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Laplace Transform Method for Solving Electric Circuit Equations

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Abstract

Laplace transform is a powerful mathematical tool for solving ordinary differential equations. Numerical solutions of ordinary differential equations for an electrical circuit are very helpful in many engineering branches. In this paper, we shall discuss the numerical solutions of some ordinary differential equations arising in electrical circuit with the help of Laplace transform method. Numerical results show the accuracy of the Laplace transform method.

Keywords: *Laplace transforms method, Electric circuits, Ordinary differential equations, Numerical examples.*

1. Introduction

Laplace transform method is a powerful method for solving ordinary differential equations arising in different branches of sciences and engineering. The knowledge of Laplace transforms has in recent years become an essential part of mathematical background required of engineers and scientists. This is because the transform methods provide an easy and effective means for the solution of many problems arising in engineering. The method of Laplace transform has the advantage of directly giving the solution of differential equations with given boundary values without the necessity of first finding the general solution and then evaluating from it the arbitrary constants. Laplace transform reduce the problem of solving differential equations to mere algebraic manipulation. Exact analytic solutions of some ordinary fractional differential equations have been presented in [1]. Haar wavelet method has been used for solving L-C-R equations in [2]. Inverse Laplace transform of some analytic functions has been used for the solutions of the Boltzmann equation in [3]. Exact solution of some linear fractional differential equations by Laplace transform has been presented in [5]. Coupling of homotopy perturbation, Laplace transform and pade Approximants have been used for solving nonlinear oscillatory systems in [6]. Laplace transform and Green function method have been used for calculation of water flow and heat transfer in fractured rocks in [8].

The formulation of differential equations for an electric circuit depends on the following two Kirchhoff's laws:

(a) The algebraic sum of the voltage drops around any closed circuit is equal to the resultant electromotive force in the circuit.

(b) The algebraic sum of the currents flowing into any node is zero.

1.1 Formulation of Differential Equations:

R, L Series Circuit:

Consider a circuit containing resistance R and inductance L in series with a voltage source (battery) E. Let i be the current flowing in the circuit at any time t. Then by Kirchhoff's first law, the sum of voltage drops across R and L is equal to E. Therefore, we get

$$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}, \qquad (1)$$

With initial condition $i(0)=0$. The exact solution of (1) is

$$i=\frac{E}{L}\left(1-e^{-\frac{Rt}{L}}\right).$$

R, L, C series circuit:

Consider a circuit containing resistance R, inductance L and capacitance C, all in series with a constant *e.m.f* E. If i be the current in the circuit at time t, then the charge q on the condenser that is

$$i=\frac{dq}{dt}.$$

Applying Kirchhoff's law, the sum of voltage drops across R, L and C is equal to E. Therefore, we get

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$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E. \qquad (2)$$

1.2 Laplace Transform:

Let $f(t)$ be a function of t defined for all positive values of t. Then the Laplace transforms of $f(t)$, denoted by $L\{f(t)\}$ is defined as

$$L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt=\bar{f}(s),(3)$$

provided the integral exists. s is a parameter which may be a real or complex number.
From (3), we get

$$L\{f(t)\}=\bar{f}(s),$$

i.e.

$$f(t)=L^{-1}(\bar{f}(s)).$$

Then, $f(t)$is called the inverse Laplace transform of $\bar{f}(s)$.

1.3 Numerical Examples

In this section, we present some numerical observations for solving electric circuit equations arising in various applications of engineering.

Example 1: Solve

$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=Esinwt, \qquad (4)$$

With initial conditions

$$Q(0)=Q'(0)=0$$

The exact solution of the problem is

$$Q(t)=-\frac{ECw}{(1-LCw^2)}\sqrt{LC}\,Sin\left(\frac{t}{\sqrt{LC}}\right)+\frac{EC}{(1-LCw^2)}Sinwt$$

Assuming $E=2v,\ w=2$radian, $C=2\mu F, L=1H$. Taking Laplace transform on both sides of (4), we get

$$L\left\{\frac{d^2Q}{dt^2}+10^6\,\frac{Q}{2}\right\}=L\{2sin2t\},$$

$$L\{Q''+k^2Q\}=\frac{4}{s^2+4},$$

where $k^2=\frac{10^6}{2}$

$$s^2\bar{Q}(s)-sQ(0)-Q'(0)+k^2\bar{Q}(s)=\frac{4}{s^2+4}$$

Applying initial conditions, we get

$$[s^2+k^2]\bar{Q}(s)=\frac{4}{s^2+4},$$

$$\bar{Q}(s)=\frac{4}{s^2+4}\times\frac{1}{s^2+k^2},$$

$$\bar{Q}(s)=\frac{4}{(s^2+4)(s^2+k^2)},$$

$$\bar{Q}(s)=\frac{4}{4-k^2}\left[\frac{1}{s^2+k^2}-\frac{1}{s^2+4}\right]$$

Taking inverse Laplace transform both sides, we get

$$Q(t)=\frac{4}{4-k^2}L^{-1}\left\{\frac{1}{s^2+k^2}-\frac{1}{s^2+4}\right\}$$

$$Q(t)=\frac{4}{4-k^2}\left\{\frac{sinkt}{k}-\frac{sin2t}{2}\right\}$$

Figure 1 shows the comparison of exact solution, Laplace transform solution and Haar wavelet solution.

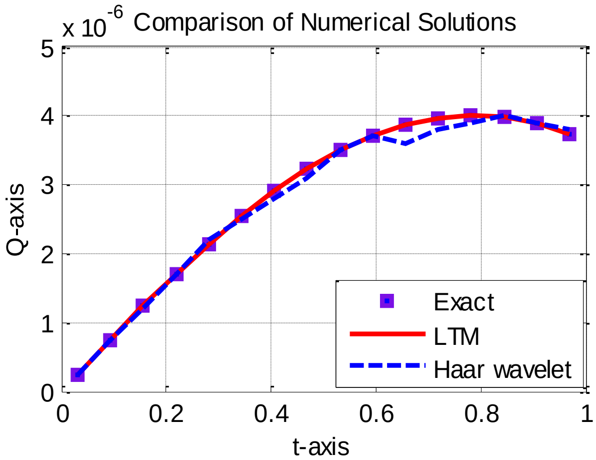

Figure 1: comparison of numerical solutions for example 1.

Example 2: Solve

$$L\frac{d^2Q}{dt^2}+\frac{Q}{C}=Esinwt, \qquad (5)$$

With initial conditions

$$Q(0)=Q'(0)=0$$

Assuming $w=1$radian $E=4v, C=0.3\mu F, L=2H$ Taking Laplace transform on both sides of (5), we get

$$L\left\{\frac{d^2Q}{dt^2}+10^6\,\frac{Q}{0.6}\right\}=L\{4sint\},$$

$$L\{Q''+k^2Q\}=\frac{2}{s^2+1},$$

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where $k^2 = \frac{10^6}{0.6}$

$$s^2\bar{Q}(s) - sQ(0) - Q'(0) + k^2\bar{Q}(s) = \frac{2}{s^2+1}$$

Applying initial conditions, we get

$$[s^2 + k^2]\bar{Q}(s) = \frac{2}{s^2+1},$$

$$\bar{Q}(s) = \frac{2}{s^2+1} \times \frac{1}{s^2+k^2},$$

$$\bar{Q}(s) = \frac{2}{(s^2+1)(s^2+k^2)},$$

$$\bar{Q}(s) = \frac{2}{1-k^2}\left\{\frac{1}{s^2+k^2} - \frac{1}{s^2+1}\right\},$$

Taking inverse Laplace transform both sides, we get

$$Q(t) = \frac{2}{1-k^2}L^{-1}\left\{\frac{1}{s^2+k^2} - \frac{1}{s^2+1}\right\}$$

$$Q(t) = \frac{2}{1-k^2}\left\{\frac{sinkt}{k} - sint\right\}.$$

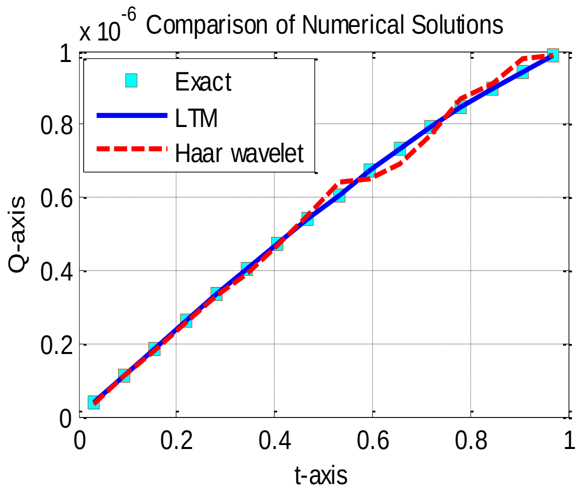

Figure 2: comparison of numerical solutions for example 2.

Figure 2 shows the comparison of exact solution, Laplace transform solution and Haar wavelet solution.

Example 3: Solve

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{E}{L}, \qquad (6)$$

with initial conditions $I(0) = 0$.

Assuming $R = L = 2,\ E = 4$. Equation (6) becomes

$$\frac{dI}{dt} + I = 2$$

Taking Laplace transform both sides of above equation, we get

$$L\{I' + I\} = L\{2\},$$

$$s\bar{I}(s) - I(0) + \bar{I}(s) = \frac{2}{s},$$

$$(s+1)\bar{I}(s) = \frac{2}{s}$$

$$\bar{I}(s) = \frac{2}{s(s+1)}$$

$$\bar{I}(s) = \frac{2}{s} - \frac{2}{(s+1)}$$

Taking inverse Laplace transform both sides, we get

$$I(t) = L^{-1}\left\{\frac{2}{s} - \frac{2}{(s+1)}\right\}$$

$$I(t) = L^{-1}\left\{\frac{2}{s}\right\} - L^{-1}\left\{\frac{2}{(s+1)}\right\}$$

$$I(t) = 2 - 2e^{-t}$$

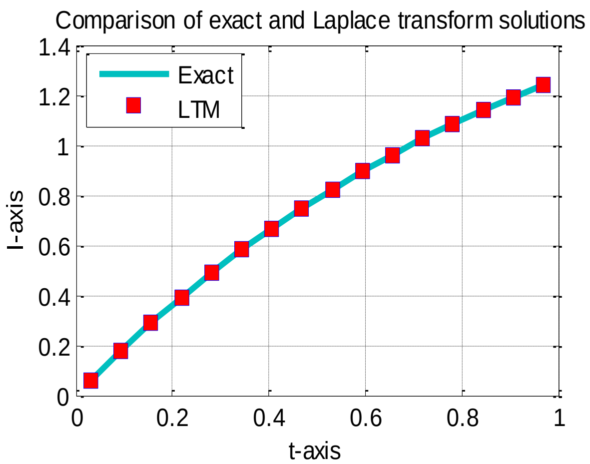

Figure 3: comparison of numerical and exact solution for example 3.

Figure 3 shows the comparison of exact solutions with Laplace transform solution.

Example 4: Solve

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{E}{L}, \qquad (7)$$

With initial conditions $I(0) = 0$.

Assuming $R = 1,\ L = 0.5,\ E = 2$. Equation (7) becomes

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$$\frac{dI}{dt} + 2I = 4$$

Taking Laplace transform both sides of above equation, we get

$$L\{I' + 2I\} = L\{4\},$$

$$s\bar{I}(s) - I(0) + 2\bar{I}(s) = \frac{4}{s},$$

$$(s+2)\bar{I}(s) = \frac{4}{s},$$

$$\bar{I}(s) = \frac{4}{s(s+2)},$$

$$\bar{I}(s) = \frac{2}{s} - \frac{2}{(s+2)},$$

Taking inverse Laplace transform both sides, we get

$$I(t) = L^{-1}\left\{\frac{2}{s} - \frac{2}{(s+2)}\right\}$$

$$I(t) = L^{-1}\left\{\frac{2}{s}\right\} - L^{-1}\left\{\frac{2}{(s+2)}\right\}$$

$$I(t) = 2 - 2e^{-2t}$$

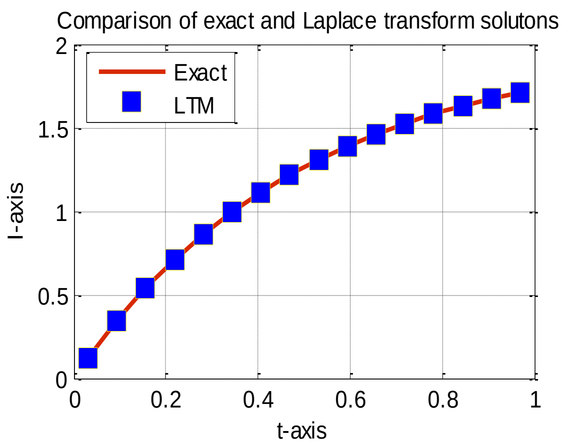

Figure 4: comparison of numerical and exact solution for example 4.

Conclusion

There are many applications of engineering and sciences that used linear and nonlinear models of differential equations. In this paper, we solve some models of ordinary differential equations that arising in some applications of engineering and sciences. Numerical results show that Laplace transform method is more reliable and efficient for solving such applications of sciences and engineering.

References

[1] Ali, E.H., Asaad, A.A., Elagan, S.K., Mawaheb El Amin and AlDien, M. Using Laplace transform method for obtaining the exact analytic solutions of some ordinary fractional differential equations. Global Journal of Pure and Applied Mathematics, 13(9): 5021-5035, (2017).

[2] Berwal, N., Panchal, D. and Parihar, C.L. Numerical solution of series L-C-R equation based on Haar wavelet. Italian Journal of Pure and applied Mathematics, 30; 157-166, (2013).

[3] Bobylev, A.V. The inverse Laplace transform of some analytic functions with an application to the eternal solutions of the Boltzmann equation. Applied Mathematics Letters, 15(7): 807–813, (2002).

[4] Doetsch, G. Introduction to the Theory and Applications of the Laplace Transformation. Springer-Verlag, Berlin (1974).

[5] Kazem, S. Exact solution of some linear fractional differential equations by Laplace transform. International Journal of Nonlinear Science, 16 (1): 3-11, (2013).

[6] Merdan, M., Yildirim, A, Gokdogan, A. and Mohyud-dins, T. Coupling of homotopy perturbation, Laplace transform and pade approximants for nonlinear oscillatory systems. World Applied Sciences Journal, 16(3), 320-328, (2012).

[7] Schiff, J.L. The Laplace Transform theory and applications, Springer, New York (1999).

[8] Xiang, Tan-yong, Guo and Jia-qi, A Laplace transform and Green function method for calculation of water flow and heat transfer in fractured rocks. Rock and Soil Mechanics, 32(2), 333-340, (2011).