IJASRM
International Journal of Advanced Scientific Research and Management, Volume 4 Issue 6, June 2019
www.ijasrm.com
ISSN 2455-6378

Tuning of PID Controller Using Bode Plot Technique towards the Dynamics of Human Cardiac Muscle Considering Dead Time

Aishwarya Banerjee[1], Soumyendu Bhattacharjee[2] and Biswarup Neogi[3]

[1]Associate Consultant, Tech Lateetud Pvt Ltd, Kolkata, W.B, India
[2]Assistant professor, Dept. of ECE, KIEM, Mankar,W.B, India
[3]Assistant professor, Dept. of ECE, JISCE, Kalyani, W.B, India

Abstract

Because of the prolong use of the system, the performance (Output parameters of the system) can change and output of the system may start deteriorating from the desired value. If the performance of a system, based on control theory is not up to the expectations as per the desired specification, then some changes in the system are required to obtain the desired performance. PID controllers are widely used in process control industries as implementation structure is simple. Cardiovascular muscle senses the force generated due to the contraction and expansion of muscle wall .This can be well understood by the analytical approach of the transfer function generated by using a mechanical model of force displacement analogy. But in practical dead time should be considered along with the derived transfer function of the model. Due to the huge popularities of tuning method in the process industries, lots of methods have been developed to find out the parameters of the PID controller such as a Z-N method, IAE, ITAE and IMC method. All the methods are surveyed and modified by Ho and Ho’s method. Here in this paper, the tuning parameters have been calculated using Bode plot technique and finally the simulation result is compared with the older tuning process.

Keywords: *Bode plot, Muscle Dynamics, PID Controller, Cardiac Muscle Mass.*

1. Introduction

Because of the prolonged use of a system (performance), the parameters of the system can change. So to improve the transient response and the steady state response of a plant, a controller (in the domain) is being introduced along with the plant in a cascading arrangement[1][2].To analyze the cardiovascular system and its effects, control system based modeling of cardiovascular muscle has been observed which is important and essential for researchers. In the previous work the transfer function of cardiac muscle has already been derived with the variation of age considering proper value of damping coefficient and natural frequency considering the system as a second order system, but in practical modeling of cardiac muscle must be represented by HOPDT model (Higher order plus dead time model) [3][4]which makes it towards the nonlinearity. PID controllers are the most important and popular controller used infield of research. The rule to tune the PID Controller had proposed by Ziegler and Nichols in the year of 1942.Tuning of PID controllers for HOPDT is not simple as this model is unable to generate peaks for monotonic (all poles lie on the negative real axis) processes. In this work, poles of higher order system are allocated in such a way that model poles are cancelled out by controller zero though proper cancellation is never possible. As a result, the higher order system is reduced into a second order system and then tuning of the PID controller has been applied [16]. In many applications might require two some specific control action to get the appropriate system and that can be found by setting the other parameters to zero. If accuracy is required for a plant, then the PI controller is sufficient to achieve the requirements [7][17]. When the speed of the system is a priority, then the PD controller is suitable and then an integral parameter is set to zero. So PID controller is used as per requirement of different processes. PID controller takes input as a difference between set point and feedback signal. Tuning of PID controller by using the root locus method is for any dynamics whether it is the high order or low order, high dead time or low dead time, the oscillatory or monotonic system. This method is also approximated on the basis of the Newton-Raphson method to bring the system very close to the exact value. The response of the system is highly improved in comparison to those older methods. The assigning of the pole is done by a root locus method to see whether pole lie on real part or imaginary part.

2. Cardiac Muscle Modeling

Human cardiovascular system and its abnormities nowadays play an important role for research. To analyze cardiovascular system and its effects, control system based modeling of cardiovascular muscle is very much needed for researchers. Consideration is focused on the fact that, the cardiovascular system is a 2nd order system having suitable parametric values of the damping coefficient & the natural frequency. The equation for cardiovascular system having damping coefficient and natural frequency can be written as follows.

$$f_d = f_n\sqrt{(1-\xi^2)} \quad (1)$$

Here f_d is the damping frequency, fn be the natural frequency and ξ is the damping coefficient. The dynamic behavior of 2nd order system can also be described by the above parameters. A cardiac muscle can be modeled with a mass (M) of it, viscous damping (B) which is proportional to the wall movement of the specimen and a torsion drag which is also proportional to the displacement of the specimen (K). f(t) is the force exerted by the cardiac muscle specimen generated by electrical and electromechanical activity effects on the cardiovascular system[5][6]. Comparing the model equation with standard 2nd order system equation it is found that viscous damping (B) is a function of cardiac muscle mass 'M' is taken based on different age to calculate the value of 'B' for modeling of cardiac muscle. Damping coefficient (ξ) has also been varied for individual 'M' to get different transfer function of the model stated above. Steady state error is calculated for different transfer function to get an optimized value of 'B'**.**As the system is under damped, it can be assumed that the (damping ratio) ξ = 0.7 and will vary the damping coefficient are for our next study .The value of ω_n for different value of ξ has been calculated using equation (2) and given in table 1 shown below.

Table 1: The value of ω_n for different value of ξ

Set	Assumed values of Damping coefficient (ξ)	Natural Frequencies (ωn)
1	0.1	7.57
2	0.2	7.73
3	0.3	7.83
4	0.4	8.24
5	0.5	8.66
6	0.6	9.37
7	0.7	10.5
8	0.8	12.5
9	0.9	17.5

Cardiac muscle mass (M) is one of the important parameter of muscle dynamics. It has been observed that mass of human cardiac muscle varies with age and given below in table 2.

Table 2: Variation of mass of heart muscle with age

Age Group(Years)	Cardiac Muscle Mass (Male)(gm)	Cardiac muscle Mass(Female)(gm)
5-20	264	196
20-35	277	251
35-50	297	267
50-65	317	256

So the model is based on the glimpses of changing transfer function with different age groups due to the variation of mass of heart muscle .The variation of mass with different age group is given above in table 2. Here M is the mass of cardiac muscle, B is the constant of viscous drag of myocardial cell. x(t) is the movement of cardiac wall which is generated due to exerted force f(t) by electrical and electrochemical activity effects on the cardiovascular system. The viscous damping is proportional to the muscle wall movement of the specimen, so that the contribution to this viscous damping may be represented by the expression $B\frac{dx(t)}{dt}$. Torsional drag is proportion to the displacement of the specimen, so that its contribution is given by the expression k.x(t), k being the constant of proportionality. So the diffential equation is given by

$$M\frac{d^2x}{dt^2} + B\frac{dx(t)}{dt} + k\,x(t) = f(t) \quad (2)$$

Taking the Laplace transform of equation (2) the following equation can be written

$$Ms2X(s) + BsX(s) + kX(s) = F(s) \quad (3)$$

Here we have taken the Laplace Transform of the equation, where X(s) = L[x(t)] and F(s) = L[f(t)]

So from equation (3) it is possible to write

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2+Bs+k} \quad (4)$$

$$\frac{X(s)}{F(s)} = \frac{K_1}{s^2+K_2s+K_3} = T(s) \quad (5)$$

$$\frac{1/M}{s^2+\frac{B}{M}s+\frac{K}{M}} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad (6)$$

Again from the previous equation (6) numerically it has calculated that

$$K_1 = 1/M \quad (7)$$

$$K_3 = k/M \quad (8)$$

Since numerically, K_1 and K_3 are equal from equation (8), it can be concluded that

$$1/M = K/M \quad (9)$$

From the above equation, it can be observed that, K = 1.

Again from equation (2) it is observed that, $\frac{B}{M} = 2\xi\omega_n$

$$B = M*2\xi\omega_n \quad (10)$$

So from the above stated equation (10) it can be observed that B is function of M and ξ. So by varying the values of M and ξ and considering two cells are connected in series, so many transfer functions has been obtained. For our simulation purpose one of them (Forth order with a delay) is taken and given below.

$$G(s) = \frac{1}{s^4+10s^3+37s^2+60s+36}e^{-.4s} \quad (11)$$

3. Applied Method for Higher Order Reduction

The transfer function of a plant is represented here by G(s). Consider a transfer function is given by

$$G(s) = \frac{e^{-st_0}}{as^2+bs+c} \quad (12)$$

Depending on the values of a, b, and c, the model can be characterized into real or complex poles. Hence it is easy to represent both non-oscillatory as well as oscillatory processes [15]. Again a PID controller can be written in the form of $K(s) = K_P + \frac{K_I}{s} + sK_D$. Putting s=jω into equation 12, and then divide into real and imaginary parts, We need four equations for finding out four unknowns. So by fitting the process gain G(s) at two nonzero frequency points it can be constructed into the equation of K(S). Now two different points are chosen like $s = j\omega_c$ and $s = j\omega_b$, where angle of $j\omega_c$ is -180 degree and angle of $j\omega_b$ is -90 degree. After calculating real and imaginary parts, the following equations are formed and given below.

$$c - a\omega_c^2 = \frac{\cos(\omega_c L)}{-IG(j\omega_c)I} \quad (13)$$

$$b\omega_c = \frac{\sin(\omega_c L)}{IG(j\omega_c)I} \quad (14)$$

$$c - a\omega_b^2 = \frac{\sin(\omega_b L)}{IG(j\omega_b)I} \quad (15)$$

$$b\omega_b = \frac{\sin(\omega_b L)}{IG(j\omega_b)I} \quad (16)$$

After solving these above equations the values of a, b and c are calculated and given below.

$$a = \frac{1}{\omega_c^2-\omega_b^2}\left[\frac{\sin(\omega_b t_0)}{IG(j\omega_b)I} + \frac{\cos(\omega_c t_0)}{IG(j\omega_c)I}\right] \quad (17)$$

$$b = \frac{\sin(\omega_c t_0)}{\omega_c IG(j\omega_b)I} \quad (18)$$

$$c = \frac{1}{\omega_c^2-\omega_b^2}\left[\omega_c^2\frac{\sin(\omega_b t_0)}{IG(j\omega_b)I} + \omega_b^2\frac{\cos(\omega_c t_0)}{IG(j\omega_c)I}\right] \quad (19)$$

Here an assumption is made to get t_0 is given below.

$$\frac{\sin(\omega_c t_0)}{\cos(\omega_b t_0)} - \frac{\omega_c}{\omega_b}\frac{IG(j\omega_c)I}{IG(j\omega_b)I} = \Omega \quad (20)$$

For tuning of the controller, the range at which system is stable is found out by using Routh-Hurwitz criterion, making 1+G(s)H(s)=0 and $k<\frac{b}{\tau_0}$. The range of k gives the stability. The speed of response of a process is inversely proportional to its equivalent time constant. Equivalent time constant has been calculated and given below.

$$\frac{1}{\tau} = \frac{c}{\sqrt{b^2-4ac}},\ b^2 - 4ac < 0 \quad (21)$$

$$\frac{1}{\tau} = \frac{b}{2a},\ b^2 - 4ac > 0 \quad (22)$$

Rewriting the form of PID controller design equation as follows

$$K(s) = k.\frac{\alpha s^2+\beta s+\gamma}{s} \quad (23)$$

where $\alpha = \frac{K_D}{K}, \beta = \frac{K_P}{K}, \gamma = \frac{K_I}{K}$.

Now to cancel out the proposed models pole, a controller is chosen whose α=a,β=b,γ=c. So the open loop transfer function can be represented as given below.

ISSN 2455-6378

$$G(s)H(s)=K\frac{e^{-st_0}}{s} \quad (24)$$

4. Tuning of PID Controller Using Bode Plot Technique

PID controllers are popularly used in process control industries. Reasons for its wide implementation are simple structure and simple formulation. Tuning of SOPDT (Second order plus dead time model) model is widely used for many applications unless they are not sufficient to fulfill specifications. For finding the value of k, Routh-Hurwitz criterion is used. The value of k is multiplied to the numerator part of the process and then different frequencies are found out. The values of a, b, c and L can be calculated and they together form the SOPDT model. The reduction of higher order model in second order is done by the Bode plot method. Bode plot method has been used in the sense that the values of ω_{gc} and ω_{pc} can be obtained. Let the higher order model is written as given below.

$$G_P(s) = \frac{K}{s^n + a_1 s^{n-1} + \cdots + a_n} e^{-t_d s} \quad (25)$$

In the above equation (12) there are four unknowns which are a, b, c and t_0. The frequency domain of the equation (26) can be written as follows.

$$G_P(j\omega)=\frac{Ke^{-t_d s}}{(j\omega)^n + a_1 (j\omega)^{n-1} + \cdots + a_n} \quad (26)$$

The magnitude part and phase part for second order cardiac muscle model can be expressed as follows.

$$A^2 + B^2 = \frac{1}{\mathrm{I}(j\omega)\mathrm{I}^2} \quad (27)$$

$$\angle G(j\omega)=-\arctan\frac{B}{A} - t_0\omega \quad (28)$$

where $A = c - a\omega^2$ and $B = b\omega$.

Now the equation (28) can be rearranged as follows.

$$(\omega^4 \ \omega^2 \ \omega^0)\begin{pmatrix} x^2 \\ y-yz2 \\ z^2 \end{pmatrix} = \frac{1}{\mathrm{I}(j\omega)\mathrm{I}^2} \quad (29)$$

Here x,y,z are unknown parameters. To find out these, bandwidth frequency, ω_{gc} and ω_{pc} are to be calculated. Basically these frequencies will be used to find the unknowns. Now if the magnitude part of the original system and phase part of the original system can be calculated, then it is expected that the original model is same as the proposed model for any frequency. Let us consider,

$$\Gamma=[x^2 \ \ y^2 \ \ -2xz \ \ z^2]^T \quad (30)$$

The three frequencies have been represented in a matrix Ω given below.

$$\begin{matrix} \omega_b^4 & \omega_b^2 & \omega_b^0 \\ \omega_{gc}^4 & \omega_{gc}^2 & \omega_{gc}^0 \\ \omega_{pc}^4 & \omega_{pc}^2 & \omega_{pc}^0 \end{matrix} = \Omega \quad (31)$$

The gain matrix M can be represented as follows.

$$\begin{matrix} \frac{1}{\mathrm{I}G(j\omega_b)\mathrm{I}^2} \\ \frac{1}{\mathrm{I}G(j\omega_{gc})\mathrm{I}^2} \\ \frac{1}{\mathrm{I}G(j\omega_{pc})\mathrm{I}^2} \end{matrix} = M \quad (32)$$

So using above equations it can be concluded that, M= $\Omega\Gamma$. Let $\Omega = \begin{matrix}\alpha \\ \beta \\ \gamma\end{matrix}$, then the solution of the variable x,y,z can be written as follows.

$$\begin{cases} x = \alpha \\ y = \sqrt{\beta + 2xz} \\ z = \sqrt{\gamma} \end{cases} \quad (33)$$

So the modified transfer function is given below.

$$\frac{e^{-t_0 s}}{As^2+Bs+C} = G(s) \quad (34)$$

where A=K.x, B=K.y ,C=K.z.

The modified PID controller with respect to Bode plot is as follows.

$$\begin{pmatrix} K_P \\ K_I \\ K_D \end{pmatrix} = K.\begin{pmatrix} B \\ C \\ 2A \end{pmatrix} \quad (35)$$

5. Simulation Result

Here by varying the values of M and ξ and considering two cells are connected in series, so many transfer functions has been obtained. For our simulation purpose two of them (Forth order with a delay) are taken and given below in table 3a & table 3b. The transfer Functions of the cardiac muscle mass have been defined in table 3a and their respective parameters have been defined in table 3b.

Model No	Transfer Function	PID Controller
1	$G(s) = \frac{1}{s^4 + 10s^3 + 37s^2 + 60s + 36} e$	$32 + \frac{24}{s} + 13s$
2	$G(s) = \frac{1}{s^3 + 6s^2 + 9s + 4} e^{-.3s}$	$.57 + \frac{1.1}{s} + .6s$

Table 3(a): Transfer function of cardiac muscle

Model No	Delay Time(sec)	Peak Time(sec)	Settling Time(sec)	Overshoot (percent)
1	3.2	7.6	11	8
2	4	10.8	12.8	7

Table 3(b): Parameters of cardiac muscle

The Bode plot diagram of these two transfer function are depicted in figure 1 and figure 2.

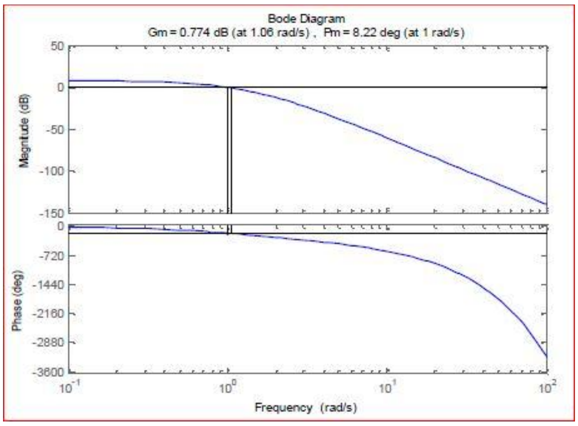

Figure 1: Bode plot diagram of model 1.

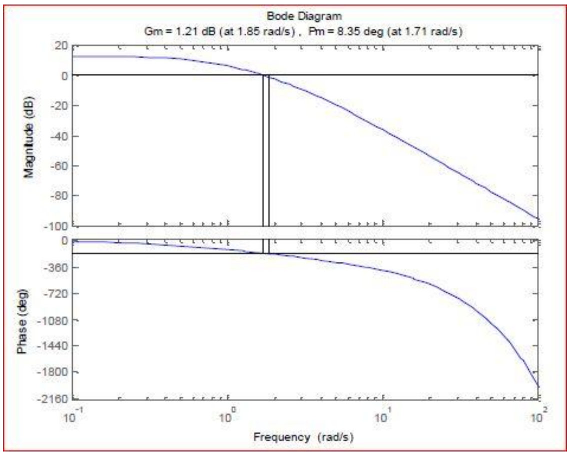

Figure 2: Bode plot diagram of model 2

Finally the step responses of proposed tuning process using Bode plot and previously used techniques are given in figure 3.

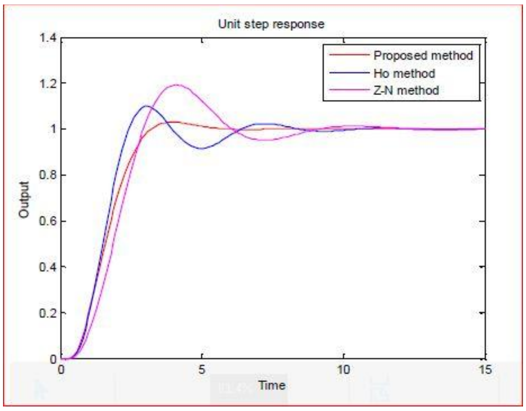

Figure 3: Step responses of proposed tuning process using Bode plot.

6. Conclusions

Because of popularity in the process industries, lots of methods have been developed to find out the parameters of the PID controller such as a Z-N method, IAE, ITAE and IMC method. All the methods are surveyed and modified by Ho and Ho's method has been improved by Wang. In spite of enormous work done, but so many methods are poorly tuned. There are so many methods which are tuned well for a particular application, but fail for other applications. So, it is required to propose a model which is universally accepted and perform a task with high speed which is inversely proportional to the time constraint. In this paper, tuning of PID controller is divided into two parts. Firstly, the higher order system is reduced in a second order system and secondly, PID controller is tuned. In the higher order reduction method, three frequencies namely, gain crossover frequency; phase crossover frequency and bandwidth frequency are required. For a stable system, it is required to get a value that gives finite ω_{gc} and ω_{pc}. Here in this paper, the tuning parameters have been calculated using Bode plot technique for human cardiac muscle and finally the simulation result is compared with the older tuning process.

References

[1] C. C. Hang and K. K. Sin, ―A comparative performance study of PID auto-tuners,‖ IEEE Control Syst. Mag., vol. 11, no. 5, pp. 41–47, Aug. 1991.

[2] Chyi Hawang, Syh-Haw Hwang and Jeng-Fan Leu, ―Tuning PID controllers for time-delay processes with maximizing the degree of stability,‖ 5th Asian control confer-ence, 2005.

[3] Dong Hwa Kim, ―Tuning of PID controller using gain/phase margin and immune al-gorithm,‖ IEEE, Helsinki University of Technology, Espoo, Finland, June 28-30, 2005.

ISSN 2455-6378

[4] Damir Vrancic, ―Simplified disturbance rejection tuning method for PID controllers,4th Asianconference, 2004.

[5] E.H. Maslen, G.B. Bearnson, P.E. Allaire, R.D. Flack, M. Baloh, E. Hilton, M.D. Noh, D.B.Olsen,P.S. Khanwilkar,J.D. Long, "Feedback Control Applications in Artificial Hearts", IEEE Control Systems Magazine, Vol.18, No.6, pp.26-34, December 1998.

[6] Firoozabadi, "Simulating of Human Cardiovascular System and Blood Vessel Obstruction Using Lumped Method"Proceedings of World Academy of Science, Engineering and Technology,31, July 2008, pp 367-375.

[7] J.G.Ziegler and N.B.Nichols, "Optimum settings for automatic controllers," Trans.ASME, voI.64, pp. 759-768, 1942.

[8] K. J. Astrom and C. C. Hang, ―Toward intelligent PID control,‖Automatica, vol. 28, no. 1, pp.1–9, 1999.

[9] K. J. Astrom and T. Hagglund, ―Automatic tuning of simple regulator with specifica-tions onphase and amplitude margins,‖ Automatica, vol. 20, no. 5, pp. 645-651.

[10] L I Mingda, WANG Jing, LI Donghai, ―Performance robustness comparison of two PID tuningmethods‖, pro. of 29th chinese control conference, pp. 3601-3605, 2012

[11] M. Zhuang and D. P. Atherton, ―Automatic tuning of optimum PID controllers,‖ Proc. Inst. Elect. Eng., vol. 140, no. 3, pp. 216–224, May 1993.

[12] Panagopoulos H, Åström K J, Hägglund T. Design of PID controllers based on con-strained optimization [J]. IEE Proc-Control Theory and Apply, pp. 32-40. 2003.

[13] Tornambe A. A decentralized controller for the robust stabilization of a class of MIMO dynamical systems [J]. Journal of Dynamic Systems, Measurement, and Con-trol, pp. 293-304.

[14] W. K. Ho, C. C. Hang, and L. S. Cao, ―Tuning of PID controllers based on gain and phasemargin specifications,‖ Automatica, vol. 31, no. 3, pp. 497–502, 1995.

[15] W. K. Ho, C. C. Hang, W. Wojsznis, and Q. H. Tao, ―Frequency domain approach to self-tuning PID control,‖ Contr. Eng. Practice, vol. 4, no. 6, pp. 807–813, 1996.

[16] W. K. Ho, O. P. Gan, E. B. Tay, and E. L. Ang, ―Performance and gain and phase margins of well-known PID tuning formulas,‖ IEEE Trans. Contr. Syst. Technol., vol. 4, no.2, pp. 473–477.

[17] Yohei Okadai, Yuji Yamakawa, Takanori Yamazaki and Shigeru Kurosu, ―Tuning Method ofPID Controller for Desired Damping Coefficient,‖ SICE annual confer-ence, Kagava University,