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Dependence of Q-Factor of the Series RLC Circuit on Inductance in Real Practice

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Abstract

The study of quality (Q-factor) of the series RLC circuit, in ac current is very fundamental especially for resonance condition and the value of quality factor of that circuit depends on the values of R, L and C for a particular range of source frequency. In the present work, for fixed values of R and C the variations of the quality of the circuit i.e. quality factors are studied experimentally with different values if inductances at resonance for a particular range of applied source frequency. Though theoretically it is well established that at resonance frequency, the quality factor of the circuit depends linearly with the value of ideal inductance but in real practice each inductance belongs to some resistance and this static resistance may or may not constant when current flows through it. There will be an experimental study of the dependence of the value of real inductances (considering the resistance of the induction coil and the total resistance of the circuit at the time of flow of current) for the measurement of quality factors in the RLC series circuit. At first it would be checked either the resistance of induction coil is the same or not at on and off conditions of the circuit. For the study of the variations of Q-factors, as the coil resistance plays a vital role, hence the total resistance including the coil resistance to be measured accurately when current flows through the circuit.

Keywords: *Resonance circuit, quality factor, resonance, frequency, ideal inductance.*

1. Introduction

EMF of a sinusoidal ac source and can be express as $E=E_0e^{jwt}$, where w is the angular frequency and w= 2π f, f is the frequency of the applied source. When the circuit is inserted simultaneously with resistance, inductance and capacitance in series the current that flows through the circuit is also sinusoidal i.e. at any instant the circuit current has two parts, one is magnitude and another is phase. For fixed values of R, L, and C, magnitude and phase of the circuit current are controlled by the applied frequency. With the increase in frequency the impedance across the inductance ($Z_L = jwL$) increases and that of across the capacitance ($Z_C = \frac{1}{jwC}$) decreases but as a whole, the sum of the two impedances decreases for low frequency region, hence the circuit current increases with frequency and this increment of current continues until the impedance of inductance and the impedance of capacitance become equal in magnitude and opposite in phase, cancelling each other. Then the circuit is completely resistive, no effect of inductance or capacitance remains present. Thus the condition of the circuit is called resonance stage. At resonance the current that flows through the circuit is the maximum and only depends on the total circuit resistance. Actually the total circuit resistance is the sum of the resistance inserted directly in the circuit and the resistance of the induction coil. But practically it is found that total resistance when current flows through the circuit is not same, rather greater than the arithmetic sum of resistances of series resistance (R) and the static resistance of coil (induction coil). At resonance condition the current and applied voltage are in the same phase. The frequency that is responsible to achieve this condition is called resonance frequency. At that time the serried RLC circuit gains maximum energy from the source. Beyond resonance the impedance across capacitance also decreases and that of across inductance increases with source frequency but the resultant of these two increases with frequency, as a result the circuit current decreases in the higher frequency region. Now, the interesting points are weather the rising part and the falling part of the current are symmetric or not and what are the different factors on which the rate of rise or fall depends and also on what way, one RLC series resonance circuit is different from another for same range of applied frequency. Theoretically it is well established that to achieve the resonance condition the values of R, L and C are very important for a given range of supplied frequency and a term called Q-factor (quality factor) is commonly used to explain the rate of rise of current from lower frequency to rate of fall of current to higher frequency range. The value of quality factor of the circuit, differentiate between two resonance circuits for given range of frequency. For a particular series RLC circuit, the Q-factor is high means rapid rise and fall of circuit current with frequency. In the present experimental work, the variation of Q-quality factors with real inductance (considering the total resistance of the circuit) would be studied.

2. Experiment with Theory

The circuit diagram and block diagram (RLC kit) of the series RLC circuit is shown in figure 1 and figure 2 respectively.

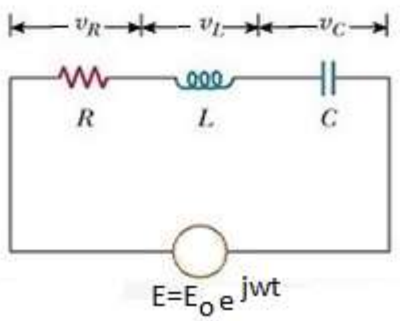

Figure: 1

Figure: 2, RLC kit

When I current that flows through the circuit at any instant,
The potential drop across resistance R,

$$V_R = IR$$

The potential drop across resistance L
(Considering the resistance of the induction coil)

$$V_L = Ir + jwL$$

The potential drop across resistance C,

$$V_C = \frac{1}{jwC}$$

Appling Kirchhoff's law

$$E_0 e^{jwt} = IR + Ir + jwL + \frac{1}{jwC}$$

$$E_0 e^{jwt} = I(R + r) + j(wL - \frac{1}{wC})$$

Solving this equation, the expression of current is

$$I = \frac{E_0 e^{j(wt+\theta)}}{\sqrt{(R+r)^2 + (wL - \frac{1}{wC})^2}},$$

$$I = \frac{E_0 e^{j(wt+\theta)}}{\sqrt{R_t^2 + (wL - \frac{1}{wC})^2}},$$

and

$$\theta = \tan^{-1}\frac{(wL - \frac{1}{wC})}{(R+r)} \quad \text{------------(1)}$$

Where R_t is the total resistance of the circuit, R_t= R + r, So the circuit current, for a particular set of R, L and C depends on source frequency and also total resistance of the circuit. The current first increases and reaches the maximum value and then decreases with source frequency. The particular value of frequency for which the current reaches the maximum is called resonance frequency (f_0) and the circuit is then termed as resonance circuit. At that time inductive impedance is equal to capacitive impedance but opposite in phase, so the resonance frequency is obtained as.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

And the circuit is purely resistive; also the current and the voltage are in the same phase. Quality or Q-factor is something that indicates the rate of rise of current of the circuit up-to the maximum point or from the peak of the current the rate of fall of current and sharpness of resonance is usually denoted with Q-factor. This rate of rise or fall of current depends on the elements connected in the circuit. In the present work four different ways are considered for the measurement of Q-factor considering fixed values of R and C and using several inductances in a given range of frequency.

1) By using the concept of band-width, quality factor

$Q = \frac{f_0}{f_2 - f_1}$, where f_0 is the resonance frequency, f_2 and f_1 are two half power frequencies where the

current falls $\frac{I_0}{\sqrt{2}}$ of the resonant current .i.e. at that situation impedances satisfy the condition,

$$\sqrt{(R+r)^2 + (wL - \frac{1}{wC})^2} = \sqrt{2}(R+r)$$

$$i.e`.(wL - \frac{1}{wC}) = \pm(R+r)$$

Considering frequencies f_1 and f_2 for rise and fall of current due to corresponding angular frequencies w_1 and w_2.

$$Q_R = \frac{f_0}{f_2 - f_1} = \frac{w_0 L}{R+r} ------(2)$$

r is the resistance of Inductance, It can be measured before the experiment starts as well as during experiment. At resonance, r can be measured accurately using the relation given below.

$$R_t = R + r$$

$$R_t = \frac{input\ .voltage}{resonance\ \ .current} = \frac{V_i}{I_{resonance}} ---(3)$$

R is the resistance connected in the circuit, so resistance (r) of the coil can be measured.

$$2)\ Q_c = \frac{Voltage\ \ .across\ \ .C\ .at\ .resonance}{Inputvolta\ \ ge} = \frac{V_c}{Vi} ---(4)$$

$$3)\ Q_L = \frac{Voltage\ \ .across\ \ `.L\ .at\ .resonance}{Inputvolta\ \ ge} = \frac{V_L}{V_i} --(5)$$

V_C , V_L and V_i are measured by ac meters attached with the kit during the resonance condition. But for an ideal inductance the coil resistance is considered to be a very small value i.e. it is treated as zero. So for an ideal inductance theoretically, Q- factor can be written as

(Putting r=0, everywhere in the earlier equations)

$$Q = \frac{w_0 L}{R} ---(6)$$

Where R is the resistance connected in series and it is assume that the resistance of the induction coil is very small. The series resistance connected with L and C is equal to the total resistance of the circuit.

First of all seventeen different standard inductances are chosen of the observation. Values of inductances are from 8.18 mH to 94.8 mH. and there coil resistances are accurately measured simultaneously with digital multi-meter.

3. Results:

First of all selecting a standard RLC kit as shown in figure: 2, R and C are connected in series. After that L is to be connected in series with R and C. On the actual experimental study, L is inserted between R and C. In the kit various provisions are there to change the components simultaneously or any one in any time. Digital voltmeters and mill-ammeter are attaches with the kit. In the kit source frequency can be adjusted up to several kilo-hertz. For the present study seventeen different inductances are considered. At first, the coil resistances of seventeen inductances are accurately measured with digital multi-meters. The values of inductances lie between 8.18mH t0 97.9mH. For first set of observation the value of R and C made fixed, these are 218ohms and 0.22 microfarad respectively and starting from the least valued inductance, other inductances are connected one by one. For each inductance variation of circuit current with frequency for a fixed input voltage (V_i) is observed and data are recorded systematically starting from minimum value to maximum value (called resonance current I_R) and again to minimum value of current. The voltages across the inductance and capacitance are simultaneously noted down accurately along with input voltage (V_i), resonance current (I_R). These observations are recorded several times for each inductance and same values of resistance and capacitance and finally mean value of these recorded data are placed in tabulation form as shown in the table no.1. Similarly considering R=318ohms and C= 0.11microfarad, for 2nd set of observations have done and recorded the data as shown in the table2. For the third set R is selected 94.8 ohms and C=0.33 microfarad. The recorded values are placed in the table no.3. Then finally R_t, Q, Q_R, Q_L and Q_C are all calculated by using different equations already marked earlier. From the observations, it is found that the coil resistances (static resistance) before the experiment and when current flows through the circuit, especially at resonance are not same. These values of coil resistances for different inductances at two conditions are shown in the table 4. Different quality factors (Q, Q_R, Q_L and Q_C) are plotted with L for three different sets of R and C, are shown in fig.3, fig.4 and fig.5 respectively.

Table No.1 Q factors for different inductance (R=218 ohms, C=0.22 microfarad)

No. of L	Induct-ance in m H	Reso-nance frequ-ency in 10^3 Hz	V_i in volts	V_C in volts.	V_L in volts.	I_R reso-nance current in 10^{-3} amps	Q for ideal L	R_t Total resist-ance	Q_R	Q_L	Q_C
1	8.18	2.93	0.78	0.57	0.43	2.65	0.690	294.34	0.511	0.551	0.730
2	10	2.69	0.78	0.55	0.40	2.30	0.774	339.13	0.498	0.512	0.705
3	16.8	2.32	0.78	0.55	0.40	2.32	1.122	336.20	0.728	0.512	0.705
4	20	1.77	0.78	0.74	0.54	1.97	1.019	395.93	0.561	0.692	0.948
5	24.87	1.84	0.78	0.70	0.62	1.95	1.318	400.0	0.718	0.794	0.897
6	25.6	2.00	0.78	1.06	0.99	3.19	1.474	244.51	1.315	1.269	1.358
7	26.6	1.97	0.78	1.08	1.18	3.36	1.509	232.14	1.417	1.512	1.384
8	40	1.66	0.78	0.87	0.72	2.03	1.912	384.23	1.085	0.923	1.115
9	48.5	1.35	0.78	0.95	0.86	1.91	1.886	408.37	1.006	1.102	1.217
10	57.3	1.172	0.78	1.14	0.89	1.76	1.934	443.18	0.951	1.141	1.461
11	60	1.314	0.78	0.97	0.91	1.89	2.271	412.69	1.199	1.166	1.243
12	65.4	1.211	0.78	1.11	0.99	1.70	2.281	458.82	1.084	1.269	1.423
13	80	1.139	0.77	0.92	0.80	1.55	2.624	496.77	1.151	1.038	1.194
14	85.1	1.00	0.78	1.73	1.98	3.01	2.451	259.13	2.062	2.538	2.217
15	90	1.104	0.78	1.90	1.89	3.1	2.862	251.61	2.479	2.423	2.435
16	94.2	1.055	0.77	1.75	2.02	3.04	2.862	253.28	2.464	2.623	2.272

Table No.2 Q factors for different inductance (R=318 ohms, C=0.11 microfarad)

No. of L	Inductance in mH	Reson-ance frequ-ency in 10^3 Hz	V_i in volts.	V_L in volts	V_C in volts	I_R reso-nance curre-nt. 10^{-3} amps	Q for ideal L	R_t Total resistance in ohms	Q_R	Q_L	Q_c
1	8.18	4.531	0.89	0.61	0.44	1.87	0.711	475.9	0.489	0.685	0.494
2	10	4.507	0.69	0.55	0.43	1.68	0.865	410.7	0.689	0.797	0.623
3	16.8	3.024	0.91	0.86	0.62	1.77	0.975	514.1	0.620	0.945	0.681
4	20	2.92	0.9	0.77	0.60	1.46	1.121	616.4	0.594	0.855	0.666
5	24.87	2.937	0.9	0.67	0.75	1.43	1.402	629.3	0.728	0.744	0.833
6	25.6	3.023	0.9	1.16	1.17	2.29	1.486	393.0	1.236	1.288	1.300
7	26.6	2.977	0.9	1.26	1.35	2.44	1.520	368.8	1.348	1.400	1.500
8	40	2.656	0.9	0.87	0.82	1.45	2.040	620.6	1.074	0.966	0.911
9	48.5	2.150	0.9	1.56	1.52	2.16	2.002	416.6	1.571	1.733	1.688
10	57.3	1.521	0.9	1.21	0.93	1.18	1.673	762.7	0.717	1.344	1.033
11	60	2.009	0.91	1.09	0.94	1.4	2.314	650.0	1.164	1.197	1.032
12	65.4	1.392	0.9	1.27	0.93	1.12	1.748	803.5	0.711	1.411	1.033
13	80	1.951	0.9	0.93	0.95	1.17	2.997	769.2	1.274	1.033	1.055
14	85.1	1.615	0.9	2.26	2.25	2.33	2.639	386.2	2.234	2.511	2.500
15	90	1.566	0.9	2.27	2.17	2.27	2.706	396.4	2.232	2.522	2.411
16	94.2	1.513	0.9	2.33	2.38	2.25	2.737	400.0	2.237	2.588	2.644
17	97.6	1.467	0.9	2.43	2.40	2.28	2.758	394.7	2.284	2.700	2.666

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Table No. 3 Q factors for different inductances (R=98.4 ohms, C=0.33 microfarad)

No. of L	Induc-tance in mH	Resonance frequency in 10^3 Hz	V_i in volts.	V_L in volts	V_C in volts	I_R reso-nance current. in10^{-3} amps	Q for ideal L	R_t total resista-nce in ohms	Q_R	Q_L	Q_c
1	8.18	2.485	1.01	1.05	0.92	5.27	1.297	191.65	0.666	1.039	0.910
2	10	1.656	1.01	1.09	1.11	3.63	1.056	278.23	0.373	1.079	1.099
3	16.8	2.273	1.01	0.97	0.79	4.38	2.437	230.59	1.039	0.960	0.782
4	20	1.709	1.00	1.04	1.12	3.62	2.181	276.24	0.777	1.040	1.120
5	24.87	1.47	1.01	1.05	1.18	3.47	2.333	291.06	0.788	1.039	1.168
6	25.6	1.730	1.01	2.11	2.12	7.2	2.826	140.27	1.982	2.089	2.099
7	26.6	1.655	0.99	2.3	2.45	7.52	2.809	131.64	2.100	2.323	2.474
8	40	1.422	1.01	1.34	1.31	3.77	3.630	267.90	1.333	1.326	1.297
9	48.5	1.246	1.00	2.57	2.6	6.34	3.856	157.72	2.406	2.570	2.600
10	57.3	0.844	1.01	1.79	1.49	2.99	3.086	337.79	0.899	1.772	1.475
11	60	1.089	1.01	1.55	1.42	3.28	4.170	307.92	1.332	1.534	1.405
12	65.4	0.809	1.01	1.78	1.56	2.85	3.376 6	354.38	0.937	1.762	1.544
13	80	0.900	1.00	1.49	1.27	2.64	4.595	378.78	1.193	1.490	1.270
14	85.1	0.918	1.00	3.92	3.97	7.05	4.985	141.84	3.458	3.920	3.970
15	90	0.900	1.00	3.70	3.82	6.52	5.169	153.37	3.316	3.700	3.820
16	94.2	0.830	1.00	3.89	3.94	6.26	4.989	159.74	3.073	3.890	3.940
17	97.6	0.812	1.00	4.12	4.06	6.53	5.057	153.13	3.249	4.120	4.060

Table No.4 Variations of circuit- resistances at resonance for different coils resistances.

Inductances L in mill-henry	coil resistance measured before expt. in ohms	Total circuit resistance measured at resonance in ohms.		
		R=218 ohms. C=0.22 microfarad	R=318 ohms. C=0.11 microfarad	R=98.4 ohms. C=0.33 microfarad
8.18	12.4	294.34	475.9	191.65
10	17.6	339.13	410.7	278.23
16.8	17.7	336.20	514.1	230.59
20	26.2	395.93	616.4	276.24
24.87	32.5	400.0	629.3	291.06
25.6	13.1	244.51	393.0	140.27
26.6	10.8	232.14	368.8	131.64
40	37.1	384.23	620.6	267.90
48.5	25.1	408.37	416.6	157.72
57.3	12.4	443.18	762.7	337.79
60	47.7	412.69	650.0	307.92
65.4	12.6	458.82	803.5	354.38
80	58.7	496.77	769.2	378.78
85.1	12.2	259.13	386.2	141.84
90	12.5	251.61	396.4	153.37
94.2	9.3	253.28	400.0	159.74
97.6	12.1	294.34	394.7	153.13

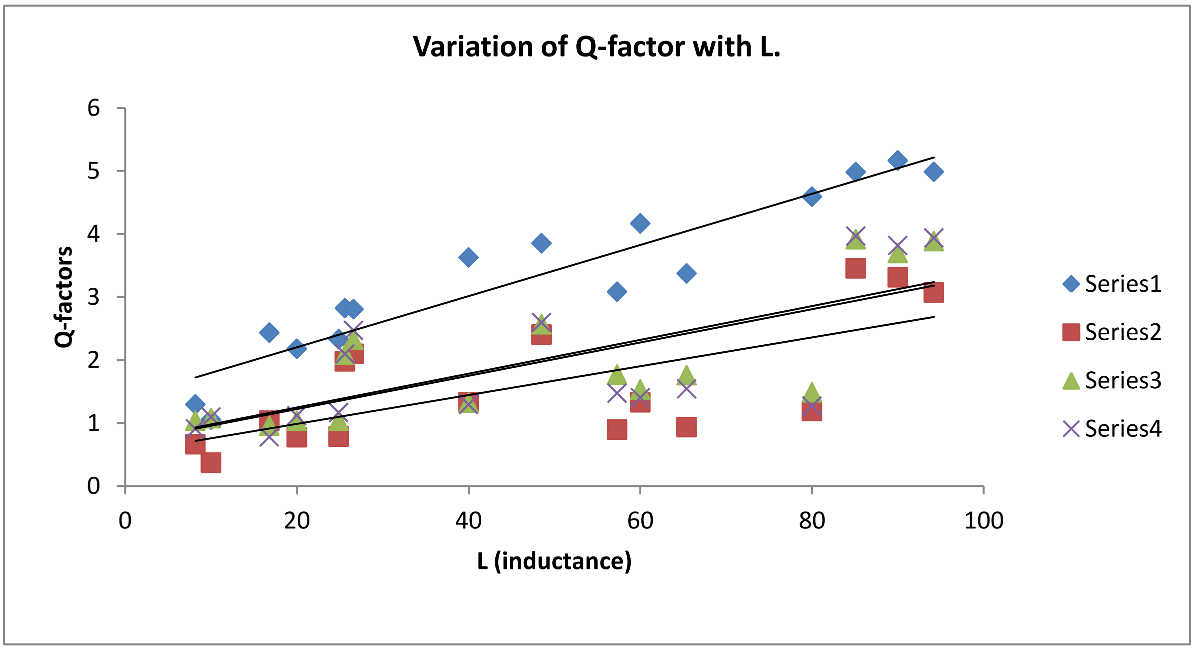

Figure.3

Fig.3 [Variation of Q-factor with inductance for R=218 ohms and C=0.22 microfarad.]. In the Q, Q_R, Q_L and Q_C are represented by series1, series2, series3 and series4 respectively.

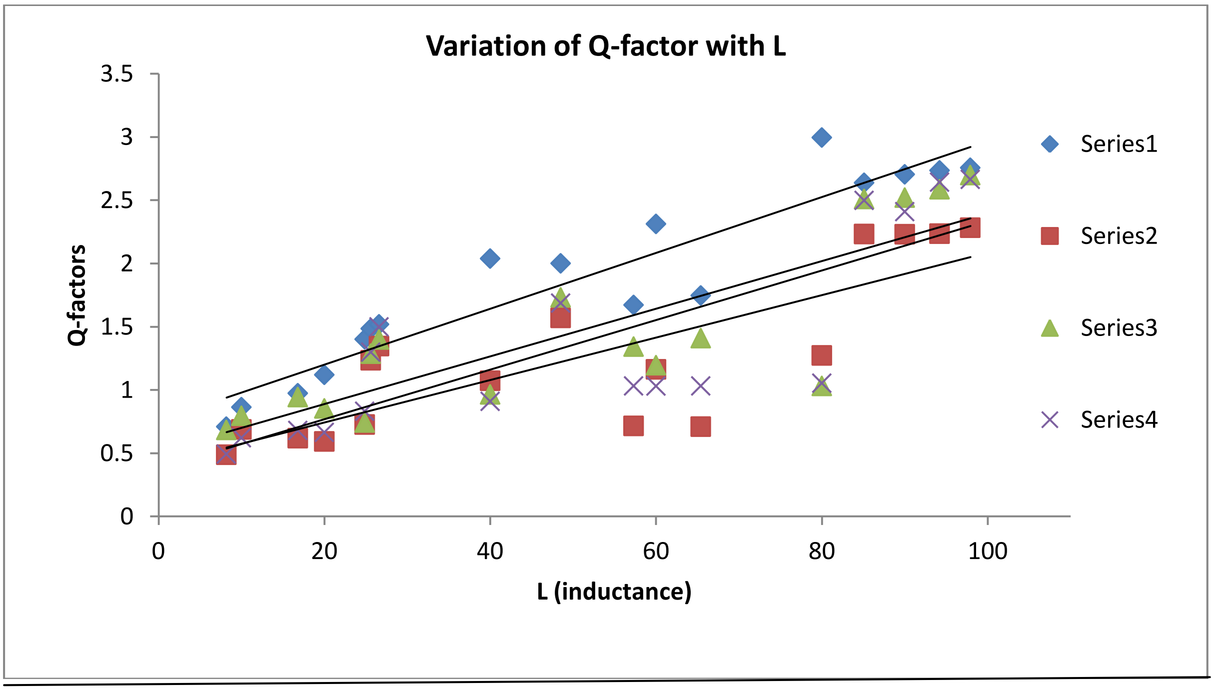

Figure.4

Fig.4 [Variation of Q-factor with inductance for R=318 ohms and C=0.11 microfarad.]. In the Q ,Q_R, Q_L and Q_C are represented by series1, series2, series3 and series4 respectively.

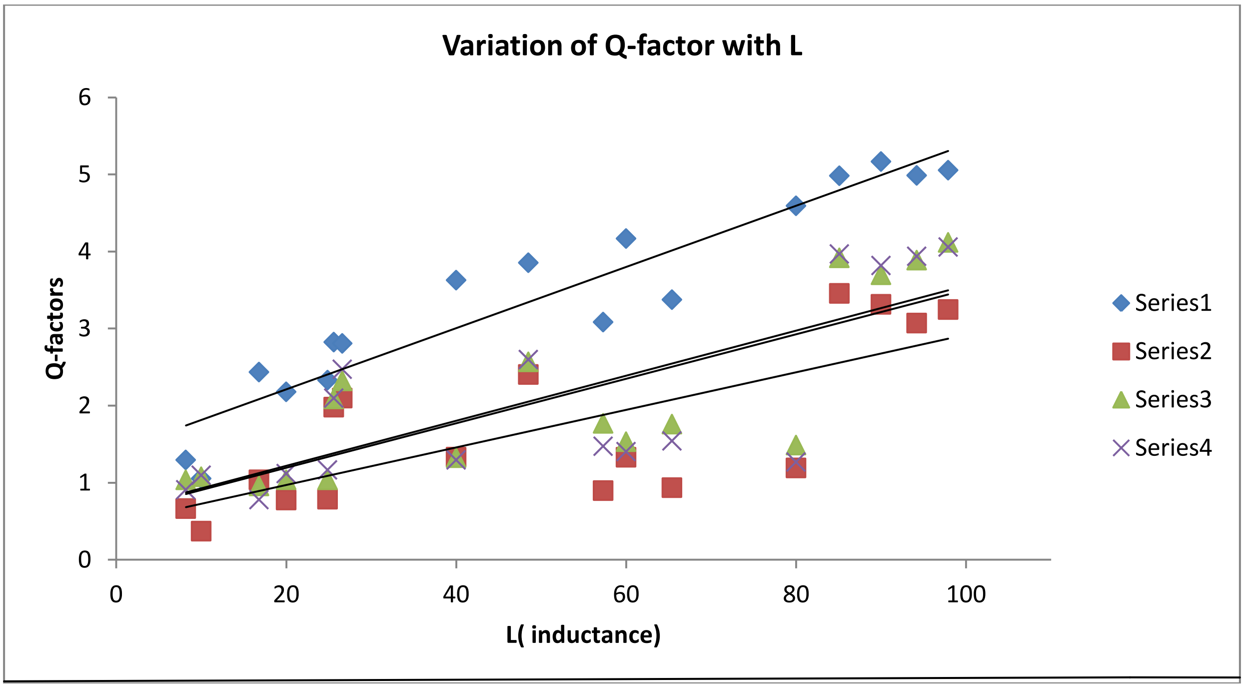

Figure.5

Fig.5 [Variation of Q-factor with inductance for R=98.4 ohms and C=0.33 microfarad.]. In the Q ,Q_R, Q_L and Q_C are represented by series1, series2, series3 and series4 respectively.

4. Discussions and Conclusion

From the fig.3, fig.4 and fig.5, it is found that the variations of Q, Q_R, Q_L and, Q_C with L are not symmetric and uniform. Q (as it is calculated, considering resistance of the coil (L) is very small) not changes systematically, but theoretically linear variation is expected and at resonance, it should be depended only on resistance(R, static resistance of the coil) that is connected in series. As Q has non uniformity in variation, so it may be predicted that resistance connected in series is not the total resistance of the circuit. Total resistance of the circuit may be the sum of resistances, connected in series and the coil resistance. But from the table 4, it is observed that sum of resistances are not the same as calculated from the ratio of input voltage to resonance current. This resistance that is calculated from the ratio of input voltage to resonance current can be termed as actual total resistance (R_t) of the circuit when the current flows. Later is greater than the sum of resistances, though coil resistances are measured just before the switch on with the help of digital multi-meter. Next Q_R is calculated by using the actual total resistance of the circuit at the time of resonance. Till the variations of Q_R with L for all three sets, are found not to be linear or symmetric. For a fixed set, R, C and input voltage (V_i) are constant and the ratios of V_L to V_i and V_C to V_i are measured to obtain Q_L and Q_C at resonance, and their variations with L again found to follow the same results as obtained from Q and Q_R. But the important observation is noticed i.e. for a fixed R, C similar asymmetry of Q, Q_R, Q_L and Q_C are found for same L. Maximum asymmetry of the various Q factors are observed in the middle portion of the linear variation curve where the values of inductances lie between ~20 to ~80 mill-henry and total circuit resistances are comparatively very high with respect to the arithmetical sum of coil resistance and series resistance at off condition of the circuit. Through it is important that the resonant frequencies for all inductances are not the same. It is practically hard enough to adjust the frequency. So, according to the obtained data, it may be predicted that Q-factors depends on inductance and simultaneously the effect of frequency on inductance not only depends on its values but depends on its coil resistance and total resistance of the circuit, but the dependence is not completely linear or symmetric. This discrepancy increases with the increase of coil resistance. So for better results the coil resistances should be very small, selection of frequency range is very important and for any comparative study the inductances should be manufactured from the same company.

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