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Triple Connected Line Domination Number For Some Standard And Special Graphs

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Abstract

The concept of triple connected graphs with real life application was introduced by considering the existence of a path containing any three vertices of a graph G. A subset S of V of a non - trivial graph G is said to be a triple connected dominating set, if S is a dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all triple connected dominating set is called the triple connected domination number and is denoted by $\gamma_{tc}(G)$. A subset D of E of a non- trivial graph G is said to be a triple connected line dominating set, if D is an edge dominating set and the edge induced subgraph <D> is triple connected. The minimum cardinality taken over all triple connected line dominating set is called the triple connected line domination number and is denoted by $\gamma_{tcl}'(G)$. In this paper, we determine this number for some standard and special graphs.

Keywords: *Triple connected, Triple connected domination number of a graph, Triple connected line domination number of a graph.*

1. Introduction

All graphs considered here are finite, undirected without loops and multiple edges. Unless and otherwise stated, the graph G = (V, E) considered here have p = $|V|$ vertices and q = $|E|$ edges. A set S of vertices in a graph G is called a dominating set if every vertex in V – S is adjacent to some vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. As an analogy to vertex domination, the concept of edge domination was introduced by Mitchell and Hedetniemi. A set F of edges in a graph G is called an edge dominating set if every edge in E – F is adjacent to at least one edge in F. The edge domination number $\gamma'(G)$ of G is the minimum cardinality of an edge dominating set of G. An edge dominating set F of a graph G is a connected edge dominating set if the edge induced subgraph $\langle F\rangle$ is connected. The connected edge domination number $\gamma'_c(G)$ of G is the minimum cardinality of a connected edge dominating set of G. The concept of connected edge domination was introduced by Kulli and Sigarkanti. The concept of triple connected graphs with real life application was introduced by considering the existence of a path containing any three vertices of a graph G. G. Mahadevan et. al., introduced the concept of triple connected domination number of a graph. A subset S of V of a nontrivial connected graph G is said to be triple connected dominating set, if S is a dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number and is denoted by γ_{tc}. A subset D of E of a nontrivial connected graph G is said to be a triple connected line dominating set, if D is an edge dominating set and the edge induced subgraph <D> is triple connected. The minimum cardinality taken over all triple connected line dominating sets is called the triple connected line domination number of G and is denoted by $\gamma_{tcl}'(G)$.

2. Exact value for some standard graphs

2.1. Proposition

For any path P_p of order $p \geq 3$,

$$\gamma_{tcl}'(P_p) = \begin{cases} 2 & \text{if } p < 5 \\ q-2 & \text{if } p \geq 5 \end{cases}$$

Proof

Let P_p be a path of order $p \geq 3$ and let D be a triple connected line dominating set of P_p.

Case (i) $p < 5$

Since the existence of a triple connected line dominating set of a connected graph G implies that $\gamma_{tcl}'(G) \geq 2$, we need to show the existence of such a set D. For P_3 it is obvious. For P_4, take any two consecutive edges to be in D. Therefore, D is an edge dominating set and <D> = P_3 which is triple connected. Hence, $\gamma_{tcl}'(P_p) = |D| = 2$, if $p < 5$.

Case (ii) $p \geq 5$

The only connected subgraph of any path are paths, which are triple connected. If $|E - D| = 2$, then there are two subcases to be considered.

Subcase 1

If E – D contains a pendant edge and its neighbor, the pendant edge is not dominated by any edge in D, and hence D is not an edge dominating set.

Subcase 2

If E – D contains the two pendant edges, then D is an edge dominating set and and <D> = P_{p-2} Hence D is a triple connected line dominating set.

2.2. Proposition

For any cycle C_p of order $p \geq 3$,

$$\gamma_{tcl}'(C_p) = \begin{cases} 2 & \text{if } p = 3 \\ q-2 & \text{if } p \geq 4 \end{cases}$$

Proof

Let C_p be a cycle of order $p \geq 3$ and let D be a triple connected line dominating set of C_p

Case (i) p=3

Then <D> = P_3 which is triple connected. Hence, $\gamma_{tcl}'(C_p) = |D| = 2$.

Case (ii) $p \geq 4$

The only connected subgraphs of any cycle are paths, which are triple connected. If $|E - D| = 2$, then D is an edge dominating set and and <D> = P_{p-2}

Hence, D is a triple connected line dominating set. If $|E - D| > 2$, then take any three consecutive edges x, y, $z \in E - D$. The edge y is not dominated by any edge in D since the only neighbors of y are x and z. Therefore, exactly two edges can be removed from C_p to obtain a triple connected line dominating set of minimum cardinality.

2.3. Proposition

For the star graph $K_{1,p-1}$ of order $p \geq 3$, we have $\gamma_{tcl}'(K_{1,p-1}) = 2$

Proof

Let $K_{1,p-1}$ be a star of order $p \geq 3$. Let v_0 be the central vertex of $K_{1,p-1}$ and let e_i, e_j be any two edges incident on the root vertex of $K_{1,p-1}$. Then, the set $D = \{e_i, e_j\}$ is a triple connected line dominating set of $K_{1,p-1}$ of minimum cardinality. Hence, $\gamma_{tcl}'(K_{1,p-1}) = |D| = 2$.

2.4. Proposition

For any bistar $B_{m,n}$ of order p, where m + n + 2 = p, m, n $\geq$ 1, and $p \geq 4$, we have $\gamma_{tcl}'(B_{m,n}) = 2$.

Proof:

Let $B_{m,n}$ be a bistar of order p, where m + n + 2 = p, m,n $\geq$ 1, and $p \geq 4$. If the edge connecting the root vertices of the bistar is in D, then D is already an edge dominating set. Thus, to obtain a triple connected line dominating set, we only need to add one more edge to D. Hence, <D> = P_3 and $\gamma_{tcl}'(B_{p;q}) = |D| = 2$.

3. Triple connected line domination number for some special graphs

3.1. The Franklin graph is a 3- regular graph with 12 vertices and 18 edges as shown in the Fig.1

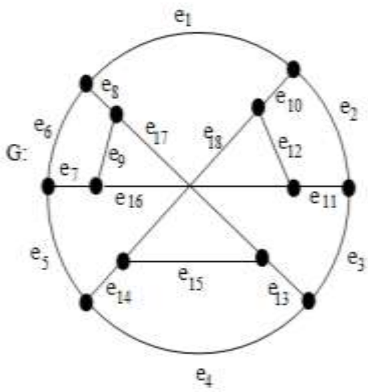

Fig. 1

For the Franklin graph G, $\gamma_{tcl}'(G) = 9$. Here D = {e_3, e_4, e_6, e_8, e_9, e_{12}, e_{14}, e_{16} , e_{18}} is a minimum triple connected line domination number of a graph.

3.2. The Truncated Tetrahedron is an Archimedean solid. It has 4 regular hexagonal faces, 4 regular triangular faces, 12 vertices and 18 edges as shown in Fig.2 Archimedean solid means one of 13 possible solids whose faces are all regular polygons whose polyhedral angles are all equal.

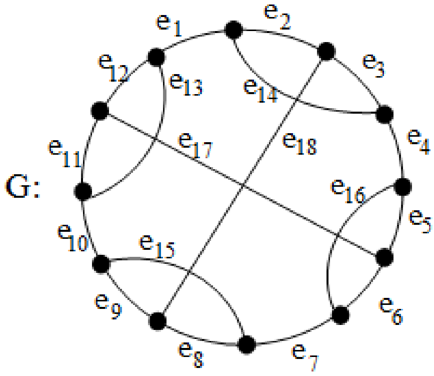

Fig. 2

For the Truncated Tetrahedron G, $\gamma_{tcl}'(G) = 8$. Here D = {e_1, e_2, e_5, e_8, e_{10}, e_{14}, e_{15}, e_{16} } is a minimum triple connected line domination number of a graph.

3.3. The **Herschel graph** is a bipartite undirected graph with 11 vertices and 18 edges as shown in Fig 3, the smallest non hamiltonian polyhedral graph. It is named after British astronomer Alexander Stewart Herschel

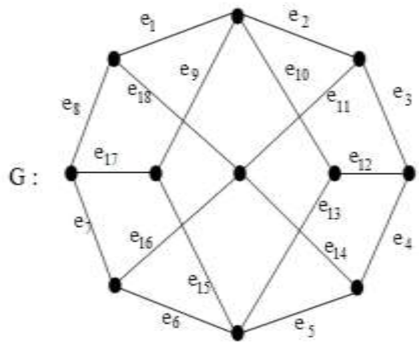

Fig. 3

For the Herschel graph G, $\gamma_{tcl}'(G) = 7$. Here D = {e_1, e_2, e_3, e_4, e_5, e_6, e_7} is a minimum triple connected line domination number of a graph

3.4. The **Golomb graph** is a unit distance graph discovered around 1960 – 1965 by Golomb with 10 vertices and 18 edges as shown in the Fig 4

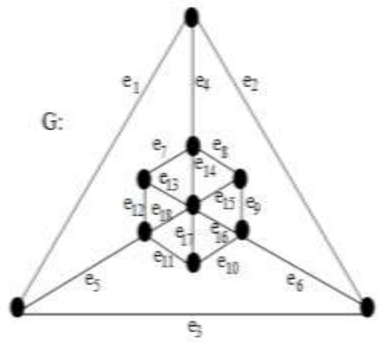

Fig. 4

For the Golomb graph G, $\gamma_{tcl}'(G) = 7$. Here $D = \{e_1, e_4, e_7, e_9, e_{10}, e_{11}, e_{12}\}$ is a minimum triple connected line domination number of a graph.

3.5 The **Goldner–Harary graph** is a simple undirected graph with 11 vertices and 27 edges as shown in Fig 5. It is named after A. Goldner and Frank Harary, who proved in 1975 that it was the smallest non hamiltonian maximal planar graph.

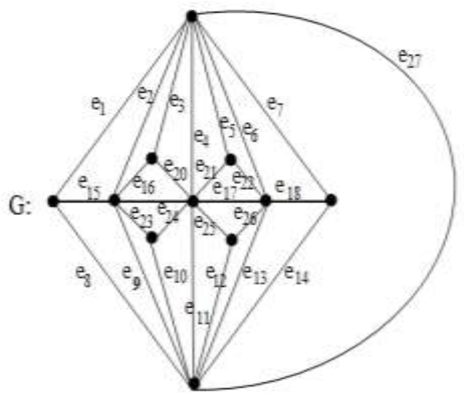

Fig. 5

For the Goldner–Harary graph G, $\gamma_{tcl}'(G) = 5$. Here $D = \{e_8, e_{15}, e_{16}, e_{17}, e_{27}\}$ is a minimum triple connected line domination number of a graph.

4.Conclusion

In this paper we provided the triple connected line domination number of some special and standard graphs. Further, the bounds for general graph and its relationship with other graph theoretical parameters can also be investigated.

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