

# Pulsatile flow of Elastico-viscous fluid with nanoparticles in a porous diseased artery subjected to external magnetic and body forces

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## Abstract

A mathematical model is proposed by assuming blood as an elastico-viscous fluid containing magnetic nanoparticles flowing through a rigid mild stenosed arterial tube. Momentum equations are framed by considering an external magnetic transverse force and periodic body acceleration on a porous tube containing fluid. The resulting flow governing equations are solved using Laplace and Hankel transforms method simultaneously. A closed form solution is derived for the flow characteristics such as flow velocity of the fluid and particles, flow resistance, effective viscosity, shear stress at the wall, and the acceleration of fluid and particles. The effects of various pertinent parameters on the flow characteristics are analysed numerically and discussed graphically. The present study asserts that the flow characteristics of blood significantly alter even with a small increase in the magnitude of elastic parameter and the Darcy number.

**Keywords:** Elastico-viscous fluid, porous medium, magnetic field, magnetic nanoparticle

## 1. Introduction

One of the fundamental causes of death across the globe is accredited to atherosclerosis which remains as the root cause for the cardiovascular diseases. The development of stenosis severely affects the arterial fluid flow rate, perfusion and thereby causes sequential malfunction of vascular network system

which disrupts the blood rheology in that region. Considering this, several modest scientific efforts have been made by various investigators ([1]-[4]) to analyze the flow physiognomies of Newtonian blood in occluded vessels. Blood constitutes formed elements like red blood cells (erythrocytes), white blood cells (leukocytes) and platelets (Thrombocytes). Therefore, typically blood is regarded as a non-Newtonian fluid model and many researchers [5-8] have investigated the flow of non-Newtonian blood through stenosed arteries assuming different shapes of stenosis. Further, the pulsatility of blood under different conditions have been discussed by numerous authors [9-12]. Silva et al. [13] have discussed the flow characteristics with respect to porosity, Darcy number, flow behavior index and shear stress jump coefficient by including the porous medium in the flow field of the Power-law model. The results proved that, for a mass flow rate the pressure drop, is a function of porosity.

Several studies analyzed and proved the significance of applied magnetic force in the blood flow. [14-16]. Assuming blood as an elastico-viscous fluid that passes through a circular tube with a periodic body acceleration in the presence of transverse applied magnetic field, the MHD flow of blood was explored by [17]. Likewise, the effect of the magnetic field and periodic acceleration on the flow of Couple stress fluid blood in a tapered stenosed artery is discussed by [18]. Sharma et al. [19] studied the effect of magnetic force on blood flow parameters, not only highlighted the presence of magnetic

nanoparticles in blood and the interaction force between fluid and particles but also stated that the velocity of fluid and particles could be reduced palpably under strong magnetic field. This phenomenon holds relevance in the healthcare processes like the reduction of bleeding during surgery and modern drug targeting delivery method. Mirza et al. [20] have presented a mathematical model for the pulsatile flow of Bingham fluid with magnetic particles that flows through a stenosed artery with oscillating pressure gradient, periodic body acceleration, applied magnetic field and slip velocity at the wall. He stated that the speed of blood and magnetic particles could be regulated by altering the magnetic parameter. Furthermore, the effect of particle concentration on the flow characteristics is examined by [21]-[22], assuming the flowing fluid as particulate suspension of a non-Newtonian model in a stenosed arterial tube. In the course of the literature survey, it is found that no one has considered the concept of elasto-viscous fluid blood with magnetic particles flows through a stenosed artery. In essence, the literature survey discloses the fact that little consideration has been given to the concept of elasto-viscous fluid blood with magnetic particles that flows through a stenosed artery.

The objective of the present study is to investigate the flow of elasto-viscous fluid blood embedded with nanoparticles in a stenosed arterial circular rigid tube, where the tube is subjected to an external magnetic field, oscillating pressure gradient, and periodic body acceleration. It is assumed that the magnetic particles are spread evenly throughout the blood and fluid flows in the axial direction of the tube. Corresponding flow governing equations are obtained and converted into a dimensionless form of non-linear partial differential equations by appropriating non-dimensional coordinates. A closed-form solution is derived for the velocity of the fluid in the axial direction, speed of particles, shear stress at the wall, fluid flow rate and flow resistance. The influence of various parameters on velocities, wall shear stress and flow resistance are analyzed numerically and their physiological significance is discussed through graphs. It is observed that the elastic parameter reduces the flow resistance for smaller magnitude and increases for greater magnitude.

**2. Mathematical model**

Let us consider the pulsatile flow of blood as an incompressible, fully developed, axially symmetric and electrically conducting fluid suspended with magnetic particles through a mild stenosed artery.

Here, we assume that the arterial wall has asymmetric and mild stenosis as shown in Fig.1 [21].

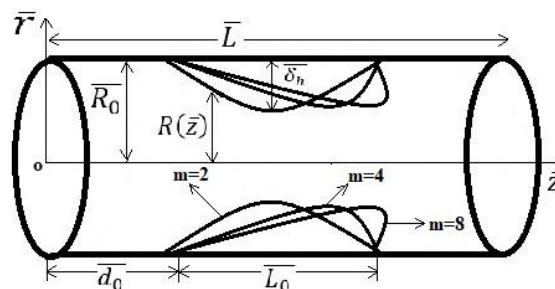


Figure1. Stenosis geometry

Blood is considered as an elasto-viscous fluid model mixed with uniformly distributed magnetic nanoparticles such that  $\bar{N}$  is the number of particles per unit volume. These particles move freely with velocity  $\bar{u}_p$  in the axial direction along with the fluid and we assume that at time  $\bar{t} \leq 0$ , fluid and particles are assumed to be motionless. An external magnetic field perpendicular to the axial direction is applied on the circular tube and that, being an electrically conducting fluid, blood engenders electromotive force which influences the magnetic particles. Hence, by using Newton's second law of motion we derive the governing equation of particles motion as

$$\bar{m} \frac{\partial \bar{u}_p(\bar{r}, \bar{t})}{\partial \bar{t}} = \bar{K}_s \bar{N} (\bar{u}_f(\bar{r}, \bar{t}) - \bar{u}_p(\bar{r}, \bar{t})) \quad (1)$$

Where the mean velocity of particle is  $\bar{m}$ ,  $\bar{u}_f$  is the fluid velocity and  $\bar{K}_s$  is the Stoke's constant.

Likewise, the flow governing equations of motion of fluid using Navier-Stokes equations can be expressed as ([16],[19],and [20]).

$$\bar{\rho} \frac{\partial \bar{u}_f(\bar{r}, \bar{t})}{\partial \bar{t}} = - \frac{\partial \bar{P}}{\partial \bar{z}} + \mathbf{G}(\bar{t}) + (\bar{\mu} + \bar{\mu}_1 \frac{\partial}{\partial \bar{t}}) \left[ \frac{\partial^2 \bar{u}_f(\bar{r}, \bar{t})}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}_f(\bar{r}, \bar{t})}{\partial \bar{r}} \right] - \bar{\sigma} \bar{B}_0^2 \bar{u}_f(\bar{r}, \bar{t}) - \bar{K}_p \bar{u}_f(\bar{r}, \bar{t}) + \bar{K}_s \bar{N} (\bar{u}_p(\bar{r}, \bar{t}) - \bar{u}_f(\bar{r}, \bar{t})) \quad (2)$$

The oscillating pressure gradient along the axial direction and the periodic body acceleration are

$$-\frac{\partial \bar{P}}{\partial \bar{z}} = \bar{A}_0 + \bar{A}_1 \cos(\bar{\omega} \bar{t}); \mathbf{G}(\bar{t}) = \bar{b}_0 \cos(\bar{\omega}_b \bar{t} + \varphi)$$

In the pressure gradient term,  $\bar{A}_0$  is the steady state part,  $\bar{A}_1$  is the amplitude of oscillatory part and  $\bar{\omega} = 2\pi \bar{f}$  where  $\bar{f}$  is the heart pulse frequency. In  $\mathbf{G}(\bar{t})$ ,  $\bar{b}_0$  is the amplitude of body force,  $\bar{\omega}_b = 2\pi \bar{f}_b$  where  $\bar{f}_b$  is the frequency of body acceleration, and  $\varphi$  is the phase angle. Let us consider the Reynolds number is small enough to negate the induced magnetic and electrical field effect on the flow.

Therefore, including the body acceleration, electromagnetic force, magnetic field and pressure gradient in the fluid momentum equation, we attain the flow governing equations of fluid as

$$\begin{aligned} \frac{\partial \bar{u}_f(\bar{r}, \bar{t})}{\partial \bar{t}} &= \bar{A}_0 + \bar{A}_1 \cos(\bar{\omega} \bar{t}) \\ &\quad + \bar{b}_0 \cos(\bar{\omega}_b \bar{t} + \varphi) \\ &\quad + \left( \bar{\mu} + \bar{\mu}_1 \frac{\partial}{\partial \bar{t}} \right) \left[ \frac{\partial^2 \bar{u}_f(\bar{r}, \bar{t})}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}_f(\bar{r}, \bar{t})}{\partial \bar{r}} \right] - \\ \bar{\sigma} \bar{B}_0^2 \bar{u}_f(\bar{r}, \bar{t}) - \bar{K}_p \bar{u}_f(\bar{r}, \bar{t}) + \bar{K}_s \bar{N} (\bar{u}_p(\bar{r}, \bar{t}) - \bar{u}_f(\bar{r}, \bar{t})) \end{aligned} \quad (2)$$

The initial and boundary conditions related to the fluid and particle motion are

$$\begin{aligned} \bar{u}_f(\bar{r}, 0) &= 0; \bar{u}_p(\bar{r}, 0) = 0 \\ \bar{u}_f(\bar{R}_z, \bar{t}) &= 0 \text{ for all } \bar{t} \geq 0; \frac{\partial \bar{u}_f(0, \bar{t})}{\partial \bar{r}} = 0 \end{aligned} \quad (3)$$

Introducing the transformation coordinates together with radial transformation  $y = \frac{r}{R}$  and  $\bar{u}_0$  as the average velocity of fluid we obtain the non-dimensional momentum equations and boundary conditions as below

Transformation coordinates:

$$\begin{aligned} \bar{u}_f^* &= \frac{\bar{u}_f}{\bar{u}_0}, \bar{u}_p^* = \frac{\bar{u}_p}{\bar{u}_0}, \bar{t}^* = \bar{t} \bar{\omega}, \bar{\omega}^* = \frac{\bar{\omega}_b}{\bar{\omega}}, \bar{R}^* = \frac{\bar{R}}{\bar{R}_0}, \\ \bar{r}^* &= \frac{\bar{r}}{\bar{R}_0}, \bar{z}^* = \frac{\bar{z}}{\bar{R}_0}, \bar{L}_0^* = \frac{\bar{L}_0}{\bar{R}_0}, \bar{A}_0^* = \frac{\bar{A}_0 \bar{R}_0^2}{\bar{\mu} \bar{u}_0}, \bar{A}_1^* = \frac{\bar{A}_1 \bar{R}_0^2}{\bar{\mu} \bar{u}_0}, \\ \bar{b}_0^* &= \frac{\bar{b}_0 \bar{R}_0^2}{\bar{\mu} \bar{u}_0} \end{aligned}$$

Non dimensional momentum equation of fluid:

$$\begin{aligned} \alpha^2 \frac{\partial \bar{u}_f(y, t)}{\partial t} &= A_0 + A_1 \cos t + \\ &\quad b_0 \cos(\omega_b t + \varphi) + \frac{1}{R^2} (1 + \beta_e \frac{\partial}{\partial t}) \\ &\quad \left[ \frac{\partial^2 \bar{u}_f(y, t)}{\partial y^2} + \frac{1}{y} \frac{\partial \bar{u}_f(y, t)}{\partial y} \right] + P_c (\bar{u}_p(y, t) - \\ &\quad \bar{u}_f(y, t) - H_2 \bar{u}_f(y, t) \end{aligned} \quad (4)$$

Non dimensional momentum equation of particle:

$$\alpha^2 G \frac{\partial \bar{u}_p(y, t)}{\partial t} = \bar{u}_f(y, t) - \bar{u}_p(y, t) \quad (5)$$

Where  $\alpha^2 = \frac{\bar{R}_0^2 \bar{\omega} \bar{\rho}}{\bar{\mu}} G = \frac{\bar{\mu} \bar{m}}{\bar{\rho} \bar{K}_s \bar{R}_0^2} M^2 = \frac{\bar{R}_0^2 \bar{B}_0^2 \bar{\sigma}}{\bar{\mu}} \beta_e =$

$$\frac{\bar{\omega} \bar{\mu}_e}{\bar{\mu}} P_c = \frac{\bar{R}_0^2 \bar{N} \bar{K}_s}{\bar{\mu}} D \alpha^2 = \frac{\bar{K}_p}{\bar{R}_0^2}$$

$$H^2 = M^2 + \frac{1}{D \alpha^2}$$

Non dimensional initial and Boundary conditions are:

$$\bar{u}_f(y, 0) = 0; \bar{u}_p(y, 0) = 0$$

$$\bar{u}_f(1, t) = 0 \text{ for all } t \geq 0; \frac{\partial \bar{u}_f(0, t)}{\partial y} = 0 \quad (6)$$

Non dimensional form of stenosis wall geometry :

$$\begin{aligned} R &= R(z) \\ &= \begin{cases} 1 - \eta \{ L_0^{m-1} (z - d_0) - (z - d_0)^m \}, \\ \text{for } d_0 \leq z \leq d_0 + L_0 \\ 1, & \text{for otherwise} \end{cases} \end{aligned}$$

Where  $\eta = \frac{\bar{\delta}_h m^{m/m-1}}{\bar{L}_0^m m-1}$  and  $d_0 = \frac{\bar{d}_0}{\bar{R}_0}$

### 3. Solution Procedure

In order to solve Eq.4 and Eq.5 subject to the initial and boundary conditions Eq.6, we operate the Laplace transform with respect to the time coordinate (t) and finite Hankel transform with respect to the radial coordinate (y) which are defined as:

$$L[\bar{u}(y, t)] = \bar{u}'(y, s) = \int_0^\infty \bar{u}(y, t) e^{-st} dt \quad (7)$$

$$\begin{aligned} H_0[\bar{u}'(y, s)] &= \int_0^1 y \bar{u}'(y, s) J_0(y \lambda_n) dy \\ &= \bar{u}'(\lambda_n, s) \end{aligned} \quad (8)$$

In the above equation  $\lambda_n, n = 1, 2, 3, \dots$  represent the positive roots of Bessel function of first kind and of order zero denoted as  $J_0(y)$

By employing Laplace transform subject to the initial conditions on Eq.4 we obtain

$$\bar{u}'_p(y, s) = \frac{\bar{u}'_f(y, s)}{(1 + \alpha^2 G s)} \quad (9)$$

Simultaneously applying Laplace transform and finite Hankel transform on Eq.5, we get

$$\begin{aligned} \left[ \alpha^2 s + H^2 + \frac{1 + s \beta_e}{R^2} \lambda_n^2 \right. \\ \left. + P_c \left( \frac{\alpha^2 G s}{(1 + \alpha^2 G s)} \right) \right] \bar{u}'_f(\lambda_n, s) \end{aligned}$$

$$= \left[ \left( \frac{A_0}{s} + \frac{s A_1}{s^2 + 1} \right) + b_0 \frac{(s \cos \varphi - \omega \sin \varphi)}{s^2 + \omega^2} \right] \frac{J_1(\lambda_n)}{\lambda_n}$$

$$\begin{aligned} \bar{u}'_f(\lambda_n, s) &= \\ \left[ \left( \frac{A_0}{s} + \frac{s A_1}{s^2 + 1} \right) + b_0 \frac{(s \cos \varphi - \omega \sin \varphi)}{s^2 + \omega^2} \right] \frac{J_1(\lambda_n)}{\lambda_n} \cdot Q(s) \end{aligned} \quad (10)$$

Where

$$Q(s) = \frac{(1 + \alpha^2 G s)}{c_{1n} s^2 + c_{2n} s + c_{3n}}$$

$$c_{1n} = \alpha^2 G \left( \alpha^2 + \frac{\lambda_n^2 \beta_e}{R^2} \right)$$

$$c_{2n} = \alpha^2 \left( 1 + G \left( H^2 + P_c + \frac{\lambda_n^2}{R^2} \right) \right) + \frac{\lambda_n^2 \beta_e}{R^2}$$

$$c_{3n} = H^2 + \frac{\lambda_n^2}{R^2}$$

Now, applying inverse Laplace and Hankel transforms simultaneously on Eq. (10), we obtain the fluid velocity distribution

$$u_f(y, t, z) = 2 \sum_{n=1}^{\infty} \frac{J_0(y\lambda_n)}{J_1^2(\lambda_n)} \{f(t) * g(t)\} \quad (11)$$

Where

$$f(t) = [A_0 + A_1 \cos t + b_0 \cos(\omega_b t + \varphi)] \frac{J_1(\lambda_n)}{\lambda_n}$$

$$g(t) = \frac{1 + \alpha^2 G r_{1n}}{r_{1n} - r_{2n}} e^{r_{1n} t} - \frac{1 + \alpha^2 G r_{2n}}{r_{1n} - r_{2n}} e^{r_{2n} t}$$

$$r_{1n}, r_{2n} = \frac{-c_{2n} \pm \sqrt{c_{2n}^2 - 4c_{1n}c_{3n}}}{2c_{1n}}$$

$$f(t) * g(t) = \int_0^t f(x)g(t-x)dx \quad (12)$$

The particle velocity distribution is derived as

$$u_p(y, t, z) = \frac{1}{\alpha^2 G} e^{\frac{1}{\alpha^2 G} t} * u_f(y, t, z) \quad (13)$$

Using Eq. (11) we derive the following expressions.

The wall shear stress distribution:

$$\tau_w(y, t, z) = \frac{1}{R} \left( 1 + \beta_e \frac{\partial}{\partial t} \right) \frac{\partial u_f}{\partial t} \Big|_{y=1} \quad (14)$$

The volumetric flow rate:

$$Q(t, z) = R^2 \int_0^1 y u_f(y, t, z) dy \quad (15)$$

The flow impedance:

$$\lambda(y, t, z) = \int_0^L \frac{A_0 + A_1 \cos t}{Q(t, z)} dz \quad (16)$$

The acceleration of fluid and article can also be calculated by using the expression

$$\text{Fluid acceleration: } F_1 = \frac{\partial u_f(y, t, z)}{\partial t} \quad (17)$$

$$\text{Particle acceleration: } F_2 = \frac{\partial u_p(y, t, z)}{\partial t} \quad (18)$$

Further, the dimensional effective viscosity  $\bar{\mu}_e$  is defined as:

$$\bar{\mu}_e = \frac{\pi \left( \frac{\partial P}{\partial z} \right) R_z^4}{Q(\bar{t}, \bar{z})} \quad (19)$$

$$\text{Taking } \mu_e = \frac{\bar{\mu}_e}{\bar{\mu}} \text{ and } Q(t, z) = \frac{Q(\bar{t}, \bar{z})}{\frac{\pi R_0^2 u_0}{8}}$$

The non-dimensional effective viscosity as:

$$\mu_e = \frac{(A_0 + A_1 \cos t) R^4}{Q(t, z)} \quad (20)$$

#### 4. Numerical Computation and Discussion

The quantitative effects of various physiological parameters on the flow characteristics are analyzed by calculating numerical values using MATLAB software whose results are depicted in graphs. It is observed that the analytic expression for fluid velocity (Eq.11) obtained agree with the formula given by [19] for  $Pc = 0$ .

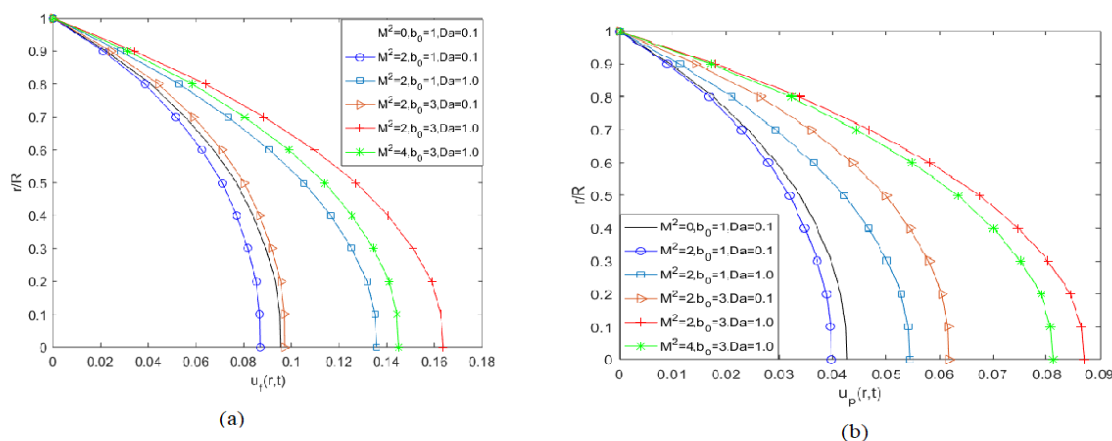


Figure 2. velocity distribution of (a) Blood and (b) Nanoparticle with axial distance for different Hartmann number, Darcy number and body acceleration number

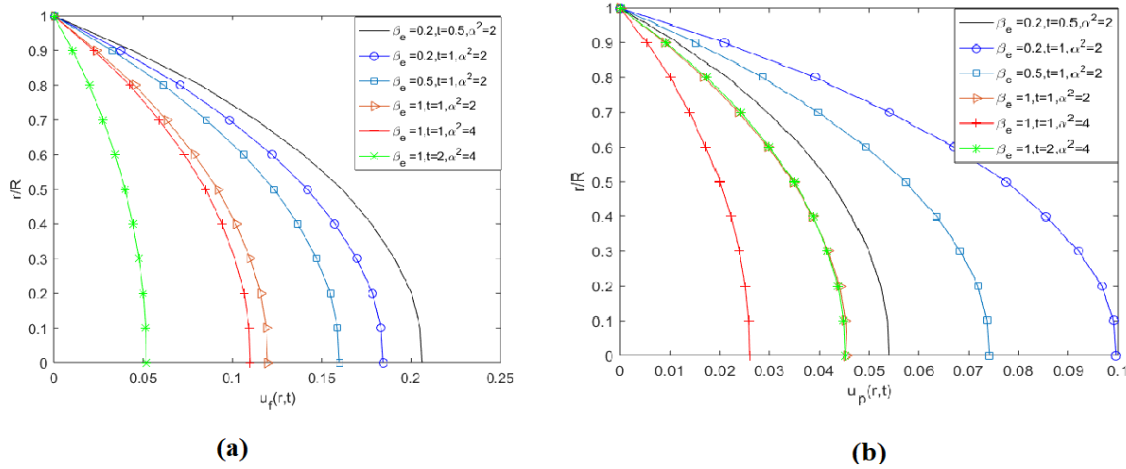


Figure 3. velocity distribution of (a) Blood and (b) Nanoparticle with axial distance for different elastic parameter, time and pulsatile Reynolds number

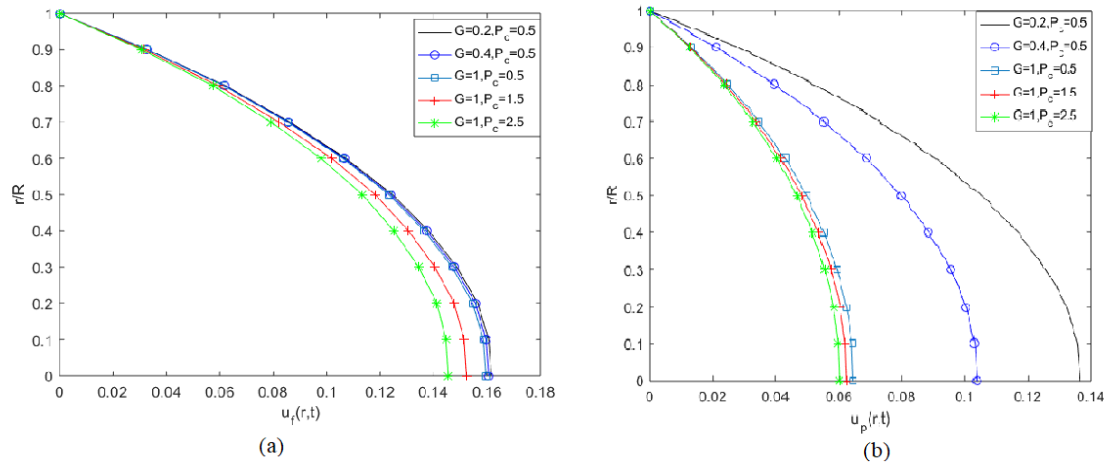


Figure 4. velocity distribution of (a) Blood and (b) Nanoparticle with axial distance for different particle mass and concentration parameter

Figure 2 illustrates the variation of both fluid and particle velocity profile for different values of magnetic number, body acceleration, and Darcy number. While the velocity profile increases with the rise of Porous number (Da) and body acceleration parameter  $b_0$ , the increase of magnetic number  $M^2$  decreases the speed of fluid. The radial distribution of velocities of both fluid and particles are similar in terms of Darcy number, periodic body acceleration number and magnetic number, the effects of these parameters are comparatively more in particles than in fluid velocity, which in turn, validates the presence of drag force. Evidently, velocity is less in a strong magnetic field and attains its maximum when the magnetic field is absent. Figure 3 explains the influence of elastic parameter, time and Reynolds number on the velocity profiles of both fluid and

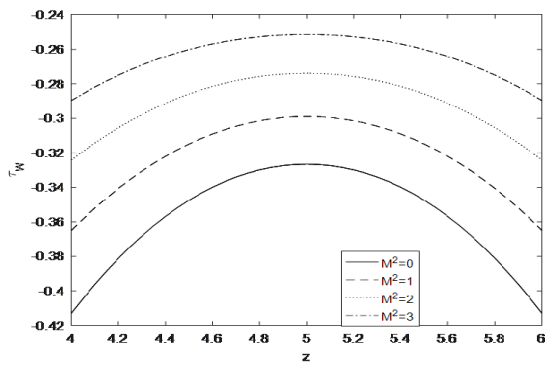
particle, reveals that the stream and particle speed decreases with the increase of above mentioned parameters. However, the space between different curves for different parameters indicates that the variation is more in particle velocity profile compared to the fluid speed field. Figure 4 reveals that the velocity profile of both fluid and particle decreases for the increase of particle mass and concentration number.

The effect of magnetic number versus axial distance on the wall shear stress is shown in fig.5(a) and noted that the amplitude of wall shear stress negatively increases with the increase of Hartmann number. Figure 5(b) exhibits that the wall shear stress increases negatively with the advancement of magnitude of body acceleration number  $b_0$ .

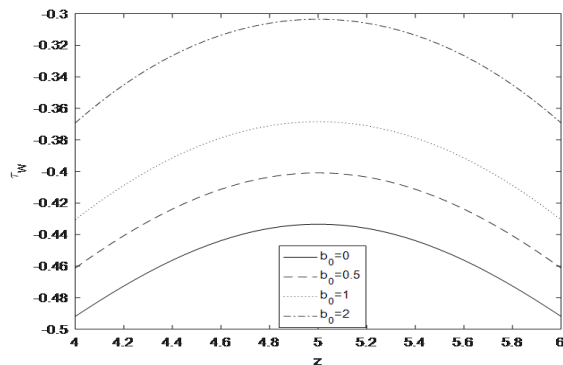
Figure 6 establish the prominence of particles presence in fluid blood by varying particles mass and concentration number respectively. It is significant that the existence of nanoparticles in blood amplifies the shear stress at the wall surface with the fluid in contact. Further, it is observed that the extent of wall shear stress increases along with the increase of number of particles and their mass in fluid.

Figure 7 depicts that the variation percentage for shear stress distribution with respect to elasticity is more for the low elastic parameter and for the high

elastic parameter the distribution curve seems to be plane explaining that the distribution is more and seems to be uniform. Figure 7(b) explicates that the profile of shear stress at wall decreases considerably to form the low porous medium and the increase of Darcy number, the wall shear stress diminishes gradually. Figure 7(c), disclose that the presence of stenosis in arterial wall enhances the shear stress, and affect the fluidic flow. It is found that the wall shear stress profile has the nature of parabolic curve for the symmetric shape ( $m = 2$ ).

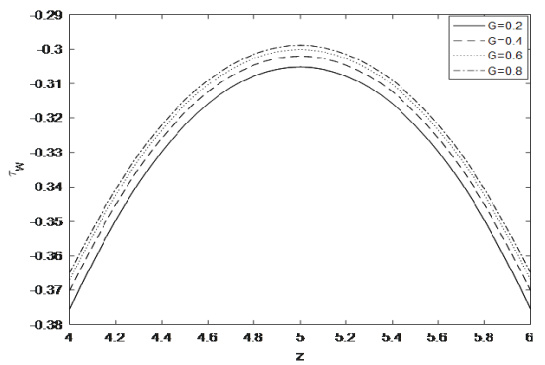


(a) Magnetic number effect

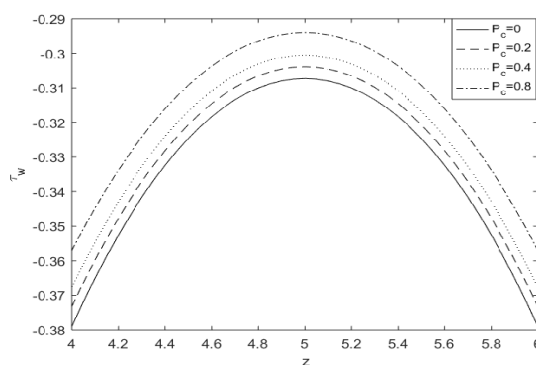


(b) Body acceleration number effect

Figure 5. Wall shear stress profile for different Magnetic number (a) and body acceleration number (b)



(a). Particle mass number effect



(b). Particle concentration number effect

Figure 6. Wall shear stress profile for different particle mass number (a) and particle concentration number (b)

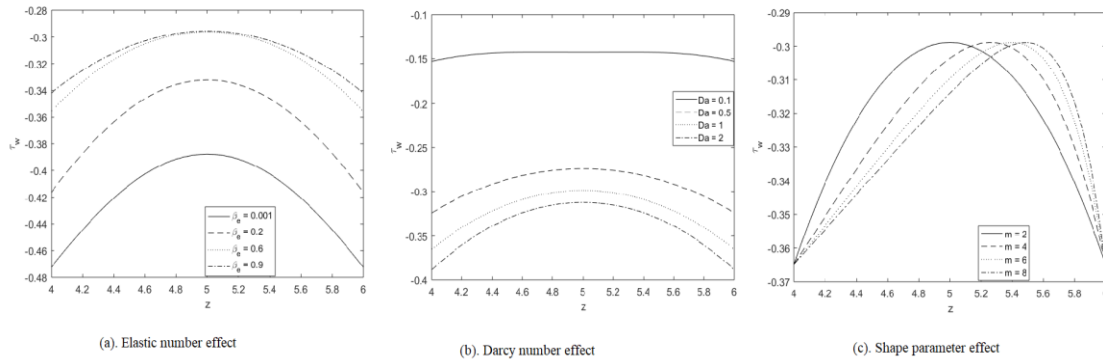


Figure 7. Wall shear stress profile for different Elastic number (a), Darcy number (b) and Shape parameter effect (c)

Figures 8-10 enumerates the variation of flow impedance for different values of appropriate parameters. Figure 8(a) shows that the flow resistance increases with the increase of magnetic intensity. The flow impedance attains its greater magnitude in low magnetic intensity and decreases as the Hartmann number increases because the magnetic particles have a rotational motion in the

presence of a magnetic field and enhance the apparent viscosity. Figure 8(b) indicates that the magnitude of flow impedance increases with the increase in body acceleration number. In fig. 9, it is reported that for different values of particle concentration number and mass parameter the flow resistance increases with the increase in particle concentration number as well mass number.

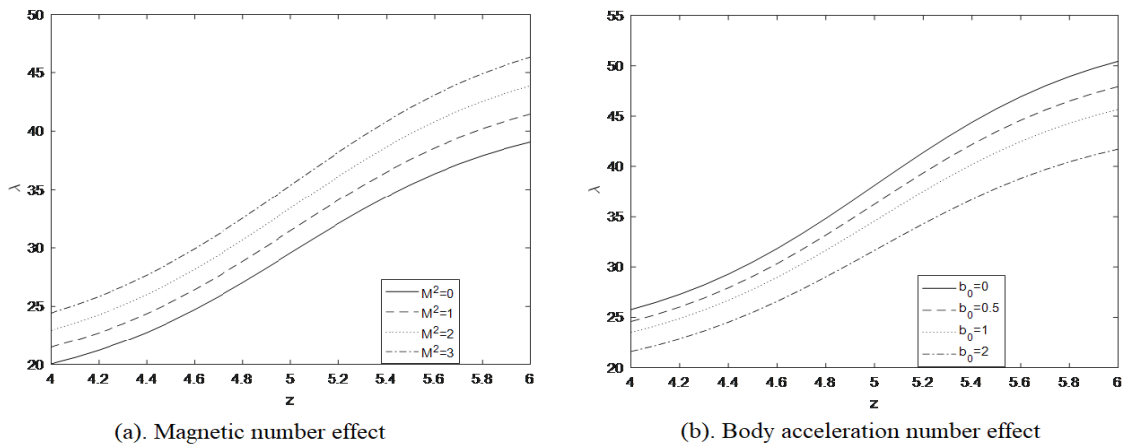


Figure 8. Variation of flow resistance for different Magnetic number (a) and Body acceleration number (b)

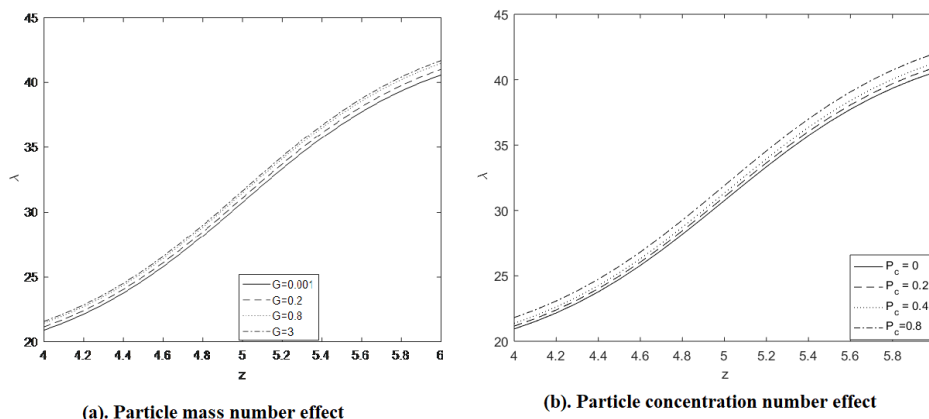


Figure 9. Variation of flow resistance for different particle mass number (a) and particle concentration number (b)

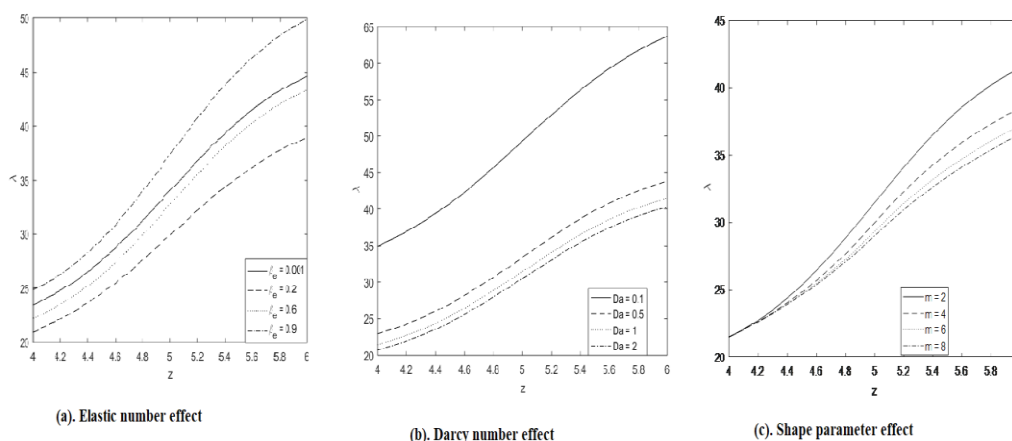


Figure 10. Variation of flow resistance for different elastic number (a), Darcy number (b) and Shape parameter effect (c)

Figure 10(a) demonstrates that the flow resistivity increases with the advancement of the elastic parameter. As the Darcy number gets amplified, the flow resistance reacts contradictorily and allows the increased flow rate is shown in Fig. 10(b). From Fig. 10(c) it is observed that the rate of flow resistance decreases as the shape parameter increases. Further, for the symmetric stenosis shape number ( $m=2$ ), the resistive flow value attains its maximum.

### 5. Conclusion

In this paper pulsatile flow of elastico-viscous fluid suspended with nanoparticles flow through stenosed artery is considered. The flow is under the applied

transverse magnetic field with periodic body acceleration through a porous circular rigid tube. Exact values for the flow variables are obtained through analytic expressions and their distribution profiles with different involved parameters are discussed and narrated graphically. This model concludes the significance of magnetic field strength on the axial velocity of both particle and fluid. The velocity decreases for the stronger magnetic field. Hence, in magneto therapy, the magnetic appliances on flow velocity of blood can be controlled by regulating the magnetic field intensity value.

## Reference

- [1] A.G. May, J.A. Dewese and C.G. Rob, Hemodynamic effects of arterial stenosis, *Surgery* 53: 513-524, (1963).
- [2] D.F. Young, Effect of a time-dependent stenosis on flow through a tube, *Journal. Eng. Ind.*, 90: 258-254, (1968).
- [3] J.H. Forrester, D.F. Young, Flow through a converging-diverging tube and its implications in occlusive vascular disease I, *Journal of Biomechanics*, 3 : 297-305, (1970).
- [4] D.A. Mac Donald, On steady flow through modeled vascular stenoses, *J. Biomech.*, 12: 13-20, (1979).
- [5] N.A. Casson, A flow equation for pigment-oil suspensions of the printing ink type. In: Mill, C.C. Ed, *Rheology of Disperse Systems*, Pergamon Press, Oxford, 84-104, (1959).
- [6] J.B. Shukla, R.S.Parihar and B.R.P Rao, Effects of stenosis on non-Newtonian flow of the blood in an artery, *Bulletin of Mathematical Biology*, 42: 283-294, (1980).
- [7] J.C. Misra, M.K. Patra and S.C. Misra, A non-Newtonian fluid model for blood flow through arteries under stenotic conditions, *Journal of Biomechanics*, 26: 1129-1141, (1993).
- [8] R. Ponalagusamy, Mathematical analysis on effect of Non-Newtonian behavior of blood on optimal geometry of microvascular bifurcation system, *Journal of The Franklin Institute*, 349: 2861-2874, (2012).
- [9] P.W. Duck, Pulsatile flow through constricted or dilated channels, *Quarterly Journal of Mechanics and Applied Mathematics*, 33: 77-92, (1980).
- [10] Cheng Tu, Michel Deville, Luc Dheur, Luc Vanderschuren, Finite element simulation of pulsatile flow through arterial stenosis,. *J. Biomech.*, 25: 1141-1152, (1992).
- [11] H.P Mazumdar, U.N. Ganguly(Habishyasi), S.Ghorai and D.C. Dalal , On the distribution of axial velocity pressure gradient in a pulsatile flow of blood through a constricted artery, *Indian Journal of Pure and Applied Mathematics*, 27: 1137-1150, (1996).
- [12] Zuhaila Ismail, IlyaniAbdullah, Norzieha Mustapha and Norsarahaida Amin, A Power-law model of blood flow through a tapered overlapping stenosed artery, *Applied Mathematics and Computation*, 195: 669-680,(2008).
- [13] R.A. Silva, M. Assato and M.S. de Lemos, Mathematical modeling and numerical results of Power-law fluid over a finite porous medium, *International Journal of Thermal Sciences* 100 (2016) 126-137.
- [14] M.K. Sharma, K. Bansal and S. Bansal, Pulsatile unsteady flow of blood through porous medium in a stenotic artery under the influence of transverse magnetic field, *Korea-Australia Rheology Journal*, 24: 181-189, (2012).
- [15] V. Mohan, V. Prasad and Varshney, MHD Oscillatory flow of Elastico-Viscous blood through porous medium in a stenosed artery , *ISOR Journal of Mathematics*, 7: 68-71, (2013).
- [16] R. Ponalagusamy and S. Priyadarshini, Numerical Modelling on Pulsatile Flow of Casson Nanofluid through an inclined artery with stenosis and tapering under the influence of magnetic field and periodic body acceleration, *Korea-Australia Rheology Journal*, 29: 303-316, (2017).
- [17] E EI-Shehawey, E.M.E. Elbarbary, N.A.S. Affi and M. EI-Shahed, MHD flow of an elastic-viscous fluid under periodic body acceleration, *Journal of Mathematical Science*, 23: 795-799, (2000).
- [18] R. Ponalagusamy and S. Priyadarshini, Couple stress fluid model for pulsatile flow of blood in a porous tapered arterial stenosis under magnetic field and periodic body acceleration, *Journal of Mechanics in Medicine and Biology*, 17: 1750109, (2017).
- [19] Shashi Sharma, Udaya Singh and V.K. Katiyar, Magnetic field effect on flow parameters of blood along with magnetic particles in a cylindrical tube, *Journal of Magnetism and Magnetic materials*, 377: 395-401, (2015).
- [20] I. A. Mirza, M.Abdulhameed, S.Shafie, Magnetohydrodynamic approach of non-Newtonian blood flow with magnetic

- particles in stenosed artery, Appl.Math.Mech.-Engl.Ed., 38: 379-392, (2017).
- [21] R. Ponalagusamy, Particulate Suspension Jeffrey Fluid flow in a Stenosed artery with a Particle-free Plasma Layer near the Wall, Korea-Australia Rheology Journal, 28: 217-227, (2016).
- [22] R. Ponalagusamy, A Two-Layered Suspension(Particle-Fluid) Model for Non-Newtonian Fluid Flow in a Catheterized Arterial Stenosis with Slip Condition at the wall of Stenosed Artery, Korea-Australia Rheology Journal, 29: 87-100, (2017).