IJASRM International Journal of Advanced Scientific Research and Management, Volume 3 Issue 6, June 2018
www.ijasrm.com
ISSN 2455-6378

Laminar Boundary Layer Flow on a Flate in a rotating system with Magnetic Field

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Abstract

An analysis of two dimensional steady state of an incompressible electrically conducting fluid, it is assumed that the magnetic Reynolds number is small and whole system behaves as in case of a solid body rotation with constant angular velocity about y axis normal to the plate. The effect of angular velocity is found to have an increasing effect on the component of the velocity which is parallel to the length of the plate while the rotation angular velocity has the effect of decreasing the displacement thickness.

Keywords: *Laminar boundary layer flow*, Porous medium, Runge-Kutta Methods ,Magnetic field, angular velocity.

1. Introduction:

Cengel and Y.A.[1] studied laminar boundary layer flow equation on moving continuous flat surface in the presence of suction and a magnetic field by using group analysis to discuss the existence and uniqueness of the concerned boundary value problem. Runge-Kutta-Merson method was used to find the effect of magnetic field on boundary layer thickness and skin friction at the surface. Shrigopal Agarawal [5] studied of MHD boundary layer flow with suction and injuction through porous medium past an oscillating plate in a rotating system. S.N. Murthy and Mohini Sapre[6] studied the effect of magnetic field on laminar boundary later flow on a flate plate. In this work we studied the effect of constant angular velocity on the laminar boundary layer flow on a flat plate with magnetic field. This work is the extension of the paper of S.N.Murthy and Mohini Sapre [6] with the application of constant angular velocity about y axis normal to the flat plate.

Fundamental Equation: We consider that the whole system is rotating with a constant angular velocity about the y axis. A uniform transverse magnetic field magnetic field is acting parallel to the axis of rotation taking the magnetic Reynolds number to be small; the induced magnetic field is neglated in comparison with the applied magnetic field $B_y(x)$.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-2\Omega v = \upsilon\frac{\partial^2 u}{\partial y^2}-\sigma\frac{B_0^2 u}{\rho}\ldots(1)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\ldots.(2).$$

The boundary conditions to be satisfied are - $u=0=v\ at\ y=0\ and\ u=U_\infty\ at\ y=\infty$ where $y=\infty$ denotes the edge of the boundary layer and where U_∞ is a constant potential flow velocity. By using the stream function ψ such that

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x}$$

$$\frac{\partial\psi}{\partial y}.\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}.\frac{\partial^2\psi}{\partial y^2}+2\Omega\frac{\partial\psi}{\partial x}=\upsilon\frac{\partial^3\psi}{\partial y^3}-s(x)\frac{\partial\psi}{\partial y}\ldots\ldots.(3)$$

Where $s(x)=\sigma\frac{B_0^2(x)}{\rho}$ taking $s(x)=s_0x^\epsilon$ and putting

$$\eta=\left(\frac{U_\infty}{2\upsilon_x}\right)^{1/2}\text{and }\ \psi=(2\upsilon U_\infty x)^{1/2}F$$

To study similarity solutions of (3) the condition that each term in the equation conditions the same degree in x gives

$\epsilon = -1$ and eqn. (3) reduces to
$F''' + FF'' - \gamma F + (\alpha - \beta)F' = 0 \ldots (4)$
Where $\alpha = \frac{2\Omega v}{U_\infty}$ and $\gamma = 2\Omega\left(\frac{2v}{U_\infty^3}\right)^{1/2}$ and $\beta = \frac{2S_0}{U_\infty}$ the boundary conditions become $F(0) = 0 = F'(0) \ldots$ (5) and $F'(\infty)$ = 1..(6)
We have to solve eqn. (4) with boundary condition (5) and (6).

Solution of the problem:
Let the solution of eqn. (4) be taken as
$F(\eta) = \frac{a_2\eta^2}{2!} + \frac{a_3\eta^3}{3!} + \cdots$ ……. (7)
To satisfied the boundary conditions (5) substituting for F, F', F'' and F''' using (7) in (4) and equating coefficient of each power of η to zero , we get,
$a_3 = 0, a_4 = (\beta - \alpha)a_2,\ a_5 = (\gamma - a_2)a_2,$
$a_6 = (\beta - \alpha)^2 a_2$, $a_7 = a_2(\beta - \alpha)(2\gamma - 8a2$,$a8=a2\gamma2-12a2\,\gamma+11a22+\beta-\alpha3$.
Hence taking $a_2 = a$ we get solution of (4) satisfying boundary condition (5) as
$F(\eta) = \frac{a\eta^2}{2!} + \frac{(\beta-\alpha)a\eta^4}{4!} + \frac{(\gamma-a)a\eta^5}{5!} + \frac{(\beta-\alpha)^2 a\eta^6}{6!} + \frac{a(\beta-\alpha)(2\gamma-8a)\eta^7}{7!} \ldots$ (8)
From equation (4) we write,
$F''(\eta) = e^{-F(\eta)}.\phi(\eta)$ ….. (9)
Where $F(\eta) = \int_0^\eta F(\eta)d\eta \ldots$ (10)
And
$\phi(n) = a + \int_0^\eta[(\beta - \alpha)F' + \gamma F]d\eta$…(11)
And a is same as that given in (8) in order to find value of a we shall evaluate the integral in (11) by the method of steepest descents. Let,
$F(\eta) = \eta^3 \sum_{n=0}^\infty C_n\eta^n = \tau$…(12)
and $\phi(n) = \sum_{n=0}^\infty b_n\eta^n \ldots$ (13)
Since $F(\eta)$ starts with η^3, we put
$\eta = \sum_{m=0}^\infty \frac{A_m}{m+1}\tau^{(4/3)(m+1)}$……(14)

So that

$\int^{0+}\frac{dn}{\tau^{(1/3)(m+1)}} = \frac{A_m}{3}\int^{0+0+0+}\frac{d\tau}{\tau} =$
$2\pi i A_m$ …(15)Here 0+ denotes a circuit in the positive direction round the zero point and the single circuit round $\eta = 0$ in the η –plane corresponds to three circuits about $\tau = 0$ in the τ plane. From (15) it follows that A_m is coefficient of n^{-1}in the expansion of $\tau^{-(1/3)(m+1)}$ in ascending and descending power of η.From (12) –

$\tau^{-\left(\frac{1}{3}\right)(m+1)} =$

$\eta^{-(m+1)}\left[\begin{matrix} C_0 + C_1\eta + C_2\eta^2 \\ + \cdots\end{matrix}\right]^{-\frac{1}{3}(m+1)}$ …(16) So that A_m is the coefficient of η^m in the expression $[C_0 + C_1\eta + C_2\eta^2 + \cdots]^{-\frac{1}{3}(m+1)}$ the term $C_1\eta$ is not considered since C_1 vanishes in the integrals where the above expression will be used. The coefficient A_m will therefore be evaluated for $C_1 = 0$ that is, we write –

$$\sum_{m=0}^{\infty} A_m\eta^m = [C_0]^{-\frac{1}{3}(m+1)}$$

$$\left[1 + \frac{C_2}{C_0}\eta^2 + \frac{C_3}{C_0}\eta^3 + \frac{C_4}{C_0}\eta^4 + \cdots\right]^{-\frac{1}{3}(m+1)} \ldots(17)$$

From (17) we get, $A_0 = [C_0]^{-\frac{1}{3}}$, $A_1 = 0$, $A_2 = -\frac{C_2}{C_0^2}$, $A_3 = -[C_0]^{-\frac{4}{3}}\left(\frac{4}{3}\frac{C_3}{C_0}\right)$, $A_4 = \frac{5}{3}[C_0]^{-\frac{5}{3}}\left[-\frac{C_4}{C_0} + \frac{4}{3}\frac{C_2^2}{C_0^2}\right]$

$$A_5 = 2[C_0]^{-2}\left[-\frac{C_5}{C_0} + \frac{3C_2C_3}{C_0^2}\right] \quad \ldots. (18)$$

Now consider the integral

$\int_0^{\tau} e^{-\tau} \emptyset(\eta) \frac{d\eta}{d\tau} d\tau$ …. (19)

And let,

$$\emptyset(\eta) \frac{d\eta}{d\tau} = \sum_{m=0}^{\infty} d_m \frac{\tau^{\frac{(m+1)}{3}}}{\tau} \quad …. (20)$$

By the procedure similar to that used in the previous case we find

$$d_m = \frac{1}{6\pi i} \int^{0+0+0+} \emptyset(\eta) \frac{d\eta}{d\tau} \tau^{\frac{-(m+1)}{3}} d\tau$$

Or $d_m = \frac{1}{6\pi i} \int^{0+} \emptyset(\eta)\, \tau^{\frac{-(m+1)}{3}} d\eta$

So that d_m is equal to 1/3 of the coefficient of η^{-1}in the expansion of $\emptyset(\eta)\tau^{\frac{-(m+1)}{3}}$ from the series expansion of $\tau^{\frac{-(m+1)}{3}}$ in (16) this means –

$$\sum_{m=0}^{\infty} d_m \eta^m = \frac{1}{3}[C_0 + C_2\eta^2 + C_3\eta^3 + C_4\eta^4 +$$

…−13m+1. b0+b2η2+b3η3+b4η4+…

Expanding and multiplying, we get

$$d_0 = \frac{1}{3}(C_0)^{-1/3}.b_0 \ , \ d_1 = 0,$$

$$d_2 = \left(\frac{1}{3}\right)(C_0)^{-1}(b_2 - \frac{C_2}{C_0} b_0),$$

$$d_3 = \left(\frac{1}{3}\right)(C_0)^{-4/3}(b_3 - \frac{4}{3}\frac{C_3}{C_0} b_0)$$

$$d_4 = \left(\frac{1}{3}\right)(C_0)^{-\frac{5}{3}}(b_4 - \frac{5}{3}\frac{C_4}{C_0} b_0 - \frac{5}{3}\frac{C_2}{C_0} b_2 + \frac{20}{9}\frac{C_2^2}{C_0^2} b_0)$$

$$d_5 = \left(\frac{1}{3}\right)(C_0)^{-2}(b_5 - 2\frac{C_5}{C_0} b_0 - 2\frac{C_3}{C_0} b_2 + \frac{6C_2C_3}{C_0^2} b_0) \quad …. (21)$$

From (9), (19) and (20) and boundary condition at ∞ , we get –

$$1= \int_0^{\infty} e^{-F(\eta)} \emptyset(\eta) d(\eta)$$

$$=\sum_{m=0}^{\infty} d_m \Gamma(m+1)/3 \ ….(22)$$

$$F(\eta) = \frac{a\eta^3}{6} + \frac{(\beta-\alpha)a\eta^5}{120} + \frac{a(\gamma-a)\eta^6}{720} + \frac{a(\beta-\alpha)^2\eta^7}{5040} + \frac{a(\beta-\alpha)(2\gamma-8a)\eta^8}{40320} + …….(23)$$

Substituting the values of $F''(\eta), F(\eta)$ and $\phi(\eta)$ in the eqn. (9) by using (8), (23) and (13) and comparing the coefficients of power of η, we obtain

$$b_0 = a \ , \quad b_1 = 0 \ , \ b_2 = \frac{(\beta-\alpha)a}{2} \ , \ b_3 = \frac{\gamma_a}{6} \ , \ b_4 = \frac{(\beta-\alpha)^2 a}{24}, , \ b_5 = \frac{(\beta-\alpha)(2a\gamma+3a^2)}{12} ……(24)$$

From eqn.(12) and (23) , we have –

$$C_0 = {}^{a}\!/_{6}, \ C_1 = 0, \ C_2 = \frac{(\beta-\alpha)a}{120}, C_3 = \frac{(\gamma-a)a}{720}, C_4 = \frac{((\beta-\alpha))^2 a}{5040}, \ C_5 = \frac{(2\gamma-8a)(\beta-\alpha)a}{40320} ….(25)$$

From eqn.(18), (25)

$$A_0 = \left({}^{a}\!/_{6}\right)^{-1/3} = \left({}^{6}\!/_{a}\right)^{1/3}, A_1 = 0,$$

$$A_2 = -3\frac{(\beta-\alpha)}{10a}, \ A_3 = -\left({}^{6}\!/_{a}\right)^{4/3} \frac{((\gamma-a))}{90},$$

$A_4 = ((\beta-\alpha))^2 (6/a)^{\frac{2}{3}} (\frac{3}{140a})$........

Hence from Equation (14)

$\eta = (6/a)^{1/3} \tau^{1/3} - \frac{(\beta-\alpha)}{10a}\tau - (6/a)^{\frac{4}{3}} \frac{(\gamma-a)}{90}.\frac{1}{4}\tau^{4/3} + \frac{(\beta-\alpha)^2 (6/a)^{2/3}\tau^{5/3}}{700a}$ +.... (26)

Also from equation (21), (24) (25) we get

$d_0 = (a/3)(6/a)^{1/3}$, $d_1 = 0$,

$d_2 = (9/10)(\beta-\alpha)$,

$d_3 = (1/45)(6/a)^{1/3}(14\gamma + a)$,

$d_4 = \frac{(\beta-\alpha)^2}{140}(6/a)^{2/3}$ $d_5 = \frac{(\beta-\alpha)}{a}.\left(\frac{448\gamma+4464a}{11200}\right)$..... (27)

Then from equation (22)

$1 = \int_0^\infty e^{-F(\eta)} \phi(\eta) d\eta$

$= \sum_{m=0}^{\infty} d_m \Gamma(m+1)/3$

$= d_0\Gamma 1/3 + d_1\Gamma(2/3) + d_2\Gamma 1 + d_3\Gamma(4/3) + d_4\Gamma(5/3) + d_5\Gamma 2 + \cdots$

On putting the values of $d_0, d_1, d_2 d_3, d_4, d_5$ .... In above equation.

$= (a/3)(6/a)^{1/3}\,\Gamma 1/3 + (9/10)(\beta-\alpha)\Gamma 1 + (1/45)(6/a)^{1/3}(14\gamma + a)\,\Gamma(4/3)$ $+ \frac{(\beta-\alpha)^2}{140}(6/a)^{2/3}\,\Gamma(5/3) + \frac{(\beta-\alpha)}{a}\left(\frac{44B\gamma+4464a}{11200}\right)\Gamma 2$ +.... In this case $\gamma \to 0$then equation becomes-

$= (a/3)(6/a)^{1/3}\,\Gamma 1/3 + (9/10)(\beta-\alpha)\Gamma 1 + (1/45)(6/a)^{1/3} a.\Gamma(4/3)$

$+ \frac{(\beta-\alpha)^2}{140}(6/a)^{2/3}\,\Gamma(5/3) + \frac{(\beta-\alpha)}{a}\frac{279}{700} a\,\Gamma 2 + \ldots$(28)

From (28) on simplification , we get-

$(a)^{4/3} + [.7797(\beta-\alpha) - .6004](a)^{2/3} + .0127((\beta-\alpha))^2 = 0$...(29)On solving equation (29) we get two values of $a^{2/3}$ as –

$a^{2/3}$ = .6004-.7797 $(\beta-\alpha)$ and $a^{2/3}$ = 0.

So that ,

a = .4562 - .9662 $(\beta-\alpha)$ ...... (30)

<u>Velocity Components and Displecement Thickness</u>:

From equation (8) the x component of velocity is given by $U = U_\infty F'$ $U = U_\infty[a\eta + \frac{(\beta-\alpha)a\eta^3}{3!} - \frac{a^2\eta^4}{4!} + \frac{(\beta-\alpha)^2 a\eta^5}{5!}$

$- \frac{8(\beta-\alpha)}{6!}a^2\eta^6$...... (30i)

The velocity component along y axis is

$V = \frac{U_\infty \nu}{2a}(\eta F' - F) = \left(\frac{U_\infty \nu}{2a}\right)^{1/2} \int_0^\eta \eta F''(\eta) d\eta$

Or $V = \left(\frac{U_\infty \nu}{2X}\right)^{1/2} \left[\frac{a\eta^2}{2} + \frac{(\beta-\alpha)a}{8}\eta^4 - \frac{a^2\eta^5}{30} + \right.$ *β−α2aη6144−8β−αa2η7840+*.......... .(31)

By use of (8) to find F'' and subsequent integration. From (31) we can find the value of v for different value of α and η the displacement thickness δ_1 is equal to

$\delta_1 = \int_0^\infty \left(1 - \frac{U}{U_\infty}\right) dy$ where U_∞ is the velocity at the edge of the boundary layer.

We have

$$\delta_1 = \int_0^\infty (1 - F')dy = \left(\frac{2\nu X}{U_\infty}\right)^{1/2} \int_0^\infty (1 - F')d\eta$$

Integrating by parts between the finite 0, we get,

$\int_0^\infty (1 - F')d\eta = \eta - \eta F'(\eta) + \int_0^n F''(\eta)\eta d\eta =$ *0∞ηF′′(η)dη* as *n→0,∞* and hence

$\delta_1 = \left(\frac{2\nu X}{U_\infty}\right)^{1/2} \int_0^\infty \eta F''(\eta) d\eta$ by

By (9) and (13) δ_1

$\delta_1 = \left(\frac{2\nu X}{U_\infty}\right)^{1/2} \int_0^\infty e^{-F(\eta)}(b_0\eta + b_1\eta^2 + b_2\eta^3 + \cdots)d\eta \ldots$ (32)

From (26)

$d\eta = \left[{}^1/_3\, (6/a)^{1/3}\tau^{-2/3} - \frac{(\beta-\alpha)}{10a} + \right.$ *1456a13τ13 +β−α2140a6a23τ23 −β−α700aτ.......dτ…..*(33)

Also, substituting the values of b_0, b_1, b_2 ….. from(24) and η, η^2…. From (26) ,

we get, $b_0\eta + b_1\eta^2 + b_2\eta^3 + \cdots$

$= a\left(\frac{6}{a}\right)^{\frac{1}{3}} \tau^{1/3} + \frac{29}{10}(\beta - \alpha)\tau +$

$(a/60)\left(\frac{6}{a}\right)^{\frac{1}{3}} \tau^{4/3} +$

$(73/700)(\beta - \alpha)^2 \left(\frac{6}{a}\right)^{\frac{2}{3}} \tau^{5/3} +$

$(1469/1400)(\beta - \alpha)\tau^2 + \ldots$.(34)

Using (33),(34) in (32) and nothing that

$F(\eta) = \tau$ we get ,

$\delta_1 = \left(\frac{2\nu X}{U_\infty}\right)^{1/2} \left[a/3 \left(\frac{6}{a}\right)^{\frac{2}{3}} \Gamma\left(\frac{2}{3}\right) + \left(\frac{6}{a}\right)^{\frac{1}{3}} \frac{13}{45}(\beta - \right.$ *αΓ13 + 6a23(a/54)Γ2/3 +927/350aβ−α2* + $(5182/28350)(\beta - \alpha)\left(\frac{6}{a}\right)^{\frac{1}{3}} \Gamma(1/3)$ + $(a/$ *24306a23Γ2/3*

$+\left(\frac{4847}{21000}\right)\frac{(\beta-\alpha)^2}{a} +$

$\frac{2(\beta-\alpha)^3}{175a}\left(\frac{6}{a}\right)^{\frac{2}{3}} \Gamma(2/3) + \ldots$ (35)

IJASRM
International Journal of Advanced Scientific Research and Management, Volume 3 Issue 6, June 2018
www.ijasrm.com
ISSN 2455-6378

α	β
.1	**.5**
.2	**.5**
.3	**.5**
.4	**.5**

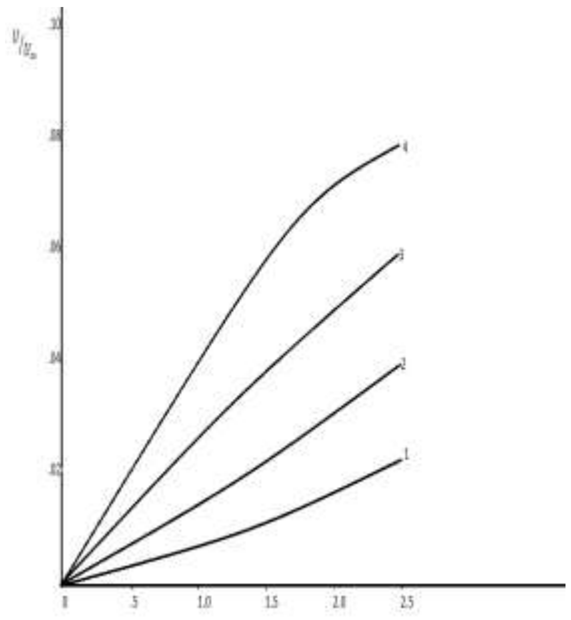

Fig.1 Velocity profile defined by eqn.(11)

Result and Discussion: Whole system behaves as in case of solid body rotation with constant angular velocity (α) about y axis normal to the plate. These velocity finds due to external forces for example –motor

The curve are given in figure for the velocity non-dimensional$\left(U/_{U_\infty}\right)$ as a function of η

For values α i.e. α = .01,0.2 ,.3 and .4 at fixed value of β=.5, the curves 1,2,3,4 show that angular velocity has increasing effect on the velocity component $\left(U/_{U_\infty}\right)$.Again as the expansion of displacement thickness δ_1 contents all positive terms it is evident that for the approximation considered in the sense of the number of terms retained in the expression for effect of decreasing the displacement thickness. It is interesting to note the fluid velocity increase with velocity. Conflicts of interest: The authors declared that there are no conflicts of interest.

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