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Non-Newtonian power law model on two phased arterial blood flow during lung cancer

Dheerendra Kumar

Department of Physical Sciences,Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot, India.

Abstract

The present paper has been developed for arterial blood flow of human circulatory system. The blood is considered to be constituted by two phases namely R.B.C. phase and plasma phase. The non-Newtonian power law model has been applied for bio-fluid mechanical set up. The tensorial formulation of the problem is realistic since some biofluid parameters are tensorial. The graphical interpretation of the solution between B.P.D. v/s Hematocrit is important for medical point of view. For that purpose clinical data during lung cancer has been analysed.

Key words: *Human blood circulation, non-Newtonian Power law model, Hematocrit, B.P.D., Lung cancer.*

I. Introduction

The most important feature of the circulation in human body; if a given volume of blood is pumped from the heart the same will return on the heart after passing through the different sub divisions of the circulatory system. According to Horsfield Keith 1977, "The human pulmonary arterial tree, the branches were divided into three zones according to their diameters; a proximal zone down to 0.8 mm in diameter, an intermediate zone down to 0.1 mm in diameter, and a distal zone of branches smaller than this. No data were presented for branches in the distal zone, but their dimensions were estimated by extrapolation".

Because according to Upadhyay V. and Pandey P. N., whenever the hematocrit increases, the effective viscosity of blood flowing in the arteries remote from the heart depends upon the strain rate. In this condition, the blood flow becomes non-Newtonian.

When strain rate is in 5 to 200 per second, the power law $\tau' = \eta_m e^n$, Where, $0.68 \leq n \leq 0.80$, holds good for blood flow. So we have applied non Newtonian power law model for in this condition the constitutive equation of blood is as follows.

$$\tau^{ij} = -pg^{ij} + \eta_m (e^{ij})^n = -pg^{ij} + \tau'^{ij}$$

Where, τ^{ij} is stress tensor and τ'^{ij} is shearing stress tensor.

II. Real model

Choice of frame of reference:

According to Upadhyay V. and Pandey P. N. 2000, we have secured a frame of reference is selected for mathematical modeling of two-phase blood flow of the state of a moving blood· It is experiential in view the difficulty and generality of the problem of blood flow, selected generalized three-dimensional orthogonal curvilinear coordinate system [Upadhyay 2000].

Hypothesis of two phase blood volume:

Blood has always held a special position in human though. The quantity of blood in the body is substantial, making up about 7% of the total body weight. Blood function in the transport of blood, oxygen, waste material and hormones in the regulation of temperature and in the control of disease [Upadhyay V., 2000]. According to Bessonov *et al*., 2016; "The human blood is a concentrated suspension of several formed cellular elements. The human blood cells volume more than 99% of all blood cells and total volume concentration of leukocytes and thrombocytes is only about 1%". Which is ignorable; so we have selected two phases where one phase-plasma and another is red blood cells phase. Plasma is a liquid, containing semi permeable packages of RBCs. The behavior of

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blood is about Newtonian at the high shear rate, while at law shear rate the blood exhibits yield stress and non-Newtonian behavior. The flow of blood is precious by the presence of blood cells. This effect is directly proportional to the volume taken by blood cells. Let the volume portion covered by blood cells in unit volume be X, X is replaced by H /100, where H is the hematocrit the volume percentage of blood cells. The hematocrit is normally about three times the hemoglobin concentration 'reported as grams per deciliter' Berkow, 1997. Then the volume portion covered by the plasma will be (1-X). The mass ratio of cells to plasma is r is given by:

$$r = \frac{X\rho_c}{(1-X)\rho_p}.$$

Where ρ_c and ρ_p are densities of blood cells and plasma respectivily.

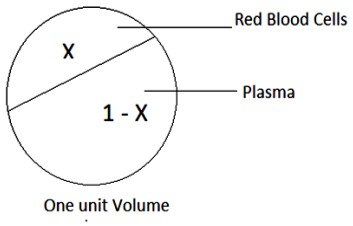

Fig. 1. Blood one unit volume

Usually, this mass ratio is not a steady. Even then this may be supposed to constant at present steady [Upadhyay, 2000].

Boundary conditions

I. The velocity of blood flow on the axis of blood vessels at $r = 0$ will be maximum and finite, say $v_0 =$ maximum velocity.

II. The velocity of blood flow on the wall of blood vessels at $r = R$, where, R is the radius of blood vessels, will be zero. This condition is well known as no-slip condition.

III. Mathematical formation

According to Upadhyay and Pandey (1999), whenever the hematocrit increases, the effective viscosity of blood flowing in the arteries remote from the heart depends upon the strain rate. In this condition, the blood flow becomes non-Newtonian. In this situation the constitutive equation of blood is as follows.

$$\tau^{ij} = -pg^{ij} + \eta_m(e^{ij})^n = -pg^{ij} + \tau'^{ij} \quad (3.1)$$

Where, τ^{ij} is stress tensor and τ'^{ij} is shearing stress tensor.

The equation of continuity in tensorial form for power law will be as follows:

$$\frac{1}{\sqrt{g}}(\sqrt{g}v^i)_{,i} = 0 \quad (3.2)$$

Again, write down the equation of motion as follows

$$\rho_m\left(\frac{\partial v^i}{\partial t}\right) + (\rho_m v^j)v^i_{,j} = \tau^{ij}_{,j} \quad (3.3)$$

Where τ^{ij} is taken from constitutive equation of power law flow (3.1). $\rho_m = X\rho_c + (1 - X)\rho_p$, is the density of blood and $\eta_m = X\eta_c + (1 - X)\eta_p$ is the viscosity of mixture of blood, $X = \frac{H}{100}$ is volume ratio of blood cells. H is hematocrit. Other symbols have their usual meanings. Since the blood vessels are cylindrical, the above major equations have to transformed the equations (3.23) and (3.24) in cylindrical form. As we know for cylindrical co-ordinates,

$$X^1 = r,\ X^2 = \theta, X^1 = r, X^3 = z$$

As we know earlier:

Matrix of metric tensor in cylindrical coordinates is fallows:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

While matrix of conjugate metric tensor is follows:

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Whereas the Chritoffel's symbol of 2nd kind is as follow-

$\left\{ {1 \atop 2\ 2} \right\} = -r$, $\left\{ {1 \atop 2\ 2} \right\} = \left\{ {1 \atop 2\ 2} \right\} = \frac{1}{r}$, Remaining others are zero.

The relation between covariant components and physical component of

$$\sqrt{g_{11}}v^1 = v_r \Longrightarrow v_r = v$$

$$\sqrt{g_{22}}v^2 = v_\theta \Longrightarrow v_\theta = rv^2$$

And $\sqrt{g_{33}}v^1 = v_z \Longrightarrow v_z = v^3$

Again the physical components of $-p_{,j}g^{ij}$ is $-\sqrt{g_{ii}}p_{,j}g^{ij}$

The matrix of physical components of sharing stress tensor $\tau'^{ij} = \eta_m(e^{ij})^n = \eta_m(g^{ik}v_{,k}{}^i + v_{,k}{}^j)^n$ will be as follows

$$\begin{bmatrix} 0 & 0 & \eta_m\left(dv/dr\right)^n \\ 0 & 0 & 0 \\ \eta_m\left(dv/dr\right)^n & 0 & 0 \end{bmatrix}$$

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The covariant derivative of $\tau^{,ij}$ is

$$\tau^{,ij}_{,j} = \frac{1}{\sqrt{g}}\frac{g}{\partial X^i}\left(\sqrt{g}\tau^{,ij}\right) + \left\{ \begin{matrix} i \\ j \end{matrix} \quad \begin{matrix} \\ k \end{matrix} \right\}\tau^{,ik}$$

Keeping in view the facts, the governing tensorial equations can be transformed into cylindrical form which is as follows:

Equation of continuity

$$\frac{\partial v}{\partial z} = 0 \qquad (3.4)$$

Equation of motion

Components of equations of motion

r- Component

$$-\frac{\partial P}{\partial r} = 0 \qquad (3.5)$$

θ – Component

$$0 = 0 \qquad (3.6)$$

Z – Component-

$$0 = -\frac{\partial P}{\partial z} + \frac{\eta_m}{r}\frac{\partial}{\partial r}\left[r\left\{\frac{\partial v_z}{\partial r}\right\}^n\right] \qquad (3.7)$$

Here this reality has been taken in see that the blood stream (flow) is pivotally (axially) symmetric in supply routes concerned (Upadhyay, 2000, pp. 54 - 56).

i.e. $v_\theta = 0$ And $v_r = 0$,

v_z and p do not depend upon θ. Also the blood flow steadily, i.e.

$$\left(\frac{\partial \mathrm{p}}{\partial \mathrm{t}}\right) = \left(\frac{\partial v_\mathrm{r}}{\partial \mathrm{t}}\right) = \left(\frac{\partial v_\theta}{\partial \mathrm{t}}\right) = \left(\frac{\partial v_\mathrm{z}}{\partial \mathrm{t}}\right) = 0 \qquad (3.8)$$

IV. Solution

Now we have integrated equation (3.3) [section III], $v_z = v(r)$ {v does not depend upon θ} and the integrating of equation of motion (3.5) yields:

Where, $P = P(z)$ {p does not depend upon θ}.

Now, from equation (3.7) and (3.8) the equations of motion (3.6) change in to the subsequent shape-

$$0 = -\left(\frac{\mathrm{dp}}{\mathrm{dz}}\right) + \left(\frac{\eta_\mathrm{m}}{\mathrm{r}}\right)\frac{\mathrm{d}}{\mathrm{dr}}\left\{\mathrm{r}\left(\frac{\mathrm{dv}}{\mathrm{dr}}\right)^\mathrm{n}\right\} \qquad (4.1)$$

We know that the pressure gradient $-\frac{\partial \mathrm{p}}{\partial \mathrm{z}} = \mathrm{P}$ of blood flow in the arteries remote the heart may be hypothetical to be steady and for this equation (3.8) the following form-

$$\frac{\mathrm{d}}{\mathrm{dr}}\left\{\mathrm{r}\left(\frac{\mathrm{d}v}{\mathrm{dr}}\right)^\mathrm{n}\right\} = -\left(\frac{\mathrm{P_r}}{\eta_\mathrm{m}}\right) \qquad (4.2)$$

Again equation (3.8), we obtain

$$\mathrm{r}\left(\frac{\mathrm{d}v}{\mathrm{dr}}\right)^\mathrm{n} = \frac{\mathrm{P_r}}{2\eta_\mathrm{m}} + \mathrm{A} \qquad (4.3)$$

The rate of the blood go with the flow at the axis of cylindrical arteries is most and constant. So that we've concern the boundary conditions at r = 0, $v = v_0$ (constant), on equation (4.2) takes the subsequent shape-

$$\mathrm{r}\left(\frac{\mathrm{d}v}{\mathrm{dr}}\right)^\mathrm{n} = \frac{\mathrm{P_r}}{2\eta_\mathrm{m}} \Longrightarrow -\frac{\mathrm{d}v}{\mathrm{dr}} = \left[\frac{\mathrm{P_r}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}} \qquad (4.4)$$

Again integrating equation (4.4), we get-

$$v = -\left[\frac{\mathrm{P}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}} \frac{r^{\frac{1}{n}+1}}{\frac{1}{n}+1} + \mathrm{B} \qquad (4.5)$$

To finish the arbitrary steady B, we are able to be applying the non-slip condition on the inner wall of the arteries at $r = R$,

$v = 0$, where R = radius of blood vessels, on equation (4.5) so as to get

$$\mathrm{B} = \left[\frac{\mathrm{P}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}} \frac{\mathrm{n}R^{\frac{1}{n}+1}}{\mathrm{n}+1}$$

Hence the equation (4.5), we take the following form-

$$\mathrm{V} = \left[\frac{\mathrm{P}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}} \frac{\mathrm{n}}{\mathrm{n}+1}\left[\mathrm{R}^{\frac{1}{\mathrm{n}}+1} - \mathrm{r}^{\frac{1}{\mathrm{n}}+1}\right] \qquad (4.6)$$

Which conclude that the velocity of the blood flow in the artery remote from heart.

V. Result and discussion:

The flow flax of blood through the arteries is-

$$\mathrm{Q} = \int_0^\mathrm{R} \mathrm{V}.2\pi\mathrm{r}\mathrm{dr} = \int_0^\mathrm{R}\left[\frac{\mathrm{P}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}}\frac{\mathrm{n}}{\mathrm{n}+1}(\mathrm{R}^{\frac{1}{\mathrm{n}}+1} - \mathrm{r}^{\frac{1}{\mathrm{n}}+1})$$

$$\mathrm{Q} = \left[\frac{\mathrm{P}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}}\frac{\mathrm{n}2\pi}{\mathrm{n}+1}\left[\frac{\mathrm{R}^{\frac{1}{\mathrm{n}}+1}.\mathrm{r}^2}{2} - \frac{\mathrm{n}\mathrm{r}^{\frac{1}{\mathrm{n}}+3}}{3\mathrm{n}+1}\right]_0^\mathrm{R}$$

$$\mathrm{Q} = \left[\frac{\mathrm{P}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}}\frac{\mathrm{n}2\pi}{\mathrm{n}+1}\left(\frac{(\mathrm{n}+1).\mathrm{R}^{\frac{1}{\mathrm{n}}+3}}{2(3\mathrm{n}+1)}\right)$$

$$\mathrm{Q} = \left[\frac{\mathrm{P}}{2\eta_\mathrm{m}}\right]^{\frac{1}{\mathrm{n}}}\frac{\mathrm{n}\pi\mathrm{R}^{\frac{1}{\mathrm{n}}+3}}{3\mathrm{n}+1} \qquad (5.1)$$

VI. Bio-physical interpretation:

We have collected clinical data (artery) of lung cancer patient; we know that the average human pulmonary blood flow flax $(Q) = 0.00708333\ m^3/$ sec ond, approximately conman radius of pulmonary artery $(\mathrm{R}_0) =$ 1.5 cm or 0.015m, according to Gustafson, Daniel R. (1980), $\eta_\mathrm{p} = 0.0013$ pascal second, according to Glenn Elert (2010), $\eta_\mathrm{m} = 0.0271$ pascal second . Approximately pulmonary artery length $(z_f - z_i) =$ 5 cm or o.05m [J.T. Ottesen *et al.*, 2006] and we get-

Examination of hematocrit v/s blood pressure in during lung cancer with respect to clinical data for patient- Q (Female), 46 years old.

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Table (I) Hemoglobin & blood pressures in clinical data

Date	HB (Hemog-lobin) in (gram/dl)	Hematocri In ($3 \times HE$ (kg/l)	Blood Pressure (BP) (mmhg)	Arteries Pressure Drop In Pascal-second $\left(\frac{S+D}{2}\right)$
10/9/2012	13.32	0.0376952	130/70	-993.96
25/11/2012	13.16	0.0372453	110/70	-662.64
19/1/2013	13.0	0.0367925	110/60	-3328.3
11/3/2013	12.8	0.0362265	100/80	-331.32
21/7/2013	12.7	0.0359434	120/90	-996.98
5/9/2013	12.9	0.0365095	100/70	-996.98
12/11/2013	12.6	0.0356604	110/80	-996.98

In according to used clinical data (Table: I) (Hematocrit) H = 0.0331133 and Pressure drop $(P_f - P_i) = 1331.32\ Pascal\ second.$

$$P(z) = \frac{P_f - P_i}{z_f - z_i}$$

And by using relation $\eta_m = \eta_c X + \eta_p (1 - X)$ …….. (A)

In according to used clinical data (Table- II. 5.3) (Hematocrit) H = 0.0367925 and Pressure drop $(P_f - Pi = 3328.3\ Pascal\ second.$

Using relation 'A' and we get η_c

$\Rightarrow 0.0271 = \eta_c(0.000367925) + 0.0013(0.999632075)$

$\eta_c =$ 70.12428702 Pascal second

Again using (A) relation and convert in to the hematocrit form-

$\eta_m =$ 0.70124287H + 0.001299522

Now using relation (B) and putting above values-

$$0.0070833 = \left[\frac{3328.3}{2\times0.0271\times0.05}\right]^{\frac{1}{n}} \frac{n\times3.14\times(0.015)^{\frac{1}{n}+3}}{3n+1}$$

$$668.3963 = \left(\frac{n}{3n+1}\right)(18422.32472)^{\frac{1}{n}}$$

By solving trial and error method we get- $n = 0.8093$

Again putting $n = 0.8093$ above equation (B) again and find ΔP

$$0.0070833 = \left[\frac{\Delta P}{2\eta_m \Delta_z}\right]^{\frac{1}{n}} \frac{0.8093\times3.14\times(0.015)^{\frac{1}{0.8093}+3}}{(3\times0.8093)+1}$$

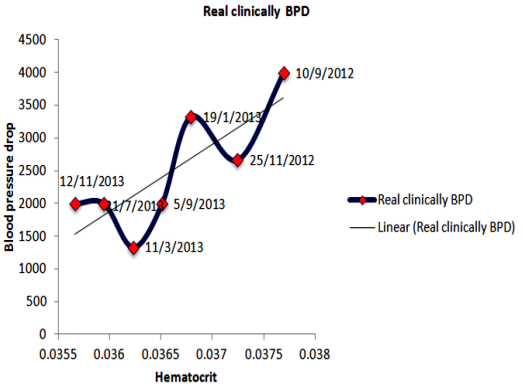
$\Delta P = (0.70124287H + 0.001299522)\ (973.3395908)$

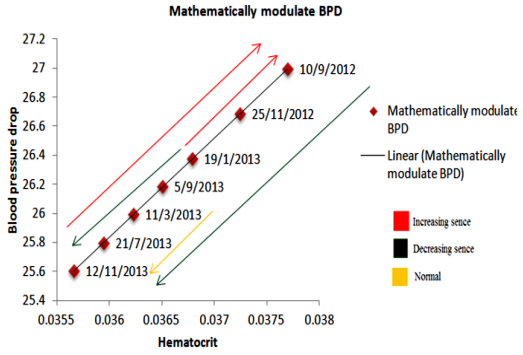
$\Delta P =$ 682.5474484H + 1.264875917

1.1 Table (II) Blood pressures drop v/s Hematocrit in Clinical data

Date	Hematocrit in $(3 \times HB)$ (kg/m^3)	BPD (Blood Pressure drop) In Pascal-second
10/9/2012	0.0376952	26.99572926
25/11/2012	0.0372453	26.68660352
19/1/2013	0.0367925	26.37754603
11/3/2013	0.0362265	25.99122418
21/7/2013	0.0359434	25.79799499
5/9/2013	0.0365095	26.1843851
12/11/2013	0.0356604	25.60483407

Graph: Table I & II. Blood pressure drop and hematocrit





Observation:

Graph (a) shows that these 7 different dates were observed minimum about 1331.32 on dated 11/3/2013 and maximum value obtain 3993.96 on dated 10/9/2010. The value from 0.0356604 to 0.0362265 via 0.0359434 of hematocrit value, the blood pressure drop upper convex in decreasing sence and the values from 0.0359434 to 0.0367925 via 0.0362265 & 0.0365095 of hematocrit value, the pressure drop shows down convex in increasing sence. Again the value from 0.0367925 to 0.0376952 via 0.0372453 of hematocrit value, the pressure drop shows down convex in increasing sence. Graph (b) shows that these 7 different dates were observed minimum about 25.60483407 on dated 12/11/2013 and maximum value obtains 26.99572926 on

dated 10/9/2012 (BDP). At the value from 0.0376952 to 0.0356604 via 0.0372453, 0.0367925, 0.0365095, 0.0362265 & 0.0359434 of hematocrit value, the blood pressure drop straightly decreases on dated 10/9/2012 to 12/11/2013 via 25/11/0/2012, 19/1/2013, 5/9/2013, 11/3/2013& 21/7/2013.

Conclusion:

From above clinical data table I a mathematically investigated and concluded; graph b (table. II) shows from 10/9/2012 to 12/11/2013 via 25/11/0/2012, 19/1/2013, 11/3/2013 & 21/7/2013 decreasing sence. Whereas from 12/11/2013 to 5/9/2013 shows increasing sence. According to this study we've concluded that designate the function of hematocrit inside the willpower of blood pressure drop. For this reason the hematocrit is extended then the blood pressure drop is likewise multiplied.

When graph shows increasing sence then we cannot suggest for serious dose and when graph shows decreasing sence then we suggest for serious dose but according to steepness of slops (triad line) at different condition (critical, middle, normal). We have suggested for successful operation but subject to the condition that the clinical data is collected in the duration of declared operation.

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