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A Mathematical Analysis of Artery with special reference to Jaundice

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Abstract:
Jaundice defined as a collection of fever within the liver parenchyma continues to remain a scourge for the population in our limited resource area. Despite radiological advancements, minimally invasive therapeutic measures and availability of effective antibiotics, morbidity and mortality continue to remain high in cases of Jaundice This study analyzes the clinical and microbiological features, mathematical methods, therapeutic management and predictive factors for recurrence and mortality of first episodes of Jaundice.

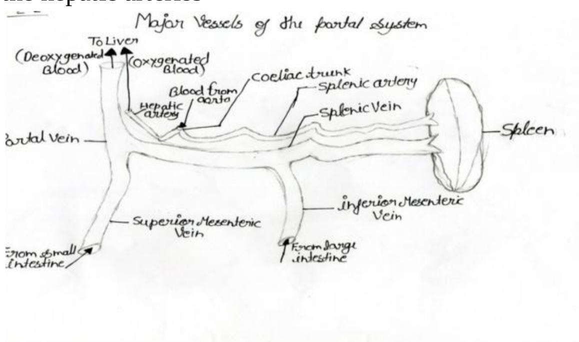
Key words:- *Mathematical model, Microbiological feature, two phase blood flow, stress and strain rate.*

1. Introduction –

1.1 Structure and function of liver: Liver is vulnerable to a variety of metabolic, toxic, microbial and circulatory insults. In some instances, the disease is primary while in others the hepatic involvement is secondary, which can be due to cardiac de-compensation, The liver is a reddish brown wedge shaped organ with four lobes of unequal size and shape. A human liver normally weighs 1.44 – 1.66 kg (3.2–3.7 lb). The liver is a dark, reddish - brown triangle - shaped organ that weighs about 3 pounds. It is both the largest internal organ and the largest gland in the human body. [1]

1.2 Introduction of Jaundice – Jaundice is term use to describe a yellowish tingle to the skin and the whites of the eye. Body fluids may also be yellow. The color of the skin and whites of the eyes will be very depending on level of bilirubin. It is the waste material found in the blood. Moderate level lead to a yellow color, while very high level will appear brown.[2]

(1.3) Structure & Functions of Hepatic circulatory System-The hepatic arteries supply arterial blood to the liver, accounting for there mainder of its blood flow. Oxygen is provided from both sources approximately half of the liver's oxygen demand is met by the hepatic portal vein, and half is met by the hepatic arteries

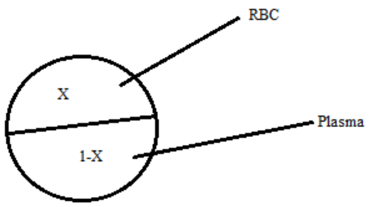


The hepatic portal system is the system of veinsco mprising the hepatic portal vein and its tributaries. I t is also the portal venous system.The portal venous system is responsible for directing blood fromparts of the gastrointestinal tract to the liver.Blood flow to the liver is unique in that it receives both oxygen ated and (partially) deoxygenated blood. [3]

2. Blood Constitution

2.1 Constitution of Blood- The average adult has a blood volume of roughly 5liter, which is composed of plasma and several kinds of cell. These blood Cells consist of erythrocytes (red blood cells), leukocytes (white blood cells) and (platelates). By 54.3% white cell about 0.7%. [5]

2.2 Plasma in Blood - Plasma is the liquid portion of blood a protein-salt solution in which red and white blood cells and platelets are suspended. platelets and other cellular components are removed.[6]. It is the single largest component of human blood, comprising about 55 percent, and contains water, salts, enzymes, antibodies and other proteins. Composed of 90% water, plasma is a transporting medium for cells and a variety of substances vital to the human body. Plasma carries out a variety of functions in the body.[7]



Red blood cells - carry oxygen around the body and remove carbon dioxide

White blood cells - help the body fight infection

Platelets - tiny cells that trigger the process that causes the blood to clot (thicken)

3. Real Model

3.1 Choice of frame of reference: The frame of reference for mathematical model of the moving blood keeping in view the difficulty and generality of the problem of blood flow. We select three dimensional orthogonal curvilinear co-ordinate system, prescribed as E^3called as 3-dimensional Euclidean space. We interpret the quantities related to blood flow in tonsorial form which is more realistic .Let the co-ordinate axes be OX^i where O is origin and i=1,2,3.The mathematical description of the state if a moving blood is affected by means of functions which give the distribution of the blood velocity $v^k = v^k (x^i,t)$.

3.2 Choice of Known Parameters- we have apply the only five known parameters and they are as follows-

(a) $\eta_c = viscosity\ cofficient\ of\ blood\ cells$

(b) $\eta_m = viscosity\ cofficient\ of\ mixture\ of\ two\ phases$

(c) $\eta_p = viscosity\ cofficient\ of\ plasma$

(d) $Q = value\ of\ flow\ flux$

(e) $\nabla p = \ pressure\ gradient$

Blood is the Non- Newtonian fluids, then using this constitutive equation for fluids.

$$\tau = \eta e^n$$

If $n = 1$ then the nature of fluid is Newtonian and if $n \neq 1$ then the nature of fluid is Non-Newtonian fluids.Where, τ is denoted by stress,e is denoted by strain rate

4. Mathematical Formulation and Analysis

4.1 Equation of Continuity for two phase blood flow– $\left(\sqrt{g}v^i\right) = 0$ - $\frac{1}{\sqrt{g}}$

(3.2) Equation of Motion-

$$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i_j = -\rho g^{ij} + \eta_m (g^{j,k}, v^i_k)$$

Where

$$\rho_m = x\rho_c + (1-x)\rho_p$$

Is the blood density of blood as mixture of blood cells and plasma.

$$\eta m = x\eta_c + (1-x)\eta_p$$

Is the viscosity of the mixture of the blood. Symbols have their usual meanings. The equation in cylindrical form for cylindrical coordinates.

$$x^1 = r\ , x^2 = \theta\ , x^3 = z$$

Determine the metric tensor is $r^2 = g$.

The components of metric tensor are $g_{11} = 1\ \ g_{22} = r^2\ , g_{33} = 1$ rest are zero. Again the components of the conjugate metric tensor are $g^{11} = 1\ , g^{22} = \frac{1}{r^2}\ , g^{33} = 1$

The value of chistoffel's symbols of second kind as follows-

$\begin{Bmatrix} 1 \\ 2,\ 2 \end{Bmatrix} = r$, $\begin{Bmatrix} 2 \\ 1,\ 2 \end{Bmatrix} = \frac{1}{r}$ rest are zero.

The relation between physical component and covariant components of the velocity of the blood flow are as follows-

$$\sqrt{g_{11}}v^1 = v_r \Rightarrow v^1$$

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$$\sqrt{g_{22}}v^2 = v_0 \Rightarrow rv^2$$
$$\sqrt{g_{33}}v^3 = v_z \Rightarrow v^3$$

Again the physical component of ρ, g^{ij} is $\sqrt{g_{ij}\rho,\ g^{ij}}$

The matrix of the physical component of $p_{,j}g^{ij}$ is $-\sqrt{g_{ii}p_{,j}\, g^{ij}}$

The matrix of component of shearing stress tensor $T^{ij} = \eta_m(e^{ij})n = \eta_m(g^{ik}v^i_k + g^{ik}v^j_k)$

$$\begin{matrix} 0 & 0 & \eta_m(\frac{dv}{dr})^n \\ 0 & 0 & 0 \\ \eta_m(\frac{dv}{dr})^n & 0 & 0 \end{matrix}$$

The covariant derivative of

$$T^{ij} = \frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}T^{ij})}{\partial x^j} + \{{}_j{}^i{}_k\}T^{ij}$$

We are in position write down the equation of blood flow in cylindrical form as follows.

The equation of continuity –

$$\frac{1}{r}\frac{\partial r_v}{\partial r} + \frac{1}{r}\frac{\partial rv_\theta}{\partial \theta} - \frac{\partial v_z}{\partial z} = 0$$

The equation of motion—

$$\rho_m(\frac{\partial v_r}{\partial t} + v_i\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v}{\partial \theta} - \frac{v^2{}_\theta}{r} + \frac{\partial v_\theta}{\partial z})$$

r-components

$$\frac{-\partial p}{\partial r} + \eta_m[\frac{\partial}{\partial r}\left\{\frac{1}{r}\frac{\partial rv_\theta}{\partial r}\right\} + \frac{1}{r^2}\frac{\partial v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} - \frac{\partial v_\theta}{\partial z^2}$$

θ –component

$$\rho = (\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_\theta v_r}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \eta_m[\frac{\partial}{\partial r}\{\frac{1}{r}\frac{\partial rv_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial y}{\partial \theta^2} + \frac{1}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial z^2}$$

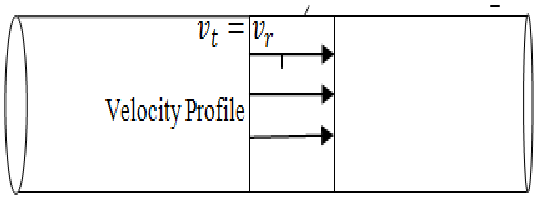
z- component

$$\rho = \{\frac{\partial v_z}{\partial x} + v_z\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\}$$

$$\frac{-\partial p}{\partial t} - \eta = [\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial v_z}{\partial r}\right\} - \frac{1}{r^2}\frac{\partial v_z}{\partial \theta^2} + \frac{\partial v_z}{\partial z^2}{}^2]$$

Boundary condition as follows-

The velocity of blood flow on the axis of artery i.e. r=0 will be maximum and finite



Figur 1.4

The blood flow is artery is symmetric with respect to axis hence $v_\theta = 0$ and also v_i, v_z and p *do not depend upon* θ.

The flow is $\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$ keeping in view these facts we obtain following results equation of continuity reduce to $\frac{\partial v_z}{\partial z} = 0 \Rightarrow v_z = v_r$

The r[th] component as the equation of motion reduce to

$$\rho_m = -\frac{\partial p}{\partial r} + \eta_m(0) \Rightarrow \frac{\partial p}{\partial r} = 0 \Rightarrow p = p(z)$$

θ component equation of motion reduce to $\rho_m(0) = 0 + \eta_m(0) = 0$

We get $r\frac{dy}{dx} = -\frac{pr^2}{2\eta_m} + A$

Where A is the constant of integration applying the first boundary condition A=0

Using 2[nd] boundary condition on equation

$$B = \frac{pR^2}{4\eta_m} \qquad v = \frac{\rho}{4\eta_m}(R^2 - r^2)$$

Patient Name - Sunita **Age -35**

S.N	Date	B.P(mmhg)	B.P.drop (S-S+D/2)	B.P Pascal	HB	H
1	10/2/16	110/70	20	2662.64	11.2	0.031698
2	15/2/16	140/90	25	3328.3	13.9	0.039339
3	18/2/16	100/60	20	2662.64	10.6	0.030094
4	19/2/16	130/80	25	3328.3	11.4	0.032264
5	20/2/16	110/60	25	3328.3	12.3	0.348113

Table 1.1

Viscosity of mixture $\eta_m = 0.0035$ *pascelsecond*

Viscosity of plasma $p = 0.0015$ *pascelsecond*

Length of common Hepatic Artery = 0.0347m

Radius of Artery = 0.0025 m

We know that $\eta_m = \eta_c X + \eta_p(1 - X)$

$$\eta_m = \frac{\eta_c H}{100} + \eta_p\left(1 - \frac{H}{100}\right)$$

$$0.0035 = \eta_c\frac{0.039339}{100} + (0.0015)(1 - \frac{0.039339}{100})$$

$$0.0035 = \eta_c 0.00039339 + 0.00149941$$

$$\eta_c = 5.085513104 \text{ p.s.}$$

Put the value of in this equation –

$[\Delta p] = Q[\Delta z]\, 8\eta_m / \pi R^4$

$$[\Delta p] = \frac{8\times0.015\times0.0347}{3.14\times(0.0025)^4}\left[\frac{\eta_c H}{100} + \eta_p\left(1 - \frac{H}{100}\right)\right]$$

$$[\Delta p] = \frac{0.004164}{1.2266\times10^{-10}}\left[\frac{5.085513104H}{100} + 0.0015\left(1-\frac{H}{100}\right)\right]$$

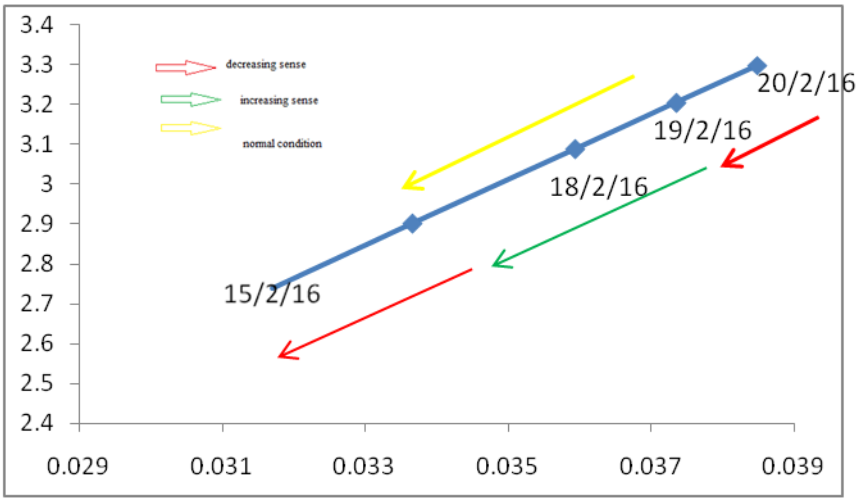
$[\Delta p]$ = 33947497.15 (5.084013104H +0.0015)/100

$[\Delta p]$ = (1725895.204H + 509.2124573)

Put the value of H in this equation and find the value of pressure drop.

Hematocrit(H)	0.033667	0.031698	0.0359434	0.03849057	0.03735849
Pressure- dropΔp	2.9015428	2.7381382	3.088191828	3.297654987	3.204037962
Date	10/2/16	15/2/16	18/2/16	19/2/16	20/2/16

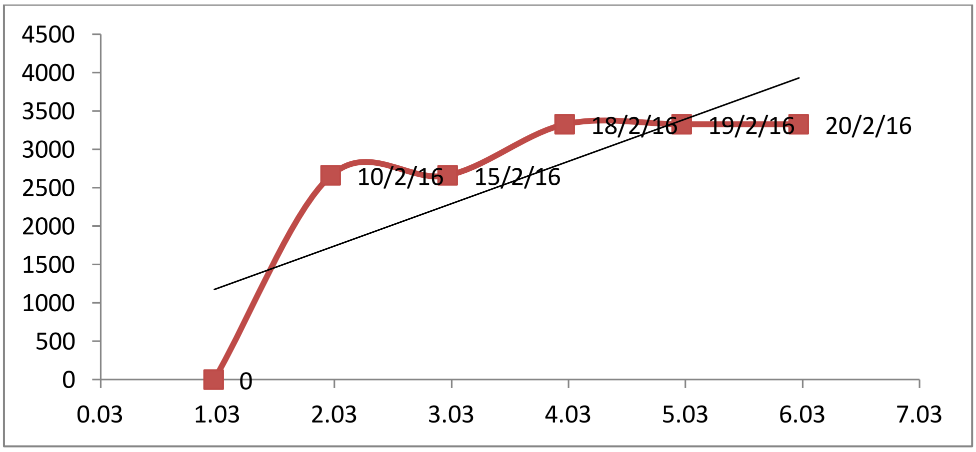
Table(1.2)



Graph (a)

Hematocrit	0.031698	0.039339	0.030094	0.032264	0.034811
B P Pascal	2662.64	3328.30	2662.64	3328.30	3328.30

Table (1.3)



Graph (b)

5. Conclusion

In graph (a) The relation between hematocrit and blood pressure. In date 10/2/16 to 20/2/16 the graph is shows the increasing sense. In increasing sense we can't suggest the operation. Also we can't suggest the high dose in this situation.

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