

Comparison of Double Sampling Plan with Single Sampling Plan in Supply Chain Management System – A Simulation Study

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Abstract

In Acceptance Sampling, there are several published research papers, which dealt with the performance of various sampling plans based on operating characteristic function and other related measures only. In this paper, an attempt is made to compare the performance of single sampling plan and double sampling plan in terms of the cost effectiveness in supply chain using the simulation software Goldsim 12.0. The study found a little difference only between those sampling plans in overall supply chain system.

Keywords: *Acceptance Sampling, Economic Order Quantity, Inspection, Quality, Simulation, Supply Chain Management*

1. Introduction

ANSI/ASQC standard A2 (1987) defines acceptance sampling as the methodology that deals with procedures by which decision to accept or not to accept a lot is based on the results of the inspection of samples. According to Dodge (1969), the general areas of acceptance sampling are Lot-by-lot sampling by the method of attributes, Lot-by-lot sampling by the method of variables, Continuous sampling of a flow of units by the methods of attributes and Special purpose plans. In acceptance sampling, inspection of items can vary from 100% of the delivery to a relatively few items from which the receiving firm draws inferences about the whole shipment.

To separate bad lots from good ones, it is to give the attention in the following aspects i) simple administrative procedures ii) economy in number of observations iii) to reduce the risk of wrong decisions by increasing lot size iv) to use accumulated sample data as a valuable source of information, v) to exert pressure on the producer or

supplier when the quality of the lots received is unreliable or not up to the standard and vi) more importantly the sample size should be reduced (Hamaker, 1960) when the quality is reliable and maintained at satisfactory level.

According to Case and Keats (1982), the sampling plan design methodologies are classified as 1) risk-based non-Bayesian, 2) risk-based Bayesian, 3) economically based non-Bayesian 4) economically based Bayesian approaches. The risk based non-Bayesian approach is applied by the vast majority of quality control practitioners due to their wider availability of tables and ease of the application. Economy based sampling plans explicitly consider certain factors as cost of inspection, accepting a non-conforming unit and rejecting a conforming unit in an attempt to design a cost-effective plan.

In this paper, non-Bayesian approach is applied for lot-by-lot sampling plan in the context of supply chain systems.

2. Review of Literature

Single Sampling Plan (SSP) is widely used for appraising the incoming product quality based on the result of a single sample. In such situations, it is logical to have a comparison between various sampling plans with single sampling plan. Wasserman (1990) presented a strategy to match operating performance of CSP-1 plans with single sampling plans based upon the long run proportion accepted about two target operating levels AQL and LQL. Also, he observed that it is not possible to match OC curves when this strategy is used.

Peach and Littauer (1946); Burgess (1948); Cameron (1952); Guenther (1969) and Golub (1953) has given a method and tables for finding the acceptance number 'c' of a single sampling plan

involving minimum sum of producer's risk and consumer's risk when the sample size 'n' is fixed. Radhakrishnan & Ravisankar (2009) presents a procedure for constructing single sampling plans for three attribute classes using acceptable quality level (AQL) as the quality standard.

Baker and Brobst (1978) have introduced the conditional Double Sampling Plan (DSP) procedures. It has operating characteristic curves identical to those of comparable Double sampling procedures when the second sample is required to make decision, it can be obtained from a related lot and not from the current lot. Soundararajan and Vijayaraghavan (1990); Soundararajan and Arumainayagam (1990) have provided tables for easy selection of double sampling plan.

This study discusses the acceptance sampling method through single and double sampling plans. It is an attempt to integrate inventory management in supply chain with acceptance sampling.

In any inventory management system, the Economic Order Quantity (EOQ) formula is probably one of the most famous formulae in the industrial management field, the composition and estimation of the cost parameters have always been vague and imprecise at best. In view of the inherently, the fuzzy aspect of the cost determination and the EOQ formula are re-examined in a fuzzy-set-theoretic perspective (Park, 1987).

In addition, Maddah & Jaber (2008) extend the model by allowing several batches of imperfect quality items to be consolidated and shipped in one lot. This is likely to be useful when there are economies of scale in shipping of imperfect quality items. The analyzed effect of screening speed and variability of the supply process on the order quantity showed that the order quantity in the model is larger than that of the classical EOQ model when the variability of the yield rate is reasonably low.

In this study, a comparative analysis is made to investigate the effects of five important parameters (inspection rate, the demand, the defective rate, the holding cost, and the ordering cost (as given in Lin, 2013).

3. Objective of the Study

The main objective of this study is to verify whether the double sampling plan is cost effective than the single sampling plan in the overall supply chain process.

4. Research Methodology

In this study, the simulation technique is used to design the models for single and double sampling plans in supply chain management to find out the Economic Order Quantity. Various costs are taken into account to compare the two plans to find out the cost effectiveness using simulation models.

Simulation is an important tool that provides a way in which alternative designs, plans and/or policies can be evaluated without having to experiment in a real system, which may be costly, time-consuming, or simply impractical to do. Moreover Simulation allows asking "What if?" questions about a system without having to experiment on the actual system which incurs the costs and delays associated with field tests, prototypes, etc.

By simulating the attached underlying forces of a manufacturing supply chain by defining the "links" in the chain (Retailer, Distributor, Manufacturer, Tier 1 supplier(s), Tier 2 supplier(s), etc.) and how organizations interact with each other. The model would simulate the movement of materials from purchase to finished product through the supply chain and could be used to identify ways in which the system could be modified (e.g., via technology or improved decision rules) to operate more efficiently.

GoldSim (12.0) is specifically designed software to quantitatively address the inherent uncertainty which is present in real-world systems. It is a user-friendly and highly supportive for graphical presentation. It also provides powerful tools for representing uncertainty in the processes, the parameters and future events, and for evaluating such systems in a computationally efficient manner. In addition, it provides powerful capabilities for superimposing the occurrence and consequences of discrete events into continuously varying systems. In this study, GoldSim is used for the realistic simulation of discrete events such as financial transactions and amount of resources utilized.

5. Supply Chain Model

In this paper, re-ordering system is having the basic assumption of the items submitted for production process or purchase by the people. However, practically there are possibilities of getting imperfect items i.e., defective or it might not meet the required standards of production process. Therefore, there is a need for rectification of the defective / imperfect items. Hence, the researcher applies acceptance sampling in-between (during the production process) the stage of purchase and production process. Generally after purchase, the entire lot is been sent for production process. But in

this proposed model, the purchased lot is being sent for sampling inspection. At this stage, if the acceptance sampling results are positive and if the lot is accepted, then the lot is being sent to production process. If the lot is rejected then it will be sent to 100% screening. And after that, the lot will be again sent to the previous stage. In some situations the lot will be sent back to the supplier itself for replacement, but this situation is not considered in this study.

In the proposed model, if there is any defective which is being found, say for example 1%, that particular defective lot is being sent for sampling inspection. Sampling inspection will decide whether the lot has to be sent for direct production or it has to undergo screening. If the lot is being sent for production directly, then the finished product will also reflect the same percentage of defective, which was detected in the raw material lot. This means the production level has to be increased, i.e. instead of producing 100 items, 101 items have to be produced.

Considering another situation, where the defective item is being sent for screening. The screening is actually good, because all the defective items will be replaced. Therefore, after the production process the finished goods will be free from defectiveness. However, 100% screening policy is a bit costlier when compared to sampling inspection. So, the decision will be taken by considering whether the lot has to be sent for inspection or not. If the lot is sent for inspection, it has to undergo rectification process which involves rectification cost. If the lot is sent for inspection, it incurs just the inspection cost. If the customer is receiving defective lot, then it incurs warrant cost, and to produce extra items it again incurs manufacturing costs. All these aspects need to be considered for having a good production process. However, if analytical models are considered for deriving at some equations, it becomes more complicated since we need to consider more parameters. Hence, the researcher is trying to solve this with the simulation model. In the simulation model, all the parameters are being fixed with numerical values to find out the total cost of implementation and optimal model is being determined.

GoldSim software is being used because this model involves inventory cost, storage cost (if the produced lot has to be kept inside the go down it involves cost for storage), back-order cost (if there is shortage of goods or if there is any delay in delivery of the lot) and ordering cost. The GoldSim has the capability of doing the computations of time durations, cost and inventory levels, etc.

5.1 100% Screen Policy Model

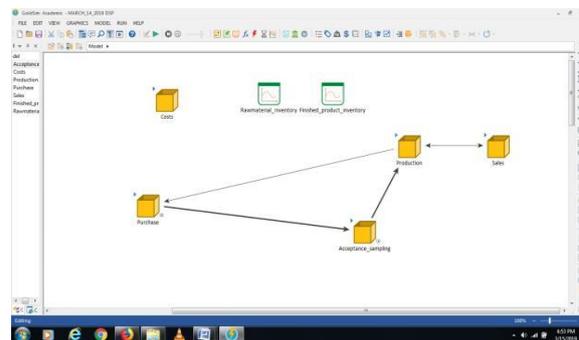


Figure-1: Operating procedure for the supply chain system under study

A supply chain process is given in the Figure-1, which shows the supply chain process i.e. from purchase the lot is being sent to sampling inspection after which the lot is being sent to production process (the arrows are used just for referencing and not to show the flow of the process). Finally, the lot will be sent to sales after production process. Purchase decisions are taken based on the production unit. The level of production will be determined based on the availability of the raw material. If there is any change, it is called as the re-order system, i.e. if stock level is less than the expected level, there will be need for new purchase order.

5.2 Operating Procedure of Acceptance Sampling Plans

The operating procedures for acceptance sampling plans are elaborated as below.

(i) Single sampling plan

Step-1: Inspect sample of size 'n'

Step-2: Count number of defects 'x', if it is less than or equal to an acceptance number 'c', then accept the lot.

Step-3: If $x > c$, then reject the lot and apply 100% screen the lot for removing the defectives.

(ii) Double sampling plan

Step-1: Inspect sample of size 'n₁'.

Step-2: Count number of defects 'x₁', if it is less than or equal to an acceptance number 'c₁', then accept the lot.

Step-3: If $x_1 > c_2$, an another constant, then reject the lot and apply 100% screen the lot for removing the defects

Step-4: If $c_1 < x_1 \leq c_2$, then take another sample of size 'n₂' and count number of defects 'x₂'.

Step-5: If $x_1 + x_2 \leq c_2$, then accept the lot, else reject the lot and apply 100% screen.

The Algorithms are programmed by the researchers in GoldSim software for single and double sampling plans with the respective formulae which are mentioned below.

Algorithm for SSP and DSP

The Probability of acceptance for single sampling plan is given by

$$Pa(p) = \sum_{k=0}^n e^{-np} np^k / k!$$

```

Script
1 Define: prob = 0.0
2 FOR (k = 0; ~k <= c1; k = ~k + 1)
3   Define: fact = 1
4   FOR (i = 1; ~i <= ~k; i = ~i + 1)
5     fact = ~fact*~i
6   END FOR
7   prob = (n*p)^~k/~fact+~prob
8 END FOR
9 Result = ~prob*exp(-n*p)
    
```

The Probability of acceptance for double sampling plan is given by

$$Pa(p) = Pa(p)_{ofSSP} + \sum_{j=c1+1}^{c2} e^{-np} np^j / j! \sum_{k=0}^{c2-j} e^{-np} np^k$$

```

Script
Statement List
1 Define: PROB = 0.0
2 Define: pr1 = 1.0
3 FOR (j = c1+1; ~j <= c2; j = ~j + 1)
4   Define: fact = 1
5   FOR (k = 1; ~k <= ~j; k = ~k + 1)
6     fact = ~fact*k
7   END FOR
8   pr1 = (n*p)^~j/~fact
9   Define: pr2 = 0
10  FOR (i = 0; ~i <= c2-~j; i = ~i + 1)
11    fact = 1
12    FOR (k = 1; ~k <= ~i; k = ~k + 1)
13      fact = ~fact*k
14    END FOR
15    pr2 = (2*n*p)^~i/~fact+~pr2
16  END FOR
17  PROB = ~pr1*~pr2+~PROB
18 END FOR
19 Result = ~PROB*exp(-3*n*p)
    
```

5.3 Cost Inputs

For this simulation process we must input certain values for the various costs involved in supply chain. Therefore from various research papers and from few qualitative interviews with experts, the researchers were able to find out the cost patterns. This pattern is taken from the Hlioui et al (2015) and is presented in the Table-1.

Table-1: Inputs for Various Costs*

Type	Item of cost	Cost (Rs)
Raw Material	Material cost	0.5
Finished product	Manufacturing cost	0.5
Raw Material	Storage cost	1
Finished product	Storage cost	1
Raw Material	Inspection Cost	12
Finished product	Backorder(Shortage) cost	40
Raw Material	Rectification cost	65
Finished product	Replacement (warranty) cost	90
Fixed	Ordering Cost	300

*Source: Hlioui et al (2015)

The material cost of raw material is 50%, manufacturing cost of finished product is 50%, which is like 50 paise, 50 paise (0.5, 0.5). The storage cost of raw material and finished product is Re.1 each, per unit per day. Inspection cost of raw materials is Rs. 12, back order cost is Rs. 40, rectification cost for raw material is Rs. 65 which involves inspection, changing cost, etc. Replacement cost for finished product is Rs. 90 which involves warranty, delivery charges, etc. The cost of the product is actually Re. 1, but when it reaches the customer, they have to pay Rs. 90, which involves all the related costs. When there is any optimization is being done in future, automatically the number of defective is expected to be low. There might be some stress to reduce the number of defectives. Hence, this is the finished cost for the product.

1.4 Comparison of SSP-DSP in Supply Chain Process

The calculated Total Cost based on the recommended parameters are presented in Table-2. In single sampling plan the acceptance number is 2, sample size is 125 ie., If the number of defective items is 0 or 1 or 2, then the lot is accepted. In double sampling, 100 items are selected as first sample initially. If the number of defective items are 0 item or 1 item, then the will be accepted. If it is more than 2, the lot is rejected and it will go for 100% inspection. In case, if the number of defectives is in-between 1 and 2, the only decision which can be taken is to take another sample of size 200 units. Out of these 200 units, if the number of defectives is 2, then the lot can be accepted.

Table-2: The Calculated Total Cost for SSP and DSP

SSP	C	2	2	2	2	2	2
	N	125	125	125	125	125	125
DSP	c ₁	1	1	1	1	1	1
	c ₂	2	2	2	2	2	2
	n ₁	100	100	100	100	100	100
	n ₂	100	100	100	100	100	100
	P	0.01	0.02	0.03	0.04	0.05	0.06
	Total Cost SSP Rs.	958715	1229203	1437751	1527174	1598496	1663804
	Total Cost DSP Rs.	989076	1324245	1422924	1552878	1596817	1675271

The total cost comparison of single and Double Sampling Plans for various 'p' values are presented in Figure – 2. It shows that the Double Sampling Plan has higher Total Cost comparatively than that of Single Sampling Plan for lower level of defective.

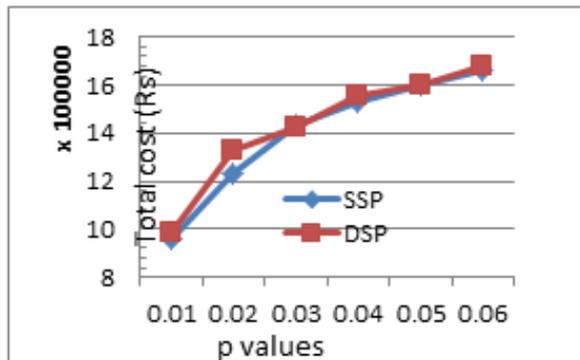


Figure – 2: Cost curve of SSP and DSP for various 'p' values

It may be infer logically that when the lot is of good quality (low proportion defective), there is no need of second sample for inspection. This will reduce the cost involved for inspection. On the other hand, for higher proportion defective, both double sampling and single sampling plan are equal in Cost effectiveness. Therefore, there is no much use of second sample in reduction of cost though there is average sample size economy due to DSP.

This argument is better to test with a hypotheses testing (paired t-test). Let us fix the null hypothesis as there is no significant difference between the means of the total costs associated with SSP and DSP. The summary of the t- Test results is presented in Table -3.

Table-3: Summary of the t-Test Results

Paired t-Test for Two Sample Means		
	Total Cost SSP	Total Cost DSP
Mean (Rs)	1402523.83	1426868.50
Variance (Rs)	70091484946.97	61734820727.50
Number of Observations	6	6
Hypothesized Mean Difference	0.00	
Df	5	
Observed value of test statistic	-1.55	
p-value (one tailed)	0.09	

As the p-value (=0.09) is greater than 0.05 the null hypothesis of equality could not be rejected at 5% level of significance with sufficient evidence. Hence it may be concluded that there is no significant difference between the means of the total costs associated with SSP and DSP.

6. Conclusion

This study reveals an interesting fact that while defective items are higher, both double sampling and single sampling plan are same in cost effectiveness. It is concluded that type of sampling plan has no big role in supply chain cost while more number of other cost parameters dominates. This study is limited to 100% inspection when the lot is rejected in sampling and it has discussed only SSP and DSP for the comparison.

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