^*$ -soft dense .

Proof. Let F_E be $(1,2)^*$ -soft b-dense set. Then $(1,2)^*$ -sbcl(F_E) = \tilde{X} . Since $(1,2)^*$ -sbcl(F_E) \subseteq $\tilde{\tau}_{1,2}$ -cl(F_E), we have $\tilde{\tau}_{1,2}$ -cl(F_E) = \tilde{X} and so F_E is $(1,2)^*$ -soft dense. \square

The converse of the Proposition 4.3 need not be true as can be seen from the following example.

Example 4.4: Consider the soft bitopological

space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$, where $\tilde{\tau}_1 = \{ \tilde{X}, \phi, F_{E_4},$

$F_{E_{10}} \}$, $\tilde{\tau}_2 = \{ \tilde{X}, \phi, F_{E_1}, F_{E_7}, F_{E_{13}} \}$ and

the soft subsets are as in the Example 2.13 . Then

$\tilde{\tau}_{1,2}$ - open set are $\{ \tilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_7}, F_{E_{10}},$

$F_{E_{13}} \}$ and $\tilde{\tau}_{1,2}$ - closed set are $\{ \tilde{X}, F_\phi, F_{E_{12}},$

$F_{E_{14}}, F_{E_{11}}, F_{E_8}, F_{E_5} \}$. $(1,2)^*$ -soft b-open sets

are $\{ \tilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_7}, F_{E_6}, F_{E_8}, F_{E_9}, F_{E_{12}},$

$F_{E_{10}}, F_{E_{13}} \}$ and $(1,2)^*$ -soft b-closed sets are $\{ \tilde{X}, \phi,$

$F_{E_1}, F_{E_2}, F_{E_7}, F_{E_3}, F_{E_8}, F_{E_5}, F_{E_{12}}, F_{E_{10}}, F_{E_{14}}$

$\}$. Take the soft subset $F_{E_7} = \{ (e_1, \{$

$x_1 \}), (e_2, \{ x_1 \}) \}$ and $\tilde{\tau}_{1,2}$ -cl(F_{E_7}) = $\{ (e_1,$

$\{ x_1, x_2 \}) \} = F_{E_6} \neq \tilde{X}$. Thus F_{E_7} is $(1,2)^*$ -soft

dense but not $(1,2)^*$ -soft b- dense set.

Theorem 4.5: If F_E is $(1,2)^*$ -soft dense in

$(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$, then F_E is $(1,2)^*$ -soft b-open in

$(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$.

Proof. Let F_E is $(1,2)^*$ -soft dense in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$.

Then $\tilde{\tau}_{1,2}$ -cl(F_E) = \tilde{X} . Thus $\tilde{\tau}_{1,2}$ -int($\tilde{\tau}_{1,2}$ -cl(

$F_E)) = \tilde{X}$, which implies that $F_E \subseteq \tilde{\tau}_{1,2}\text{-int}(\tilde{\tau}_{1,2}\text{-cl}(F_E)) \subseteq \tilde{\tau}_{1,2}\text{-int}(\tilde{\tau}_{1,2}\text{-cl}(F_E)) \cup \tilde{\tau}_{1,2}\text{-cl}(F_E)$. Therefore F_E is $(1,2)^*\text{-soft b-open}$ in $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$. \square

Remark 4.6 : In soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$, $(1,2)^*\text{-soft b-open}$ set F_E need not be (soft dense set as shown in the following example.

Example 4.7: Consider the soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$, where $\tilde{\tau}_1 = \{ \tilde{X}, \phi, F_{E_4}, F_{E_{10}} \}$, $\tilde{\tau}_2 = \{ \tilde{X}, \phi, F_{E_1}, F_{E_7}, F_{E_{13}} \}$ and the soft subsets are as in the Example 2.13. Then $\tilde{\tau}_{1,2}\text{-open}$ set are $\{ \tilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_7}, F_{E_{10}}, F_{E_{13}} \}$ and the collection of $(1,2)^*\text{-soft b-open}$ sets are $\{ \tilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_7}, F_{E_6}, F_{E_8}, F_{E_9}, F_{E_{12}}, F_{E_{10}}, F_{E_{13}} \}$. The collection of $(1,2)^*\text{-soft dense}$ sets of \tilde{X} are $\{ \tilde{X}, F_{E_7}, F_{E_9}, F_{E_{13}} \}$.

The $(1,2)^*\text{-soft b-open}$ sets are $\phi, F_{E_1}, F_{E_4}, F_{E_7}, F_{E_6}, F_{E_8}, F_{E_9}, F_{E_{12}}, F_{E_{10}}$ are not soft dense set.

Definition 4.8 : A soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called $(1,2)^*\text{-soft submaximal}$ if every $(1,2)^*\text{-soft dense}$ subset is $\tilde{\tau}_{1,2}\text{-open}$ set in \tilde{X} .

Definition 4.9 : A soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is called $(1,2)^*\text{-soft b-submaximal}$ if every $(1,2)^*\text{-soft dense}$ subset is $(1,2)^*\text{-soft b-open}$ set in \tilde{X} .

Example 4.10 : Let us consider the soft bitopological space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ as in the Example 2.13. Define $\tilde{\tau}_1 = \{ \tilde{X}, \phi, F_{E_1}, F_{E_7} \}$, $\tilde{\tau}_2 = \{ \tilde{X}, \phi, F_{E_3} \}$ $\tilde{\tau}_{1,2}\text{-open}$ set are $\{ \tilde{X}, \phi, F_{E_1}, F_{E_7}, F_{E_8}, F_{E_{13}} \}$ and $\tilde{\tau}_{1,2}\text{-closed}$ set are $\{ \tilde{X}, \phi, F_{E_{11}}, F_{E_6}, F_{E_5}, F_{E_{12}} \}$. Then the collection of

$(1,2)^*\text{-soft open}$ sets $\{ \tilde{X}, \phi, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{14}}, F_{E_{13}} \}$. The collection of $(1,2)^*\text{-soft dense}$ sets of \tilde{X} are $\{ \tilde{X}, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{14}}, F_{E_{13}} \}$. Here all $(1,2)^*\text{-soft dense}$ sets are $(1,2)^*\text{-soft b-open}$ set and so

$(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a $(1,2)^*\text{-soft b-submaximal}$ space.

Proposition 4.11 : Every $(1,2)^*\text{-soft submaximal}$ space is $(1,2)^*\text{-soft b-submaximal}$.

Proof. Let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a $(1,2)^*\text{-soft submaximal}$ space and F_E be a $(1,2)^*\text{-soft dense}$ subset of \tilde{X} . Then F_E is $\tilde{\tau}_{1,2}\text{-open}$ set in \tilde{X} . But every $\tilde{\tau}_{1,2}\text{-open}$ set is $(1,2)^*\text{-soft b-open}$ set and so F_E is $(1,2)^*\text{-soft b-open}$ set. Therefore $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is $(1,2)^*\text{-soft b-submaximal}$. \square

The reverse implication of Proposition 4.11 is not true as it can be seen in the following example.

Example 4.12 : Let $(1,2)^*\text{-soft b-submaximal}$ space $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ given in Example 4.10. Here the $(1,2)^*\text{-soft dense}$ set $F_{E_8} = \{ (e_1, \{x_1\}), (e_2, \{x_2\}) \}$ is $(1,2)^*\text{-soft b-open}$ set but not $\tilde{\tau}_{1,2}\text{-open}$ set in \tilde{X} . Therefore $(1,2)^*\text{-soft b-submaximal}$ space is not $(1,2)^*\text{-soft submaximal}$ space.

Proposition 4.13 : Every soft bitopological space is a $(1,2)^*\text{-soft b-submaximal}$ space.

Proof. Let $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ be a soft bitopological space and F_E be any $(1,2)^*\text{-soft dense}$ set of \tilde{X} . Then by Theorem 4.5, F_E is $(1,2)^*\text{-soft b-open}$ set. Therefore $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ is a $(1,2)^*\text{-soft b-submaximal}$ space. \square

5. Conclusion

In this Paper, the characterizations of a $(1,2)^*\text{-soft b-extremally disconnected}$ spaces in soft bitopological spaces. Also it is proved that all $(1,2)^*\text{-soft dense}$ is $(1,2)^*\text{-soft b-open}$ set. Finally, the properties of $(1,2)^*\text{-soft b-submaximal}$ space using $(1,2)^*\text{-soft dense}$ set.

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