

A study of a parallel fan standby redundant system operating in a power plant

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Abstract

The study deals with the reliability and profit evaluation of a two unit parallel fan standby redundant system operating in a power plant system. The present paper deals with a subsystem of a power plant system which consists of fans: PA (Primary Air) fan, FD (Forced Draft) fan and ID (Induced Draft) fan. A standby redundant parallel system of fans is provided to increase the efficiency of the power plant system. Whenever failure occurs in any of the primary set of fans, the standby redundant set of fans attached parallel comes into operation in order to pursue unhindered working of power plant system. There is a single repairman facility available. Various measures of system effectiveness such as MTSE, Availability, and Profit etc. have been computed for the model. Graphical study with their interpretation has also been done for the present study.

Keywords— Standby systems, Semi Markov process, Regenerative point technique.

1 Introduction

Redundancy is the technique of achieving higher reliability or it can be called as a backup system. Standby redundant system refers to a system where one or more identical unit(s) remains idle until the failure occurs in the primary unit. The standby systems play a vital role while studying reliability engineering. To increase the reliability of any system, standby systems are required in order to pursue unhindered operation of the system. The literature of reliability engineering holds numerous studies regarding standby systems being considered under different circumstances with related aspects. Researchers [1-7] have dealt with systems regarding different failure modes, repair and inspection being done to them. But there are few studies related to the fans of the power plant system. Our motive is to fill this gap.

The system comprises of a primary set of fans: one PA fan, one FD fan and one ID fan. There is an identical set of fans which acts as cold standby unit. It becomes operative whenever failure occurs in any of the fan in primary set. When failure occurs in any of the primary set of fans, other two fans in the primary

set gets switched off and the cold standby unit comes into operation in order to continue the working of power plant system. The system completely shuts down when failure occurs in any of the fan in main as well as in any of the fan in cold standby unit. There is single repairman available to do the required job. The job is done on First-cum-First-serve (FCFS) basis. Various measures of system effectiveness have been computed considering the particular case. Also, the graphical study has been done for the model.

2 Notations

λ_i	Constant failure rate of main PA/FD/ID fan ($i = 1,2,3$)
λ_j	Constant failure rate of cold standby PA/FD/ID fan ($j = 4,5,6$)
$g_i(t)/ G_i(t)$	pdf/ cdf of repair time of main PA/FD/ID fan ($i = 1,2,3$)
$g_j(t)/ G_j(t)$	pdf/ cdf of repair time of cold standby PA/FD/ID fan ($j = 4,5,6$)
$PA_0/ ID_0/ FD_0$	Main PA/ FD/ ID fan is in operative state
$CS_P/ CS_F/ CS_I$	PA/ FD/ ID fan is in cold standby state
$PA_S/ FD_S/ ID_S$	PA/ FD/ ID fan is in switched off state
$O_P/ O_F/ O_I$	PA/ FD/ ID fan become operative from cold standby state
$F_{rP}/ F_{rF}/ F_{rI}$	Cold standby PA/ FD/ ID fan is under repair
$F_{wrP}/ F_{wrF}/ F_{wrI}$	Cold standby PA/ FD/ ID fan is waiting for its repair respectively
$PA_r/ FD_r/ ID_r$	Main PA/ FD/ ID fan is under repair respectively
$PA_R/ FD_R/ ID_R$	Main PA/ FD/ ID fan under repair from previous state respectively

3 Model Description and State Transition Diagram

A state transition diagram in fig. 1 shows various transitions of the system. The epochs of entry into states 0, 1, 2, 3, 13, 14 and 15 are regenerative points and thus these are regenerative states. The states 4, 5, 6, 7, 8, 9, 10, 11 and 12 are failed states.

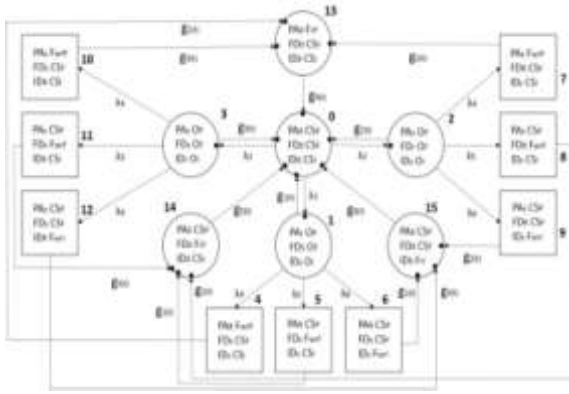


Fig. 1

○ Operating State □ Failed State

4 Transition Probabilities and Mean Sojourn Times

The non-zero elements p_{ij} , are obtained as under:

$$\begin{aligned}
 p_{01} &= \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} & p_{02} &= \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3} \\
 p_{03} &= \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} & p_{10} &= \\
 &= g_1^*(\lambda_4 + \lambda_5 + \lambda_6) \\
 p_{14} &= \frac{\lambda_4 [1 - g_1^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{1,13}^{(4)} \\
 p_{15} &= \frac{\lambda_5 [1 - g_1^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{1,14}^{(5)} \\
 p_{16} &= \frac{\lambda_6 [1 - g_1^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{1,15}^{(6)} \\
 p_{20} &= \frac{g_2^*(\lambda_4 + \lambda_5 + \lambda_6)}{\lambda_4 + \lambda_5 + \lambda_6} \\
 p_{27} &= \frac{\lambda_4 [1 - g_2^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{2,13}^{(7)} \\
 p_{28} &= \frac{\lambda_5 [1 - g_2^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{2,14}^{(8)} \\
 p_{29} &= \frac{\lambda_6 [1 - g_2^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{2,15}^{(9)} \\
 p_{30} &= \frac{g_3^*(\lambda_4 + \lambda_5 + \lambda_6)}{\lambda_4 + \lambda_5 + \lambda_6} \\
 p_{3,10} &= \frac{\lambda_4 [1 - g_3^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{3,13}^{(10)} \\
 p_{3,11} &= \frac{\lambda_5 [1 - g_3^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{3,14}^{(11)} \\
 p_{3,12} &= \frac{\lambda_6 [1 - g_3^*(\lambda_4 + \lambda_5 + \lambda_6)]}{\lambda_4 + \lambda_5 + \lambda_6} = p_{3,15}^{(12)} \\
 p_{4,13} &= g_1^*(0) = p_{5,14} = p_{6,15} \\
 p_{7,13} &= g_2^*(0) = p_{8,14} = p_{9,15} \\
 p_{10,13} &= g_3^*(0) = p_{11,14} = p_{12,15} \\
 p_{13,0} &= g_4^*(0) & p_{14,0} &= g_5^*(0) & p_{15,0} &= g_6^*(0)
 \end{aligned}$$

By these transition probabilities, it can be verified that

$$\begin{aligned}
 p_{01} + p_{02} + p_{03} &= 1 \\
 p_{10} + p_{14} + p_{15} + p_{16} &= 1
 \end{aligned}$$

$$\begin{aligned}
 p_{10} + p_{1,13}^{(4)} + p_{1,14}^{(5)} + p_{1,15}^{(6)} &= 1 \\
 p_{20} + p_{27} + p_{28} + p_{29} &= 1 \\
 p_{20} + p_{2,13}^{(7)} + p_{2,14}^{(8)} + p_{2,15}^{(9)} &= 1 \\
 p_{30} + p_{3,10} + p_{3,11} + p_{3,12} &= 1 \\
 p_{30} + p_{3,13}^{(10)} + p_{3,14}^{(11)} + p_{3,15}^{(12)} &= 1 \\
 p_{4,13} = 1 = p_{5,14} = p_{6,15} = p_{7,13} = p_{8,14} = p_{9,15} \\
 &= p_{10,13} = p_{11,14} = p_{12,15} \\
 p_{13,0} = 1 = p_{14,0} = p_{15,0}
 \end{aligned}$$

The unconditional mean time taken by the system to transit for any regenerative state j , when it is counted from epoch of entrance into that state i , is mathematically stated as –

$$\begin{aligned}
 m_{ij} &= \int_0^\infty t dQ_{ij}(t) = -q_{ij}'(0), \text{ Thus -} \\
 m_{01} + m_{02} + m_{03} &= \mu_0 \\
 m_{10} + m_{14} + m_{15} + m_{16} &= \mu_1 \\
 m_{10} + m_{1,13}^{(4)} + m_{1,14}^{(5)} + m_{1,15}^{(6)} &= f_1 \\
 m_{20} + m_{27} + m_{28} + m_{29} &= \mu_2 \\
 m_{20} + m_{2,13}^{(7)} + m_{2,14}^{(8)} + m_{2,15}^{(9)} &= f_2 \\
 m_{30} + m_{3,10} + m_{3,11} + m_{3,12} &= \mu_3 \\
 m_{30} + m_{3,13}^{(10)} + m_{3,14}^{(11)} + m_{3,15}^{(12)} &= f_3 \\
 m_{13,0} = c_1 & & m_{14,0} = c_2 & & m_{15,0} \\
 &= c_3
 \end{aligned}$$

Where,

$$\begin{aligned}
 f_1 &= \int_0^\infty \overline{G_1}(t) dt & f_2 &= \int_0^\infty \overline{G_2}(t) dt \\
 f_3 &= \int_0^\infty \overline{G_3}(t) dt \\
 c_1 &= \int_0^\infty \overline{G_4}(t) dt & c_2 &= \int_0^\infty \overline{G_5}(t) dt \\
 c_3 &= \int_0^\infty \overline{G_6}(t) dt
 \end{aligned}$$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have -

$$\begin{aligned}
 \mu_0 &= \frac{1}{\lambda_1 + \lambda_2 + \lambda_3} \\
 \mu_1 &= \frac{1 - g_1^*(\lambda_4 + \lambda_5 + \lambda_6)}{\lambda_4 + \lambda_5 + \lambda_6} \\
 \mu_2 &= \frac{1 - g_2^*(\lambda_4 + \lambda_5 + \lambda_6)}{\lambda_4 + \lambda_5 + \lambda_6} \\
 \mu_3 &= \frac{1 - g_3^*(\lambda_4 + \lambda_5 + \lambda_6)}{\lambda_4 + \lambda_5 + \lambda_6} \\
 \mu_4 &= -g_1^*(0) = \mu_5 = \mu_6 \\
 \mu_7 &= -g_2^*(0) = \mu_8 = \mu_9 \\
 \mu_{10} &= -g_3^*(0) = \mu_{11} = \mu_{12} \\
 \mu_{13} &= -g_4^*(0) & \mu_{14} &= -g_5^*(0) & \mu_{15} \\
 &= -g_6^*(0)
 \end{aligned}$$

5 Mean Time to System Failure

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \frac{N}{D}$$

Where

$$N = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03}$$

$$D = 1 - p_{01} p_{10} - p_{02} p_{20} - p_{03} p_{30}$$

6 Expected Up-Time of the system

The steady state availability of the system is given by:

$$A_0 = \frac{N_1}{D_1}$$

Where

$$N_1 = M_0 + M_1 p_{01} + M_2 p_{02} + M_3 p_{03} + p_{01} [M_{13} p_{1,13}^{(4)} + M_{14} p_{1,14}^{(5)} + M_{15} p_{1,15}^{(6)}] + p_{02} [M_{13} p_{2,13}^{(7)} + M_{14} p_{2,14}^{(8)} + M_{15} p_{2,15}^{(9)}] + p_{03} [M_{13} p_{3,13}^{(10)} + M_{14} p_{3,14}^{(11)} + M_{15} p_{3,15}^{(12)}]$$

$$D_1 = \mu_0 + p_{01} [f_1 + c_1 p_{1,13}^{(4)} + c_2 p_{1,14}^{(5)} + c_3 p_{1,15}^{(6)}] + p_{02} [f_2 + c_1 p_{2,13}^{(7)} + c_2 p_{2,14}^{(8)} + c_3 p_{2,15}^{(9)}] + p_{03} [f_3 + c_1 p_{3,13}^{(10)} + c_2 p_{3,14}^{(11)} + c_3 p_{3,15}^{(12)}]$$

7 Busy Period of Repairman

The steady state busy period of the system is given by:

$$B_R = \frac{N_2}{D_1}$$

Where

$$N_2 = W_1 p_{01} + W_2 p_{02} + W_3 p_{03} + p_{01} [W_{13} p_{1,13}^{(4)} + W_{14} p_{1,14}^{(5)} + W_{15} p_{1,15}^{(6)}] + p_{02} [W_{13} p_{2,13}^{(7)} + W_{14} p_{2,14}^{(8)} + W_{15} p_{2,15}^{(9)}] + p_{03} [W_{13} p_{3,13}^{(10)} + W_{14} p_{3,14}^{(11)} + W_{15} p_{3,15}^{(12)}]$$

And D_1 is already specified.

8 Expected No. of Visits of Repairman

The steady state expected no. of visits of the repairman is given by:

$$V_R = \frac{N_3}{D_1}$$

Where

$$N_3 = p_{01} + p_{02} + p_{03} = 1$$

And D_1 is already specified.

9 Profit Analysis

The expected profit incurred of the system is –

$$P = C_0 A_0 - C_1 B_R - C_2 V_R$$

C_0 = Revenue per unit up time of the system
 C_1 = Cost per unit up time for which the repairman is busy in doing repair
 C_2 = Cost per visit of the repairman

10 Graphical Interpretation and Conclusion

For graphical analysis following particular cases are considered:

$$g_1(t) = \beta_1 e^{-(\beta_1)t} dt \quad g_2(t) = \beta_2 e^{-(\beta_2)t} dt$$

$$g_3(t) = \beta_3 e^{-(\beta_3)t} dt \quad g_4(t) = \beta_4 e^{-(\beta_4)t} dt$$

$$g_5(t) = \beta_5 e^{-(\beta_5)t} dt \quad g_6(t) = \beta_6 e^{-(\beta_6)t} dt$$

$$p_{01} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \lambda_3} \quad p_{02} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$p_{03} = \frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \quad p_{10} = \frac{\beta_1}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_1}$$

$$p_{14} = \frac{\lambda_4}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_1} = p_{1,13}^{(4)}$$

$$p_{15} = \frac{\lambda_5}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_1} = p_{1,14}^{(5)}$$

$$p_{16} = \frac{\lambda_6}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_1} = p_{1,15}^{(6)}$$

$$p_{20} = \frac{\beta_2}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_2}$$

$$p_{27} = \frac{\lambda_4}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_2} = p_{2,13}^{(7)}$$

$$p_{28} = \frac{\lambda_5}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_2} = p_{2,14}^{(8)}$$

$$p_{29} = \frac{\lambda_6}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_2} = p_{2,15}^{(9)}$$

$$p_{30} = \frac{\beta_3}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_3}$$

$$p_{3,10} = \frac{\lambda_4}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_3} = p_{3,13}^{(10)}$$

$$p_{3,11} = \frac{\lambda_5}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_3} = p_{3,14}^{(11)}$$

$$p_{3,12} = \frac{\lambda_6}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_3} = p_{3,15}^{(12)}$$

$$\mu_0 = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3} \quad \mu_1 = \frac{1}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_1}$$

$$\mu_2 = \frac{1}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_2} \quad \mu_3 = \frac{1}{\lambda_4 + \lambda_5 + \lambda_6 + \beta_3}$$

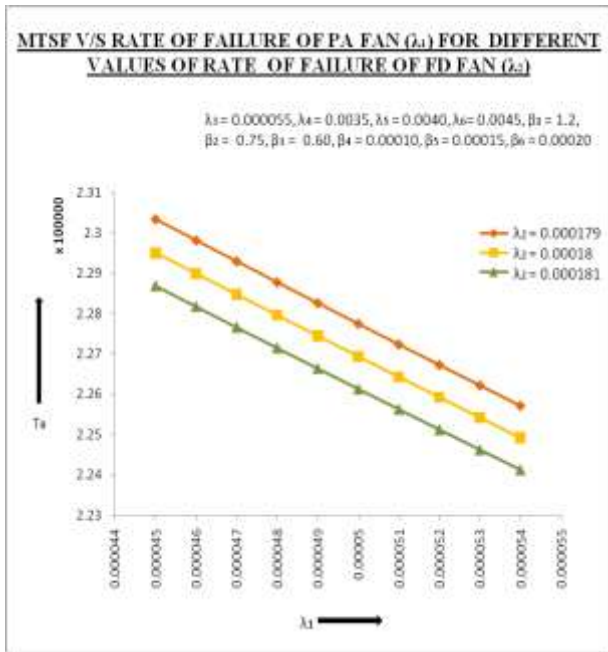


Fig. 2

Fig. 2 shows the behaviour of MTSF w.r.t. failure rate of PA fan (λ_1) for different values of rate of failure of FD fan (λ_2). It is clear from the graph that MTSF gets decreased with the increase in the values of the failure rate of PA fan (λ_1). Also, the MTSF decreases as failure rate of FD fan (λ_2) increases.

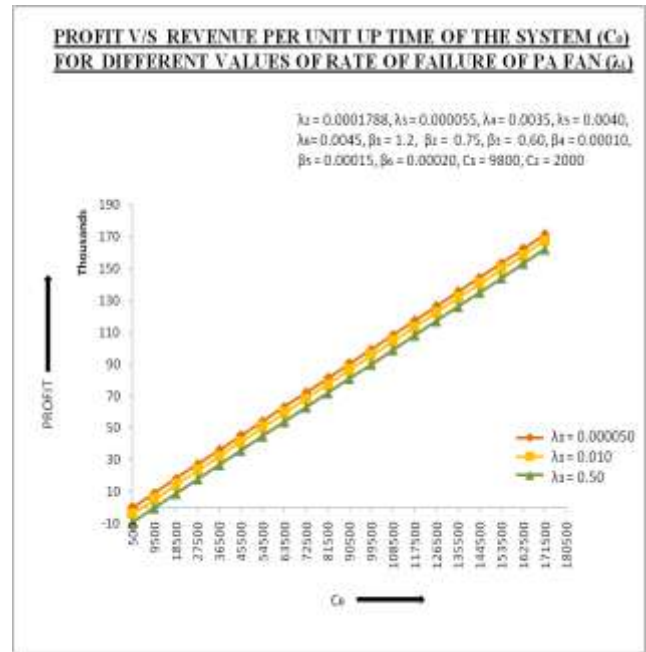


Fig. 4

Fig. 4 depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate of failure of PA fan (λ_1). It can be interpreted that the profit increases with increase in the values of C_0 . Following conclusions can be drawn from the graph:

1. For $\lambda_1 = 0.000050$, profit is $>$ or $=$ or $<$ according as $C_0 >$ or $=$ or $<$ 297, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 297 to get positive profit.
2. For $\lambda_1 = 0.010$, profit is $>$ or $=$ or $<$ according as $C_0 >$ or $=$ or $<$ 4152, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 4152 to get positive profit.
3. For $\lambda_1 = 0.50$, profit is $>$ or $=$ or $<$ according as $C_0 >$ or $=$ or $<$ 9565, i.e., i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 9565 to get positive profit.

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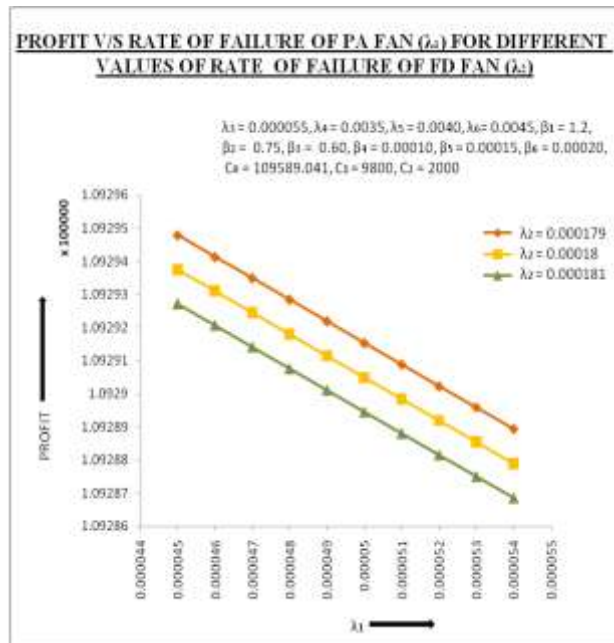


Fig. 3

The above graph shows the behaviour of profit w.r.t. to failure rate of PA fan (λ_1) for different values of failure rate of FD fan (λ_2). The figure depicts that as the values of failure rate of PA fan (λ_1) increases, profit decreases. It is also seen that the profit decreases as failure rate of FD fan (λ_2) increases.

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