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Prime Labeling of Split Graph of Coconut Tree CT(m,n)

LAVANYA. S[1] and GANESAN.V[2]

[1]PG and Research Department of Mathematics,
Thiru Kolanjiappar Government Arts College (Grade I), Vridhachalam, Tamil Nadu, India

[2] Assistant Professor, PG and Research Department of Mathematics,
Thiru Kolanjiappar Government Arts College (Grade I), Vridhachalam, Tamil Nadu, India

Abstract

In this paper, we discuss about the prime labeling of Split graph of Coconut tree CT(m,n). Here m is the number of vertices in the path and n is the number of pendent vertices in the Coconut tree. A graph G with vertex set V is said to have a prime labeling if its vertices are labeled with integer 1,2,….,|V| such that for every edge *xy* the labels assigned to *x* and *y* are relatively prime. A graph which admits prime labeling is called a prime graph.

Key Words: *Graph labeling, Prime labeling, Coconut tree, Split graph of a graph G.*

1. Introduction

In labeling of graphs, we consider only simple, finite, undirected and non-trivial graph G = (V(G), E(G)) with the vertex set V(G) and edge set E(G). According to Beineke L.W. and Hegde S.M. [1], graph labeling is described as a frontier between number theory and structure of graph. Labeling in graph was initiated by Rosa.A [9] in 1967 and an enormous body of literature has grown around the subject, especially in the last three decades, which is regularly updated by J.A.Gallian [4] and it is published by The Electronic Journal of Combinatorics. For notation and terminology, we refer to Bondy J.A and Murthy U.S.R [2]. Many researchers have studied prime graphs. For example, Fu Hung-Lin and Huany Kuo-Ching [3] have proved that the Path P_n on n vertices is a prime graphs. Dr.Meena. S and Vaithilingam. K [8] have proved the prime labeling for some fan related graph. Dr.V.Ganesan *et.al* [6] proved that the Split graph of the path P_n admits prime labeling. Dr.V.Ganesan, *et.al* [5] has proved prime labeling of triangular book and cycle - cactus graphs. Lavanya. S and Dr.Ganesan. V [7] have proved prime labeling of Split graph of Cycle C_n.

2. Definitions

Definition 2.1

If the vertices are assigned values subject to certain condition(s) then it is known as graph labeling.

Definition 2.2

Let G = (V, E) be a graph with n vertices. A bijection $f : V(G) \rightarrow \{1, 2,\ldots, n\}$ is called a prime labeling if for each edge e = *uv*, gcd(*f*(*u*),*f*(*v*)) = 1. A graph which admits prime labeling is called prime graph.

Definition 2.3

For a graph G, the Split graph, Spl(G) of a graph G is obtained by adding a new vertex *v*' corresponding to each vertex *v* of G such that N(*v*) = N(*v*').

Definition 2.4

A Coconut tree CT(m,n) is the graph,for all positive integer n and $m \geq 2$ is obtained from the path P_m by appending 'n' new pendant edges at an end vertex of P_m.

3. Preposition

The Coconut tree CT(m,n) is a prime graph for all positive integer n and $m \geq 2$.

Proof

Let G = CT(m,n) be the coconut tree.

Let $u_1,u_2,\ldots,u_m;v_1,v_2,\ldots,v_n$ be the vertices of Coconut tree V(G) = { $u_1,u_2,\ldots,u_m;v_1,v_2,\ldots,v_n$}. Let $u_1,u_2,\ldots,u_m$ be the vertices of the path and $v_1,v_2,\ldots v_n$ be the pendant vertices attached with the end vertex of the Path P_m.

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Since |V(G)| = m + n, define a mapping $f : V \rightarrow \{1, 2, \ldots..,m + n\}$ as follows

$f(u_m) = 1$

$f(u_i) = i + 1$ for $1 \leq i \leq m-1$

$f(v_j) = m + j$ for $1 \leq i \leq n$

In view of above labeling pattern, G admits prime labeling.

Hence, the coconut tree CT(m,n) is a prime graph for all positive integer n and $m \geq 2$.

3.1 Illustration

Coconut tree CT(5,5) is a prime graph.

Now, Consider following CT(5,5)

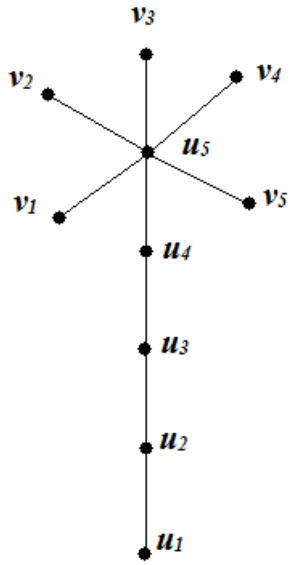

Fig. 1 Coconut tree CT(5,5)

Labeling the vertices of Coconut tree CT(5,5), using the mapping

$f(u_m) = 1$

$f(u_i) = i + 1$ for $1 \leq i \leq m-1$

$f(v_j) = m + j$ for $1 \leq i \leq n$

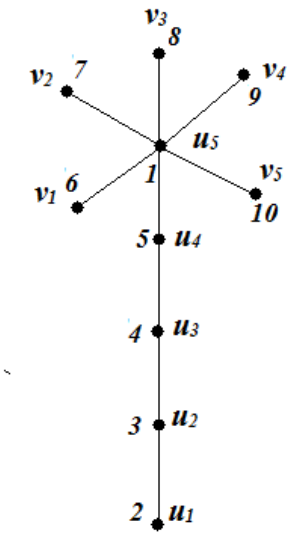

Fig. 2 Labeling the vertices of CT(5,5)

4. Preposition

Algorithm: Prime labeling of Split graph of Coconut tree CT(m,n).

Step: 1

Let CT(m,n) be the given Coconut tree with vertex set $V(CT(m,n)) = \{u_1,u_2,\ldots,u_m;v_1,v_2,\ldots,v_n\}$ where $u_1,u_2,\ldots,u_m$ be the vertices of Path P_m and $v_1,v_2,\ldots,v_n$ be the pendant vertices attached with the end vertex of the Path P_m.

Step: 2

Let $u'_1,u'_2,\ldots,u'_m$ and $v'_1,v'_2,\ldots.v'_n$ be the new vertices associated with each u's and v's respectively.

Let G = Spl (CT(m,n)) be the Split graph of the coconut tree CT(m,n).

Clearly |V(CT(m,n))| = m + n , |V(G)| = 2(m + n).

Step: 3

Since |V(G)| = 2(m + n),define a labeling $f : V(G) \rightarrow \{1, 2, \ldots..,2(m + n)\}$

$f(u_i) = 2i + 1$ for $1 \leq i \leq m-1$ and $f(u_m) = 1$.

$f(v_j) = 2m - 1 + 2j$ for $1 \leq j \leq n$

$f(u'_i) = 2i + 2$ for $1 \leq i \leq m-1$ and $f(u'_m) = 2$

$f(v'_j) = 2m + 2j$ for $1 \leq j \leq n$.

Step: 4

Let G_1 be the labeled graph as defined above. If all the adjacent vertices in G_1 are relatively prime then G_1 is a prime labeling and

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hence G is a prime graph. If not, we do the following Step 5.

Step: 5

Here, we need to check only the relative prime of the edges $u_iu'_{i+1}$ for i = 1,2,3,…,m-1.

If gcd($f(u_i)$, $f(u'_{i+1})$) = 1 for all i = 1,2,3,…,m-1 then G admits prime labeling .

If gcd($f(u_i)$, $f(u'_{i+1})$) ≠ 1 for some u_i and u'_{i+1} then encircle the pair(s) within the circle.

Step: 6

Interchange the lable of u'_i and u'_{i+1}.Where u'_{i+1} encircled vertex and u'_i is not. Repeat this step until all such pair exhausted. Let G_2 be the new labeled graph. Clearly G_2 admits prime labeling. Hence, G admits prime labeling and thus G is a prime graph.

4.1 Illustration

Split graph of CT(5, 5) is a prime graph Now, Consider following the Split graph of Coconut tree CT(5,5) is given below

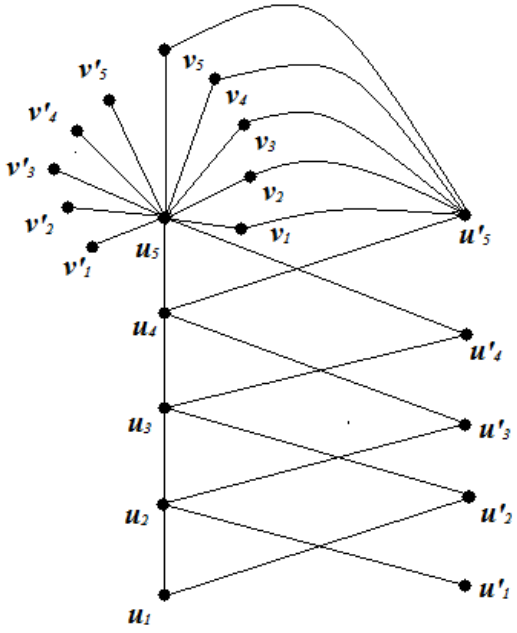

Fig. 3 Spilt graph of CT(5,5)

Labeling the vertices of Spl(CT(5,5)) by using the mapping,

$f(u_i) = 2i + 1$ for $1 \le i \le m-1$ and $f(u_m) = 1$.

$f(v_j) = 2m - 1 + 2j$ for $1 \le j \le n$

$f(u'_i) = 2i + 2$ for $1 \le i \le m-1$ and $f(u'_m) = 2$

$f(v'_j) = 2m + 2j$ for $1 \le j \le n$.

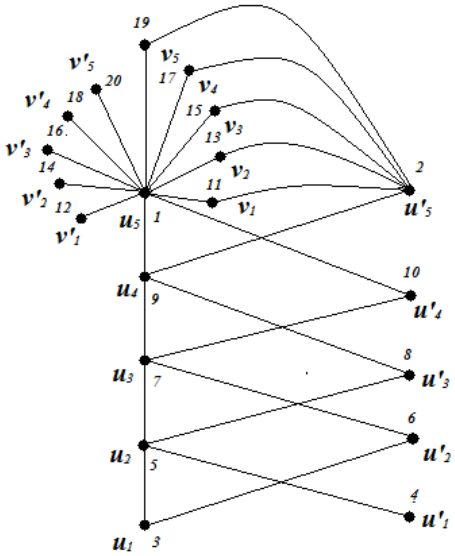

Fig. 4 Labeling the vertices of Spl(CT(5,5))

Check the relative prime of each pair of vertices u_i and u'_{i+1} and mark the vertex u_i and u'_{i+1} within the circles which are not relatively prime. We get following graph,

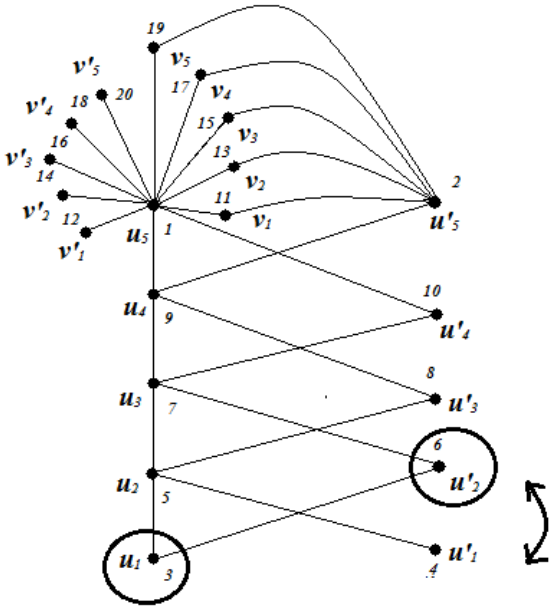

Fig. 5 Checking and Encircling the non-prime pairs.

Interchanging the labels of u'_{i+1} (which is encircled) and u'_i (which is not encircled). The procedure is continued for all such pair(s).

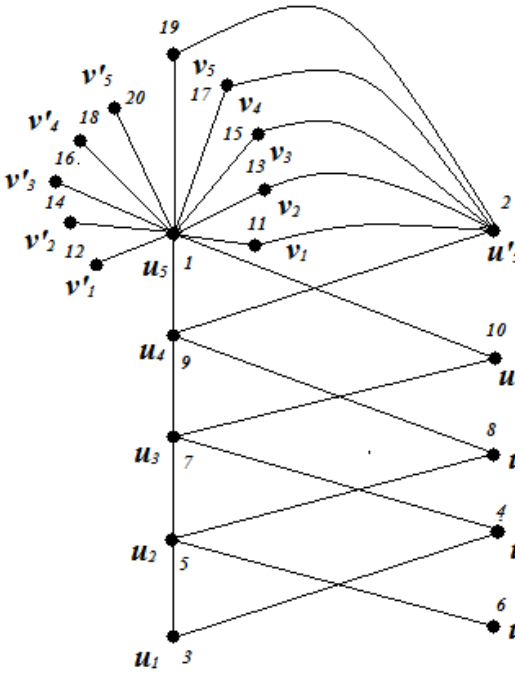

Fig. 6 Prime labeling of Spl[CT(5,5)]

5. Conclusion

We have presented an algorithm for prime labeling on some classes of graph such as Splitting graph of Coconut tree CT(m,n) and illustrate with an example. Obviously, the present work will motivate the researcher to investigate the prime labeling of Split graph of other families like Star, Bistar, Wheel and Sun let Graph etc.

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