IJASRM International Journal of Advanced Scientific Research and Management, Volume 3 Issue 8, Aug 2018
www.ijasrm.com
ISSN 2455-6378

Two Fluid Cosmological Model in (2+1) Dimensional Space-Time

G. R. Avachar

S.S.E.S. Amt's Science College,
Nagpur,Maharastra, India

Abstract

In this paper we have investigated two fluid cosmological model in the frame work of (2+1) dimensional space-time. One fluid is the perfect fluid representing the matter content of the universe and another fluid is the radiation distribution, which represents the cosmic microwave background. The model is obtained using the special law of variation for Hubble's parameter proposed by Berman (1983) and the 'gamma- law' equation of state for pressure and energy density. The physical and kinematical parameters of the model are discussed. It is observed that the model is expanding, nonsingular and non-rotating.

Keywords: *Cosmology, (2+1) dimension, Two fluid.*

1. Introduction

Lower dimensional models have been of enormous use in practically every other branch of physics. Such models are important because they help to generate new ideas, and to stimulate new insights into their higher dimensional counterparts. Moreover, they provide a simple setting in which certain basic physical phenomena can be easily demonstrated, while avoiding the mathematical complexities often encounter in four dimensions. Therein lies the motivation for studying gravity in three space-time dimensions.

(2+1) dimensional gravity does contain interesting features in common with four dimensional gravity. Einstein gravity in three space-time dimensions exhibits some unusual features, which can be deduced from the properties of the Einstein field Equations and the curvature tensor.

Deser et. al. (1984) have obtained the solutions to three dimensional Einstein gravity with massless, spinning point source, and Clement (1981) has generalized their results to include many massive spinning sources. The generalization to coupled Einstein-Maxwell theory has been considered by Deser and Mazur (1985), Melvin (1986) and Gott et.al (1986). The Regge calculus version of three dimensional gravity with point masses has been developed by Rocek and Williams (1985). Many of the basic aspects of classical Einstein gravity in three dimensions are covered in the article by Giddings et.al (1984), Gott and Alpert (1984) and Barrow et. al. (1986). They discussed the lack of correspondence between Einstein and Newtonian gravity in three dimensions; the conic geometry associated with a point mass and also includes cosmological solutions for perfect fluids. In addition, Barrow, Burd and Lancaster present two cosmological solutions containing scalar field that produce inflation, and discussed cosmological singularities for three dimensional space-time. Deser and Laurent (1986) have studied the interior and exterior solutions to various matter distributions assuming the space-time is axially symmetric and stationary. Deser (1984) has shown that there are no nontrivial statics solutions to the coupled Einstein gravity-Yang Mills system in three dimensions. Edward Witten (1988) has shown that (2 + 1) dimensional gravity (with or without a cosmological constant) is exactly soluble at the classical and quantum levels and it is closely related to Yang-Mills theory with purely the Chern-Simons action. N.J. Cornish and N.E. Frankel (1991) have investigated gravitational field theories in (2+1) space-time dimensions and reviewed the consequences of the lack of a Newtonian limit to general relativity. The cosmic holographic principle suggested by Fischler and Susskind has been examined in (2+1) dimensional cosmological models by Wang B and Abdalla E. (1999). Ranjan Sharma et.al (2015) have investigated Gravitational collapse of a circularly symmetric star in an (2+1) anti-deSitter space-time and analyzed the impacts of various factors on the evolution of the star, which begins its collapse from an initial static configuration. Yun He and Meng-Sen Ma (2017) have constructed (2 +1)-dimensional regular black holes with nonlinear electrodynamics sources and studied the thermodynamic properties of the regular black holes.

There has been considerable interest in the study of cosmological models for which the source of the gravitational field consists of two fluids [Coley, (1988), Coley and Tupper (1986)]. In these models, one fluid is chosen to be that of a radiation field, modeling the cosmic microwave background, while the other is that of a matter field, modeling the material content of the universe. Cosmologies of this type can be found for which all physical quantities are well behaved and which satisfy the laws of thermodynamics. In addition, these models are in agreement with current observations [Coley and Tupper(1986)]. Two-fluid solutions to the Einstein field equations are known for other geometries as well. For example, Letelier (1980) has found a two-fluid model for a plane-symmetric geometry, and Dunn (1989) has found two-fluid solutions to the field equations for Godel-type space-time. Coley and Dunn (1990) presented exact solutions of the spatially, homogenous, anisotropic Binachi type VI_0 field equations in which the source of the gravitational field consists of two comoving perfect fluids. Pant and Oli (2002) presented a class of solutions of Einstein's field equations describing two fluid models of the universe in a locally rotationally symmetric Bianchi type II space-time. Anisotropic, homogeneous two-fluid cosmological models in a Bianchi type I space–time with a variable gravitational constant *G* and cosmological constant Λ has been investigated by Oli S (2008). Oli [2008] presented anisotropic, homogeneous two-fluid cosmological models in a Bianchi I space-time. Anisotropic, homogeneous two-fluid cosmological models using Bianchi type-V space-time have been presented by Adhav et. al. (2011). Adhav et. al. (2011) has studied anisotropic, homogeneous two-fluid cosmological models in a Bianchi type III space-time. Katore et. al (2011) investigated cosmological models with perfect fluids and dark energy .Mete et. al. (2012) have used plane symmetric metric to present anisotropic, homogeneous two-fluid cosmological models. Anisotropic, homogeneous two-fluid cosmological models using Bianchi type-V space-time have been presented by Mete et. al. (2012). Pawar and Dagwal (2013) have studied Bianchi Type IX two fluids cosmological models with matter and radiating source. Mete et. al (2013) presented Bianchi type-I metric of the Kasner form describing two-fluid source of the universe in general relativity. R. Venkateswarlu (2013) has investigated five dimensional Kaluza-Klein space-time describing two-fluid sources in the context of zero-mass scalar field. Samanta and Debata (2013) studied a class of solutions of Einstein's field equations describing two-fluid models of the universe in a five dimensional spherical symmetric spacetime. Evolution of Bianchi type-V cosmological model is studied in the presence of two-fluid distribution with negative constant deceleration parameter have been investigated by Singh et.al. (2013), Mete et. al. (2013) considered anisotropic, homogeneous two-fluid plane symmetric cosmological models in higher dimensions. Two-fluid anisotropic Bianchi type-III cosmological model is investigated by Samanta (2013) with variable gravitational constant *G* and cosmological constant Λ in the framework of Einstein's general relativity.

Recently Katore and Shaikh have (2015) presented a class of solutions of Einstein's field equations describing two-fluid models of the universe in Hypersurface-Homogenous space time. Motivating with this work, we have presented two-fluid cosmological model in (2+1) dimensional space-time. The physical behavior of the model has been discussed in detail.The paper is organized as follows. Section 2 deals with the derivation of the field equations in (2+1) dimensional Robertson-Walker space-time, when the source for energy momentum tensor is two fluids. Section 3 is devoted to the solutions of the field equations under some physical conditions. In Section 4, we discuss some physical and kinematical properties of the cosmological cosmological model and Section 5 contains some conclusions.

2. Field Equations

We consider (2+1) dimensional Robertson-Walker line element [Cornish and Frankel (1991)]

$$ds^2 = dt^2 - R^2(t)(dr^2 + r^2 d\theta^2), \qquad (1)$$

where, the spatial curvature is taken to be zero. The Einstein's field equations for a two fluid source in natural units (gravitational units) are

$$R_{ij} - \frac{1}{2} g_{ij}\boldsymbol{R} = -8\pi T_{ij} \ (i,j = 0,1,2), \quad (2)$$

where, R_{ij} is the Ricci tensor, $\boldsymbol{R}$ is the Ricci scalar. The energy momentum tensor T_{ij} for a two fluid source is given by

$$T_{ij} = T_{ij}{}^{(m)} + T_{ij}{}^{(r)}, \qquad (3)$$

where, $T_{ij}{}^{(m)}$ is the energy momentum tensor for matter field and $T_{ij}{}^{(r)}$ is the energy momentum tensor for radiation field [3] which are given by

$$T_{ij}{}^{(m)} = (p_m + \rho_m)u_i{}^m u_j{}^m - p_m g_{ij} \quad (4)$$

and

$$T_{ij}{}^{(r)} = \frac{4}{3}\rho_r u_i{}^r u_j{}^r - \frac{1}{3}\rho_r g_{ij} \qquad (5)$$

together with

$$g^{ij}u_i{}^m u_j{}^m = 1, \ g^{ij}u_i{}^r u_j{}^r = 1 \qquad (6)$$

where ρ_m is matter density, p_m is matter pressure and ρ_r is radiation density. The off diagonal equation of (2) together with energy conditions imply that the matter and radiation are both co-moving, we get

$$u_i{}^m = (1,0,0,), u_i{}^r = (1,0,0) \qquad (7)$$

Using the co-moving coordinate system, the non-vanishing components of $T_i^{j(m)}$ and $T_i^{j(r)}$ can be obtained as

$$T_0^{0(m)} = \rho_m, T_1^{1(m)} = T_2^{2(m)} = -p_m,$$
$$T_i^{j(m)} = 0 \text{ for } i \neq j \quad (8)$$

and

$$T_0^{0(r)} = \rho_r, T_1^{1(r)} = T_2^{2(r)} = -\frac{\rho_r}{3},$$
$$T_i^{j(r)} = 0, \text{ for } i \neq j \quad (9)$$

Thus the field equation (2) for the metric (1) with energy momentum tensor (3) are given as

$$\frac{\dot{R}^2}{R^2} = 8\pi(\rho_m + \rho_r) \quad (10)$$
$$\frac{\ddot{R}}{R} = -8\pi\left(p_m + \frac{\rho_r}{3}\right) \quad (11)$$

Here, dot $(\cdot)$ denotes differentiation with respect to time t only and expansion scalar θ is given by

$$\theta = u^i_{;i} = 2\frac{\dot{R}}{R} \quad (12)$$

3. Solution of field equations

This section must contain specific details about the There are two field equations (10) and (11) with four unknowns namely one scale factor R and three physical quantities ρ_m, ρ_r and p_m. Therefore to obtain an exact solution of the field equations, we need two more relations connecting these variables. Hence we use the following plausible physical conditions.

i) Variation of Hubble's parameter proposed by Berman [39] that yields a constant deceleration parameter models of the universe which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \text{ constant.} \quad (13)$$

ii) The relation between pressure and energy density of matter field through the 'gamma-law' equation of state which is given by

$$p_m = (\gamma - 1)\rho_m, \; 1 \leq \gamma \leq 2 \quad (14)$$

Now, the Eqn. (13) admits the solution

$$R = (At + B)^{1/q+1}, \quad (15)$$

where A and B are constants of integration. Eq. (15) implies that the condition for expansion of the universe is $q + 1 > 0$.
With suitable choice of coordinates and constants i.e. taking $A = 1$ and $B = 0$, the metric (1) can be written as

$$ds^2 = dt^2 - t^{2/q+1}(dr^2 + r^2 d\theta^2) \quad (16)$$

Eq. (16) represents the two-fluid cosmological model in (2+1) dimensional gravity theory.

4. Physical and Kinematical Properties and Discussion

The physical and kinematical quantities for the model (19) have the following expressions.
The scale factor R is given by

$$R = t^{1/q+1} \quad (17)$$

Spatial volume :

$$V_2 = R^2 = t^{2/q+1} \quad (18)$$

Expansion scalar :

$$\theta = u^i_{;i} = 2\frac{\dot{R}}{R} = \frac{2}{(q+1)t} \quad (19)$$

Hubble's parameter :

$$H = \frac{1}{2}(H_1 + H_2) = \frac{1}{2}\left(\frac{\dot{R}}{R} + \frac{\dot{R}}{R}\right) = \frac{\dot{R}}{R}$$
$$= \frac{1}{(q+1)t} \quad (20)$$

Matter Energy density :

$$8\pi\rho_m = \left[\frac{3q-1}{3\gamma+2}\right]\frac{1}{[(q+1)t]^2} \quad (21)$$

Radiation Energy density:

$$8\pi\rho_r = \left[\frac{3\gamma-q-3}{3\gamma-4}\right]\frac{1}{[(q+1)t]^2} \quad (22)$$

Total density :

$$8\pi\rho = 8\pi(\rho_m + \rho_r)$$
$$= \left[\frac{9\gamma^2+6q\gamma-14q-6\gamma-2}{9\gamma^2-6\gamma-8}\right]\frac{1}{[(q+1)t]^2} \quad (23)$$

The density parameter for matter and radiation respectively are given as

$$\Omega_m = \frac{1}{3}\left(\frac{3q-1}{3\gamma+2}\right) \quad (24)$$
$$\Omega_r = \frac{1}{3}\left(\frac{3\gamma-q-3}{3\gamma-4}\right) \quad (25)$$

and total energy density parameter

$$\Omega = \Omega_m + \Omega_r = \frac{1}{3}\left(\frac{9\gamma^2+6q\gamma-14q-6\gamma-2}{9\gamma^2-6\gamma-8}\right) \quad (26)$$

Shear scalar and anisotropic parameter are given as

$$\sigma^2 = \frac{1}{2}\left[\sum_i H_i^2 - \frac{1}{2}\theta^2\right] = 0 \quad (27)$$

$$A_m = \frac{1}{2}\sum_i\left(\frac{H_i - H}{H}\right)^2 = 0 \quad (28)$$

Using the above results, we now discuss the behavior of the cosmological model (16).

The result (18) shows that the model is expanding with time since $q + 1 > 0$. It can be observed that the model given by (16) has no initial singularity at $t = 0$. It can also be observed that, the Hubble parameter H, expansion scalar θ, matter density ρ_m and radiation density ρ_r decreases with time and approach zero as $t \to \infty$ and all diverges at $t = 0$. Also shear scalar σ and anisotropic parameter A_m vanishes, which indicates that shape of the universe remains unchanged during the evolution and universe becomes isotropic and shear free.

5. Conclusions

In this paper we have investigated two fluid cosmological model in the frame work of (2+1) dimensional space-time. Here one fluid is the perfect fluid which represents the matter content of the universe and another fluid is the radiation distribution which represents the cosmic microwave background. The model is obtained using the special law of variation for Hubble's parameter proposed by Berman [33] and the the 'gamma- law' equation of state for pressure and energy density. It is observed that the model is expanding, nonsingular and nonrotating. It is also observed that all the physical and kinematical parameters of the model diverges when $t = 0$ and vanish when t is infinitely large. Also we have $\frac{\sigma^2}{\theta^2} = 0$ and anisotropic parameter $A_m = 0$, which indicate that the model is not anisotropic in nature and remains isotropic throughout the evolution.

References

[1] Adhav K. S, Borikar S. M, Desale M. S and Raut R.B, (2011), Int. J. Theor. Phys, **50**; 1846–1851.

[2] Adhav K. S, Borikar S. M, Desale M. S and Raut R. B, (2011), EJTP 8, No. **25**, 319–326.

[3] Barrow J. D, Burd A. B and Lancaster D, (1986), Class. Quantum Grav. 3, 551.

[4] Berman M. S, (1983) 'A special law of variation for Hubble's parameter' *Nuvo Cimento B,* vol. 74, no.2, pp.182–186.

[5] Clement G, (1981), Int. J. Theor. Phys. A: Math. Gen. **14,** 2353.

[6] Cornish N. J and Frankel N. E, (1991), Physical Review D, Volume 43, Number **8**, 2555-2565.

[7] Cornish N. J and Frankel N.E, (1991), Phys. Rev. **D 43**, 2555.

[8] Coley A. A and Tupper, B. O.J, (1986), J. Math. Phys., J. Math. Phys., 27,406.

[9] Coley A. A, (1988), Ap. Space Sci., 140, 175.

[10] Coley A and Dunn K, (1990), The Astrophysical Journal, **348**; 26-32

[11] .Deser S, Jackiw R and Hooft G, (1984), Ann. Phys. **152,** 220.

[12] Deser S, (1984), Class. Quantum Grav. **1,** L1.

[13] Deser S and Mazur P.O, (1985), Class. Quantum Grav. **2,** L 51.

[14] Deser S and Laurent B, (1986), Gen. Rel. Grav. **18,** 617.

[15] Dunn K. A. (1989), Gen. Rel. Grav., 21,137.

[16] Gott J. R, Simon J. Z and Alpert M, (1986), Gen. Rel. Grav. **18,** 1019

[17] Giddings S, Abbott J. and Kuchar K, (1984), Gen. Rel. Grav. **16,** 751.

[18] Gott J. R and Alpert M, (1984), Gen. Rel. Grav. **16** 243

[19] Katore S. D, Adhav K. S, Shaikh A.Y, Sancheti M.M, (2011), Astrophysics and Space Science, **333 (1)**, 333-341

[20] Katore, S. D and Shaikh A. Y, Pre-spacetime Journal,(2015),Volume, Issue 3, pp. 207-217

[21] Letelier P. S. (1980), Phys. Rev. **D**, 22,807.

[22] Melvin M. A, (1986), Class Quantum Grav. **3,** 117

[23] Mete V. G, Murade P. B, Umarkar V. M and Ugale M. R, (2012), Sci. Revs. Chem. Commun., 2(3), 497-502.

[24] Mete V.G, Umarkar V. M. and Pund A. M, (2012), Prespacetime Journal|, Vol. 3, Issue 2, pp. 170-177.

[25] Mete V.G, Umarkar V.M and Pund A.M, (2013), Int J Theor Phys. 52:2446–2454.

[26] Mete V.G, Umarkar V.M and Pund A.M, (2013),Int. J. Theor. Phys. DOI 10.1007/s10773-013-1763-4.

[27] Oli S, (2008), Astrophys Space Sci. **314**, 89–94.

[28] Oli S, (2008), Astrophys Space Sci. **314**, 95–103.

[29] Pant D.N. and Oli S, (2002), Astrophysics and Space Science **281**, 623–631.

[30] Pawar D. D and Dagwal V. J, (2013), International Journal of Scientific & Engineering Research, Volume 4, Issue 4.

[31] Rocek M and Williams R, (1985), Class. Quantum Grav. **2** , 701

[32] Samanta G.C, Debata S, Int. J. Theor. Phys. DOI 10.1007/s10773-013-1713-1

[33] Samanta G.C, Int. J. Theor. Phys. (2013).DOI 10.1007/s10773-013-1715-z.

[34] Singh M.K, Verma M.K and Shri Ram, (2013), Int. J. Theor. Phys., 52:227–232.

[35] Sharma R, Das S, Farook Rahaman and Shit G. C, (2015), Astrophys Space Sci, 359:40.

[36] Venkateswarlu R, Pre-space-time Journal, (2013),Volume 4, Issue 8, pp. 801-806.

[37] Witten E, (1988/89)Nuclear Physics, **B 311**, 46-78.

[38] Wang B and Abdalla E, (1999), Physics Letters **B 466**, 122–126.

[39] Yun He and Meng-Sen Ma, (2017), Physics Letters **B, 774,** 229–234.