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A study on g*α-Compact and g*α-Connected Spaces

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Abstract

In this paper, we introduce the concept of g*α-compact space and g*α-connected space. We also investigate their basic properties. We also discuss their relationship with already existing concepts.

Keywords: *g*α-compact, g*α-separated, g*α-connected*

1. Introduction:

The notion of Compactness and connectedness are useful for fundamental notions of not only general topology but also of other advanced branches of Mathematics. Many researchers have investigated the basic properties of compactness and connectedness. In 1974,Das defined the concept of Semi connectedness in topology and investigated its properties. In 1981 Dorsett introduced and studied the concept of Semi compact spaces. Since then, Hanna and Dorsett , Ganster and Mohammad S. Sursak investigated the properties of semi compact spaces. The aim of this paper is to introduce the concept of g*α-connectedness and g*α-compactness in topological spaces.

2. Preliminaries:

2.1. Definition A subset A of (X,τ) is called

(i) g-closed**[5]** if cl(A)⊆U whenever A⊆U and U is open in (X,τ)

(ii) g**-closed**[9]** if cl(A)⊆U whenever A⊆U and U is g*-open in (X,τ)

2.2. Definition A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is said to be

(i) g*α-continuous**[10]** if $f^{-1}(V)$ is g*α Closed (g*α-open)set of (X,τ) for every Closed (open) set V of (Y,σ).

(ii) g*α-irresolute**[10]** if $f^{-1}(V)$ is g*α closed set of (X,τ) for every g*α closed set V of (Y,σ)

(iii) g*α-resolute**[10]** if f(U) is g*α-open in Y whenever U is g*α-open in X.

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2.3. Definition Let A be a subset of X. Then the g*α-closure of A is defined as the intersection of all g*α-closed sets containing A and it is denoted by g*α-cl(A)

2.4[10].Theorem

(i)Every open set is g*α-open and every

closed set is g*α-closed

(ii)Every g**-open set is g*α-open and

every g**-closed set is g*α-closed

(iii)A Space (X,τ) is ${}_{\alpha}T^{**}_{1/2}$ space if every

g*α-closed set is closed.

2.7. Definition A topological space X is said to be connected **[4]** if X cannot be expressed as the union of two disjoint nonempty open sets in X.

2.8.Definition A collection B of open sets in X is called open cover**[6]** of A⊆X if A⊆∪{$\cup_{\alpha}$: $\cup_{\alpha}$∈B}holds.

2.9. Definition A Space X is said to be compact**[4]** if every open cover of X has finite subcover.

3. g*α-Compactness:

In this section we introduce the notion of g*α-compactness and investigate its properties.

3.1. Definition A subset S of a topological space X is g*α-compact relative to X if for every collection {A_{α} :α∈ Ω}of g*α-open subsets of X such that S⊆ $\bigcup_{\alpha\in\Omega} A_{\alpha}$ there exists a finite subset Δ of Ω such that S ⊆ $\bigcup_{\alpha\in\Delta} A_{\alpha}$ .If S=X and if S is g*α-compact relative to X then X is g*α-compact.

A subset S of a topological space X is g*α-compact if it is g*α-compact as a subspace of X.

3.2. Proposition A g*α-closed subset S of a g*α-compact space X is g*α-compact relative to X. Then

Proof: Let S be a g*α-closed subset of a g*α-compact space X. Then X\S is g*α-open. Let ξ be a cover for S by g*α-open subsets of X.Since X is g*α-Compact it has finite subcover say{B_1,B_2, B_3.... B_n}.Then {B_1,B_2, B_3.... B_n}∪{X\S} is a finite subcover of ξ for S.Thus S is g*α-compact relative to X.

3.3. Theorem Let (X,τ) and (Y,σ) be topological spaces and f:(X,τ)→(Y,σ) be a function. Then

(i) f is g*α-irresolute and A is g*α-compact subset of X ⇒ f(A) is g*α-compact subset of Y.

(ii)f is one-one, g*α-resolute map and B is a g*α-compact subset of Y⇒ f^{-1}(B) is g*α-compact subset of X.

Proof: Proof follows from definition 2.2((ii) and (iii)).

3.4. Theorem Let (X,τ) and (Y,σ) be topological spaces and f:(X,τ)→(Y,σ) be a function.

(i)f is onto,g*α-irresolute and X is g*α-

Compact⇒Y is g*α-Compact

(ii)f is bijection and g*α-resolute then Y is

g*α-Compact⇒X is g*α-Compact

Proof (i) Let {Vα: α∈J } be a g*α-open cover for Y. Then {f^{-1}(Vα): α∈J }is a g*α-open cover for X. Since X is g*α-compact, there existsVα_1 , Vα_2 , Vα_3 ,... ,Vα_n such that f^{-1}(Vα_1)∪ f^{-1}(Vα_2)∪)∪ f^{-1}(Vα_3)....... f^{-1}(Vα_n)=X.
Now f(f^{-1}(Vα_1) ∪ f^{-1}(Vα_2) ∪ f^{-1}(Vα_3)....... f^{-1}(Vα_n))= f(X).
f(f^{-1}(Vα_1) ∪ f^{-1}(Vα_2) ∪ f^{-1}(Vα_3)....... f^{-1}(Vα_n))=f(X).That is

f(X) = ff^{-1}(Vα $_1$) ∪ ff^{-1}(Vα $_2$) ∪ ff^{-1}(Vα $_3$)……. ff^{-1}(Vα $_n$)) ⊆ Vα $_1$ ∪ Vα$_2$∪ Vα $_3$……. Vα $_n$
since f is onto, f(X)=Y.Therefore Y is g*α-compact.
(ii)Proof is similar to that of (i)

3.5. Theorem Let (X,τ) be a topological space.X is g*α-compact iff any family of g*α-closed subsets of X with finite intersection property has nonempty intersection.

Proof: Suppose X is g*α-compact.Let {A_α} be a family of g*α-closed subsets of X with finite intersection property.We claim that $\cap_\alpha A_\alpha \neq \emptyset$.Suppose $\cap_\alpha A_\alpha = \emptyset$ .Then X\($\cap_\alpha A_\alpha$) =X. Therefore $\cup_\alpha(X\setminus A_\alpha)$ =X.Also since each $A_\alpha$is g*α-closed,X\ A_α is g*α-open.Therefore $\{X\setminus A_\alpha\}$ is a cover for X by g*α-open sets of X.Since X is g*α-compact,this cover has a finite subcover say {X\$A_1$, X\$A_2$…..X\ A_n}.Therefore $\cup_\alpha(X\setminus A_\alpha)$ =X. It follows that X\($\cap_\alpha A_\alpha$) =X which implies $\cap_{\alpha=1}^{n} A_\alpha$=∅ which is a contradiction to the finite intersection property.Hence $\cap_\alpha A_\alpha \neq \emptyset$.

Conversely suppose that each family of g*α-closed sets with finite intersection property has nonempty intersection.we wish to prove that X is g*α-compact.Let $\{A_\alpha : \alpha \in \Omega\}$be a cover of X by g*α-open sets.Then $\cup_{\alpha\in\Omega} A_\alpha$ =X that implies X\ $\cup_{\alpha\in\Omega} A_\alpha$ = ∅ .Hence $\cap_{\alpha\in\Omega}(X\setminus A_\alpha)$= ∅ .since each A_α is g*α-open, X\ A_α is g*α-closed for each α.Therefore(X\$A_\alpha$: $\alpha \in \Omega$}is a family of g*α-closed sets whose intersection is empty.By hypothesis there exists a finite subcollection of g*α-closed subsets of X say {X\ A_1 ,X\ A_2 ….. X\$A_n$ } such that $\cap_{\alpha=1}^{n} A_\alpha$=∅ that implies X\($\cup_{\alpha=1}^{n} A_\alpha$)=∅. Which implies $\cup_{\alpha=1}^{n} A_\alpha$ =X.This proves that X is compact.

4.g*α-connectedness:

4.1. Definition Let (X,τ) be a topological space.Two nonempty subsets A and B of X are said to be g*α-separated if A∩g*αcl(B)=∅ and g*α-cl(A)∩B=∅.

4.2. Example Consider the topology

τ = { ∅, X,{1},{2},{1,2},{2,3},{1,2,3}}on X={1,2,3,4}.Let A={2},B={1,3},C={1,2,4} be the subsets of X.Then g*αcl-(A)={2,4}, g*α-cl(B) ={1,3,4} and g*α-cl(c)={1,2,4}.A∩g*α-cl(B)=∅ and g*α-cl(A)∩B=∅.This shows that A and B are g*α-separated sets.But B∩ g*α-cl(C)={1} and g*α-cl(B)∩C={1,4}

Hence B and C are not g*α-separated sets.

4.3. Proposition If A and B are g*α-separated then they are disjoint.

Proof: A∩B ⊆ g*α-cl(A)∩B=∅.

4.4. Proposition If A and B are g*α-separated subsets of a space X and C ⊆ A and D ⊆B then C and D are also g*α-separated.

Proof: Suppose A and B are g*α-separated subsets of a space X, by using Definition 4.1 A∩g*α-cl(B)=∅ and g*α-cl(A)∩B=∅. since C ⊆ A, we have g*α-cl(C) ⊆ g*α-cl(A) and since D ⊆ B, g*α-cl(D) ⊆ g*α-cl(B). Therefore C∩g*α-cl(D)= A∩g*α-cl(B) = ∅ and g*α-cl(C) ∩D ⊆ g*α-cl (A)∩B= ∅ .Hence C and D are g*α-separated sets.

4.5.Proposition If A is g*α-open and A∩B=∅ then A∩g*α-cl(B)=∅

Proof: Suppose A∩g*α-cl(B) ≠ ∅.Choose x ∈ A∩g*α-cl(B).Then x∈ A and x∈ g*α-cl(B).It follows that x∈ B' where B' is a g*α –closed superset of B.In particular x∈ X\A that is x∉A which is a contradiction to x∈A.Hence A∩g*α-cl(B)=∅.

4.6 Definition A topological space (X,τ) is said to be g*α-connected if X is not expressed as a union of two nonempty g*α-separated subsets of X.otherwise X is said to be g*α-disconnected

4.7.Example (i) Let X={1,2,3,4}.Consider the topology τ={∅,X,{1},{1,2}{1,4},{1,2,3},{4},{1,2,4}}.Here X is not g*α-connected and not connected.

(ii) Consider the Sierpinski topology

τ = {∅,X,{1}}.Then g*α-C(X,τ)={∅,X,{2}}. Here X is both g*α-connected and connected.

(iii) Consider the finite excluded point topology τ ={∅,X,{3},{1,3,4}}on X={1,2,3,4}.Theng*α-C(X,τ)= {∅,X,{1},{2},{4},{1,2},{1,4},{2,3},{2,4},

{1,2,3},{2,3,4},{1,2,4}} Here X is connected but not g*α- connected.

4.8. Theorem For a topological space X the following are equivalent.

(i)X is g*α-connected.

(ii)X cannot be expressed as the union of two nonempty disjoint g*α-open sets.

(iii) The only subsets of X which are both g*α-open and g*α-closed are the empty set ∅ and X.

(iv) Each g*α-continuous function of X into discrete space Y with atleast two points is a constant map.

Proof: Suppose (i) hold. Let X=A∪B where A and B be the nonempty disjoint g*α-open sets.By prop4.5 A∩g*α-cl(B)=∅ and B∩g*-cl(A)=∅.By Definition 4.1 A and B are g*α-separated sets of X.Therefore X is not g*α-connected.This is a contradiction to our assumption. This proves (i) ⇒(ii)

Now to prove (ii)⇒(i)Suppose (ii) holds.If X is not g*α-connected then by defn3.6,X can be expressed as a union of two nonempty disjoint g*α-separated sets. This proves (ii)⇒(i)

Now to prove(ii) ⇒ (iii).Suppose (ii) holds.Let A be a subset of X which is both g*α-open and g*α-closed.Then X\A is both g*α-open and g*α-closed. Suppose A ≠ ∅ and A ≠ X.Since A ≠ ∅ we have X\A ≠ X.And since A ≠ X,we have X\A ≠ ∅.Therefore X=A∪(X\A) is a disjoint union of nonempty g*α-open sets.This contradicts to(ii).Hence A= ∅ or X.This proves (ii) ⇒(iii).

Suppose (iii) holds. Let f:X→Y is a g*α-continuous function where Y is a discrete space with atleast two points. Fix y_0 ∈ Y such that f(x_0) = y_0 for some x_0 ∈ X.Since Y is discrete, {y_0} is both closed and open in Y.since f is g*α-continuous,$f^{-1}(\{y_0\})$ is both g*α-closed and g*α-open by using Definition 2.2and by (iii) $f^{-1}(\{y_0\})$ = ∅ or Since $f^{-1}(\{y_0\}) \neq \emptyset$,therefore$f^{-1}(\{y_0\})$ = X.That is f(x) = y_0 for x∈X.This implies that f is a constant map.This implies (iii) ⇒(iv)

Suppose (iv) holds: Suppose X is not g*α-connected. Let X=A∪B where A≠ ∅,B≠ ∅, A∩B=∅,A and B are g*α-open set. Let Y be discrete space and |y| >1.Fix y_0 and $y_1$in Y and $y_0 \neq y_1$.Define f:X→Y with

$$f(x) = \begin{cases} y_0 \; if \; x \in A \\ y_1. \text{if x } \in B \end{cases}$$

Let V be an open set in Y.

$$\text{Then } f^{-1}(V) = \begin{cases} \varphi \; if \; y_0 \notin V, y_1 \notin V \\ X \; if \; y_0 \in V, y_1 \in V, \\ A \; if \; y_0 \in V, y_1 \notin V \\ B \; if \; y_0 \notin V, y_1 \in V \end{cases}$$

Therefore f is g*α-continuous but f is not a constant map. This is a contradiction to (iv).

This proves (iv)⇒(i)

4.9. Definition A subset A of X is g*α-connected if A cannot be written as the union of two nonempty disjoint g*α-open subsets of X.

4.10. Proposition (i) If a space X is g*α-connected then it is connected.

(ii) If a space X is g*α-connected then it is g**connected.

Proof: (i) Let X be g* α-connected. Suppose X is not connected there exists disjoint nonempty open sets A and B such that X=A∪B. By Theorem 2.4(i) A and B are g*α-open sets. This is a contradiction to X is g*α-connected. This proves (i)

(ii) Let X be g* α-connected. Suppose X is not g**connected there exists disjoint nonempty open sets A and B such that X=A∪B. By Theorem 2.4(ii) A and B are g*α-open sets. This is a contradiction to X is g*α-connected. This proves (ii)

4.11.Remark The converse of the above theorem need not be true as shown by the following example**.**

Consider the space (X, τ) where X={1,2,3,4} and τ = { ∅, X,{3},{1,3,4}}.Then (X, τ) is connected and g**-connected but not g*α-connected.

4.12. Theorem If f: X→Y is g*α-continuous surjection and X is g*α-connected then Y is connected.

Proof: Suppose that Y is not connected. Let Y=A∪B where A and B are disjoint nonempty open sets in Y.Since f is g* α-continuous and onto,X= f^{-1}(A)∪ f^{-1}(B) where f^{-1} (A) and f^{-1} (B) are disjoint nonempty g*α-open sets in X. This contradicts the fact that X is g*α-connected. Hence Y is connected.

4.13. Theorem If f: X→Y is g*α-irresolute surjection and X is g*α-connected then Y is g*α-connected.

Proof: Suppose that Y is not g*α-connected. Let Y=A∪B where A and B are disjoint nonempty g*α-open sets in Y. Since f is g*α-irresolute and onto, X= f^{-1} (A) ∪ f^{-1} (B) where f^{-1} (A) and f^{-1} (B) are disjoint nonempty g*α-open sets in X.This contradicts the fact that X is g*α-connected.Hence Y is g*α-connected.

4.14. Theorem Suppose that X is $_{\alpha}T^{**}_{1/2}$ then X is connected iff it is g*α-connected.

Proof: Suppose that X is connected. Then X cannot be expressed as disjoint union of two nonempty proper subsets of X. Suppose X is not a g*α-connected space. Let A and B any two g*α-open subsets of X such that X=A∪B where A∩B=∅ and A⊂X,B⊂X.Since X is $_{\alpha}T^{**}_{1/2}$ space and A,B are g*α-open,A,B are open subsets of X which contradicts that X is g*α-connected.

Conversely every open set is g*α-open. Therefore every g*α-connected space is connected.

4.15. Definition A subset A of X is g*α-connected if A cannot be written as the union of two nonempty disjoint g*α-open subsets of X.

4.16. Theorem Let (X,τ) be a topological space and let E⊆A∪B be g*α-connected where A and B are g*α-separated sets in X. Then E⊆A or E⊆B that is E cannot intersect both A and B.

Proof: since A and B are g*α-separated sets by defn 3.1, A∩g*α-cl(B)=∅ and g*α-cl(A)∩B=∅.Now E⊆ A∪B that implies E=E∩(A∪B) =(E∩A)∪(E∩B).We wish to prove that one of the sets E∩A and E∩B is empty. suppose none of these sets is empty that is E∩A ≠ ∅ and E∩B ≠ ∅.Then

(E ∩ A) ∩ g*α-cl(E ∩ B) ⊆ (E∩A)∩(g*α(cl(E)∩g*α-cl(B)) =(E∩g*α-cl(E))∩(A∩g*α-cl(B))=(E∩g*α-cl(E))∩∅ = ∅ similarly g*α-cl(E ∩ A) ∩ (E∩B)=∅.Hence E∩A and E∩B are g*α-separated sets. Thus E has been expressed as the union of two nonempty g*α-separated sets. Thus E is g*α-disconnected subset of X. But this is a contradiction to our assumption. Hence one of the sets E∩A and E∩B is empty. If E∩A=∅ then E = E∩B which implies that E ⊆B.Similarly if E∩B=∅ then E ⊆A.Therefore E⊆A or E⊆B.

4.17. Proposition If Y=∪ A_α where each A_α is a g*α-connected subset of X and ∩ $A_\alpha \neq \emptyset$ then Y is g*α-connected.

Proof: Let P be a point of ∩ A_α. Suppose Y = A ∪ B where A and B are g*α-separated by g*α-open sets in X. The point P is in one of the sets A or B. Suppose p∈A.since each A_α is g*α-connected by theorem 4.15, A_α ⊆A or A_α ⊆B.It cannot lie in B because it contains the point p of A.Therefore A_α ⊆A for all α. It follows that ∪ A_α ⊆A.That implies B= ∅.This is a contradiction. This proves that Y is g*α-connected.

4.18. Theorem Let A be a g*α-connected subset of X. If B is a subset of X such that A⊆B⊆g*α-cl(A) then B is g*α-connected. In particular g*α-cl(A) is g*α-connected set of X provided A is g*α-connected.

Proof: Suppose B is not a g*α-connected set of X.Then there exists a nonempty g*α-open sets C and D in X such that C∩g*α-cl(D) = ∅ ,g*α-cl(C) ∩ D = ∅ and C∪D=B.Since A⊆B =C∪D by theorem 4.15 we have

A⊆C or A⊆D. Let A⊆C which implies that g*α-cl(A) ⊆ g*α-cl(C) that implies

g*α-cl(A) ∩ D ⊆ g*α-cl(C) ∩ D = ∅.Therefore g*α-cl(A)∩D = ∅.Also C∪D =B ⊆ g*α-cl(A).That is D⊆B⊆ g*α-cl(A) which implies g*α-cl(A)∩D =D.Hence D=∅ which is a contradiction since D≠ ∅.Hence B must be g*α-connected.Again since A⊆B⊆g*α-cl(A) we have g*α-cl(A) is g*α-connected.

4.19. Theorem If every two points of a subset E of a topological space X are contained in some g*α-connected set of E, then E is g*α-connected.

Proof: Suppose E is not g*α-connected. By Defn 4.1 there exists two nonempty open sets A and B of X such that A∩g*α-cl(B) = ∅ , g* α -cl(A) ∩ B = ∅ and E=A∪B.Since A and B are nonempty there exists a,b with a∈A and b∈B.Since E ⊆ A ∪ B by theorem 4.15 E ⊆ A or E⊆B.Therefore a,b are both in A or both in B which is a contradiction since A,B are disjoint sets. Hence E must be g*α-connected.

4.20. Definition Let (X,τ) be a topological space.A maximal g*α-connected subset of X is called a g*α-component in X.

4.21.Example Let X={1,2,3,4} τ={∅,X,{2},{3}{2,3},{1,2} {1,2,3},{2,3,4}}.Then g*α-O(X,τ)=

{ ∅,X,{2},{3},{2,3},{1,2},{1,2,3},{2,3,4}}.

The g*α-components are {1,2,3},{2,3,4}

4.22. Proposition Let A be a g*α-component set in a topological space (X,τ).Then A= g*α-cl(A)

Proof: Let A be a g*α-component set of (X,τ).Since A is g*α-connected by thm 3.17 g*α-cl(A) is also g*α-connected. Further since A is a g*α-component, by definition 4.19 A is a maximal g*α-connected set. Hence g*α-cl(A) ⊆ A.But A⊆g*α-cl(A).Therefore A = g*α-cl(A).

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4.23. Proposition Let (X, τ) be a topological space.Then

(i)Each point in X is contained in exactly one g*α-component of X.

(ii)The g*α-component of X form a partition of X that is any two g*α-components are either disjoint or identical and the union of all g*α-components is X.

(iii)Each g*α-connected subset of X is contained in a g*α-component of X.

(iv)Each g*α-connected subset of X which is both g*α-open and g*α-closed is a g*α-component of X.

Proof: Let x be any arbitrary point of X. Let { A_α } be a collection of all g*α-connected subsets of X which contain x. This collection is nonempty. Since {x} is g*α-connected. Further $\cap A_\alpha \neq \emptyset$ since x is a point of each A_α.Hence $A_x = \cup A_\alpha$ is a g*α-connected by prop 4.16.Also A_x is maximal and contains x. For let B be any g*α-connected set such that $A_x \subseteq B$.Then $x \in B$.Therefore B is the one of the members of the collection $\{A_\alpha\}$.Hence $B \subseteq A_x$. Thus A_x=B. Hence A_x is a g*α-component of X containing x. Let be any other g*α-component of X containing x. Then A_x^* is one of the members of A_α's and so $A_x^* \subseteq A_x$. But since A_x^* is maximal as a g*α-connected subset of X, $A_x^* = A_x$. This proves (i)

Let C={ A_x:x∈X}where A_x is defined as in (i).We claim that C contains all the g*α-components of X.

By (i) each A_x∈C is a g*α-component.And if E is any other g*α-component, then E being nonempty contains some point x_0 ∈X and by (i), we have $E = A_{x_0} \in C$.Our aim is to prove that C forms a partition of X.Let Y=∪{ A_x:x∈X}.Let A_p and A_q be any two g*α-components such that $A_p \cap A_q \neq \emptyset$.we wish to prove that $A_p = A_q$.Let $r \epsilon A_p \cap A_q$.Then $r \epsilon A_p$ and $r \epsilon A_q$. Since A_p and A_q are g*α-connected subsets containing r and A_r is a g*α-component containg r.we have $Ap \subseteq Ar$ and $Aq \subseteq Ar$.But since A_p and Aq are g*α-components we have Ap = Ar = Aq.Hence g*α-components of X form a partition of X. This proves (ii)

Now to prove (iii) Let C be any g*α-connected subset of X. If c=∅ then C is contained in every g*α-component.If C≠ ∅ then C contains a point x_0 ∈X and so $C \subseteq A_{x_0}$by (i).This proves (iii).

Now to prove (iv) Let D be a g*α-connected subset of X which is both g*α-open and g*α-closed. By (iii) D ⊆E where E is a g*α-component of X and hence E is nonempty. We wish to prove that D=E. For if D≠E, then D is a proper subset of E which is both g*α-open and g*α-closed in E.Therefore by theorem 3.12 E is g*α-disconnected which is a contradiction.Hence D=E and so D is g*α-component.

4.24 Defintion Let (X,τ) be a topological space. Then X is (i)g*α-hyper connected if there exists no disjoint nonempty g*α-open sets

(ii)g*α-ultra connected if there exists no disjoint nonempty g*α-closed sets.

4.25.Example Consider the topology τ={ ∅ ,X,{1},{1,2}{1,3}} on X={1,2,3}.Now

g*α-O(X, τ)={ ∅ ,X,{1},{1,2},{1,3}}. Hence (X,τ) is g*α-hyper connected.

4.26. Example Consider X=(0,1/n) with usual topology. This space has no disjoint nonempty g*α-closed sets. Therefore X is g*α-ultra connected.

4.27. Proposition A space X is g*α-hyper connected if the g*α-closure of every g*α-open set is the entire space X.

Proof: Suppose X is not g*α-hyper connected. Let A and B be the nonempty g*α-open sets of X. Then A∩B=∅ that implies g*α-cl (A)∩B=∅.It follows that X∩B = ∅ .Hence B= ∅ which is a contradiction to B ≠ ∅.This proves the proposition.

4.28. Proposition A space X is g*α-ultra connected if g*α-closures of distinct points always intersect.

Proof: Suppose X is not g*α-ultra connected. Let A and B be the nonempty g*α-closed sets of X. Then A∩B=∅.Since A and B are nonempty there exists a,b with a ∈ A and b ∈ B.That implies g*α-cl({a}) ∩ g*α-cl({b})= ∅ which is a contradiction.This proves the proposition.

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