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Reliable Analysis of Riemann Solver in Ideal Magnetogasdynamicsusing Arithmetic Averaging

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Abstract:
In this paper, the arithmetic averaging in a Riemann solver for a quasilinear hyperbolic system of equations governing inviscid and perfectly conducting compressible fluid, subjected to a transverse magnetic field in an ideal magnetogasdynamics (MGD) is studied.Some cases are discussed for transformed jacobian matrices using symbols obtained after applying arithmetic averaging on the flux function and the vector of conservative variables and disscussed the behaviour of the flow variables by analyzing the eigenvectors of these transformed jacobian matrices.

Keywords: *Riemann problem, an ideal magnetogasdynamics (MGD), arithmetic averaging.*

1. Introduction

The governing equations of ideal magnetogasdynamics (MGD) describe the physics of a conducting fluid in which the conservative system of these equations defines a quasilinear hyperbolic system of equations.The hyperbolic terms characterize the convective effects.Many problems related to Astrophysics, Engineering physics, nuclear science, Plasma physics and many other aspects are d *Riemann problem, an ideal magnetogasdynamics (MGD), arithmetic averaging* ealt with MGD.Lax[1] analyzed the hyperbolic systems of conservation laws. Jeffrey and Taniuti [2] discussed the non-linear wave propagation. A generalized Riemann problem for quasi one-dimensional gas flow is studied by Glimm et al. [3]. In the case of the Euler equations, the Riemann problem corresponds to the so-called shock-tube problem, a basic physical problem in gasdynamics; for its detailed discussion, the reader is referred to the book by Courant and Friedrichs [4].Chorin [5] is determined the Random choice solutions of hyperbolic systems. Toro [6] presented an efficient solver for computing the exact solution of the Riemann problem for ideal and co-volume gases; for detailed methodologies, the reader is referred to the book by Toro [7]. Glaister [8-10] is analyzed the Riemann solver in one, two and three dimensional Euler equations using arithmetic averaging. The systems of conservation laws representing MGD are more complex and highly nonlinear than the conventional gasdynamics system, where the velocity and the magnetic field are orthogonal to each other. The systems of conservation laws representing MGD are more complex and highly nonlinear than the conventional gasdynamics system, where the velocity and the magnetic field are orthogonal to each other.

Discontinuities in MHD are studied by many authors, the integral form of MHD studies in recent literature. Powell [11] discussed an approximate Riemann solver for MGD. Woodward and Dai [12] analyzed the Riemann solver in ideal MGD. Zachary and Colella[13]studied a multidimensional scheme for the conservative equation in ideal MHD based on higher-order Godunov scheme to examine the degeneracies. Yujin Liu andWenhuaSun[14] discussed the Riemann problem for one dimensional ideal isentropic MGD with traversed magnetic field. Shengtai Li[15] investigated a modern code for solving the magneto hydrodynamics (MHD) or hydrodynamics (HD) equationsbythefinite volume method and finite difference method.The eigensystem of MHD Riemann problem is analyzed by Roe and Balsara [16]. Recently, Shen [17] is studied the limits of Riemann solutions to the isentropic MGD. Singh et al. [18] is discussed the Riemann problem in MGD. Raja Sekhar and Sharma [19-20] obtained the solution to the Riemann problem in isentropic and one-dimensional ideal MGD respectively.We have extended this work from two-dimensionsl Euler equations to three-dimensional Euler equations under traverse magnetic field, this increases allowance of degree freedom during the averaging of new termslaw presenting Riemann problem be composed of seven waves, namely one entropy wave moving with the speed u_x, two Alfven waves moving with the speed $u \pm c_1^*$ and four magneto-acoustic waves rising with the speed $u \pm c_i^*; i = 2,3$. We examine the Jacobian matrices constitute with the help of fluxand conservative variables. We discuss the identities components of flux and conservative variables. The aim of the present paper is to determine the eigenvalues and their corresponding eigenvectors of the Jacobian matrices in three cases.

2. Govering Equation of Ideal MGD

The ideal magnetogasdynamics (MGD) equations describe the macroscopic dynamics of perfectly conducting plasma. The system expresses conservation of mass, momentum, energy and magnetic flux as follows [11-12].

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0, \tag{1}$$

where the vector of conserved flow variables U, the flux vectors $F(U)$ and $G(U)$ are defined respectively, as

$$U = \left(\rho, \rho u_x, \rho u_y, \rho u_z, B_y, \mathrm{B}_z, E\right)^T, \tag{2}$$

$$F(U) = \begin{pmatrix} \rho u_x, \rho u_x^2 + p^*, \rho u_x u_y - B_x B_y, \\ \rho u_x u_z - B_x B_z, B_y u_x - B_x u_y, u_x B_z - u_z B_x \\ (E + p*) u_x - B_x \left(B_x u_x + B_y u_y + B_z u_z\right) \end{pmatrix}, \tag{3}$$

$$G(U) = \begin{pmatrix} \rho u_y, \rho u_x u_y - B_x B_y, \rho u_y^2 - B_y^2 + p + \frac{B^2}{2} \\ , u_y B_x - B_y u_x, 0, \left(e + p + \frac{1}{2} B^2\right) u_y - (B.u) B_y \end{pmatrix}, \tag{4}$$

where

$$\left.\begin{aligned} p* &= p + \frac{1}{2} B^2,\ e = \frac{p}{\rho(\gamma - 1)}, \\ E &= \frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1} + \frac{1}{2} B^2, \end{aligned}\right\} \tag{5}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2 \text{ and } u^2 = u_x^2 + u_y^2 + u_z^2. \tag{6}$$

The vector $\vec{u} = u\left(\rho, u_x, u_y, u_z, p, B_y, B_z\right)^T$, the vector of physical variables, where ρ, p, $p*$, e, E, u, B, u_x, u_y, u_z, B_x, B_y, B_z, represent the density, the pressure, total pressure, internal energy, total energy, resultant velocity, resultant magnetic field, velocity components and magnetic field componentsrespectively in the direction of x, y and z at a time t, and γ shows the ratio of specific heat capacities of the ideal gas.

Their work is not based on the system of eight conservation laws, but instead of the closely related system that comes from assuming B_x equals to constant and dropping the evolution equation for B_x, Powell [11].

3. The Jacobian Matrices and Structure

The Jacobian matrices are constructed as

$$P = \frac{\partial U}{\partial \vec{u}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ u_x & \rho & 0 & 0 & 0 & 0 & 0 \\ u_y & 0 & \rho & 0 & 0 & 0 & 0 \\ u_z & 0 & 0 & \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{2}u^2 & \rho u_x & \rho u_y & \rho u_z & \frac{1}{\gamma - 1} & B_y & B_z \end{bmatrix}, \tag{7}$$

$$Q = \frac{\partial F}{\partial \vec{u}} = \begin{bmatrix} u_x & \rho & 0 & 0 & 0 & 0 & 0 \\ u_x^2 & 2\rho u_x & 0 & 0 & 1 & B_y & B_z \\ u_x u_y & \rho u_y & \rho u_x & 0 & 0 & -B_x & 0 \\ u_x u_z & \rho u_z & 0 & \rho u_x & 0 & 0 & -B_x \\ 0 & B_y & -B_x & 0 & 0 & u_x & 0 \\ 0 & B_z & 0 & -B_x & 0 & 0 & u_x \\ \frac{1}{2}u^2 u_x & \rho u_x^2 + \frac{1}{2}\rho u^2 + \frac{\gamma p}{\gamma - 1} + B_y^2 + B_z^2 & \rho u_x u_y - B_x B_y & \rho u_x u_z - B_x B_z & \frac{\gamma u_x}{\gamma - 1} & 2B_y u_x - B_x u_y & 2B_z u_x - B_x u_z \end{bmatrix}. \tag{8}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{u_x}{\rho} & \frac{1}{\rho} & 0 & 0 & 0 & 0 & 0 \\ -\frac{u_y}{\rho} & 0 & \frac{1}{\rho} & 0 & 0 & 0 & 0 \\ -\frac{u_z}{\rho} & 0 & 0 & \frac{1}{\rho} & 0 & 0 & 0 \\ (\gamma - 1)\frac{u^2}{2} & -(\gamma - 1)u_x & -(\gamma - 1)u_y & -(\gamma - 1)u_z & -(\gamma - 1)B_y & -(\gamma - 1)B_z & (\gamma - 1) \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \tag{9}$$

The usual Jacobian matrix $M = QP^{-1}$ is

$$\frac{\partial F}{\partial U} = \frac{\partial F}{\partial \vec{u}} \left(\frac{\partial U}{\partial \vec{u}}\right)^{-1} = QP^{-1} = M$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_1 & (3-\gamma)u_x & -(\gamma-1)u_y & -(\gamma-1)u_z & (2-\gamma)B_y & (2-\gamma)B_z & (\gamma-1) \\ -u_x u_y & u_y & u_x & 0 & -B_x & 0 & 0 \\ -u_x u_z & u_z & 0 & u_x & 0 & -B_x & 0 \\ v_2 & \frac{B_y}{\rho} & -\frac{B_x}{\rho} & 0 & u_x & 0 & 0 \\ v_3 & \frac{B_z}{\rho} & 0 & -\frac{B_x}{\rho} & 0 & u_x & 0 \\ v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & \gamma u_x \end{bmatrix}, \quad (10)$$

where

$$v_1 = \frac{\gamma-3}{2}u_x^2 + \frac{\gamma-1}{2}\left(u_y^2 + u_z^2\right), \; v_2 = \frac{-u_x B_y + B_x u_y}{\rho},$$

$$v_3 = \frac{-u_x B_z + B_x u_z}{\rho},$$

$$v_4 = -u_x\left(\frac{u^2}{2} + \frac{1}{\gamma-1}\frac{\gamma p}{\rho} + \frac{B^2}{\rho} - \frac{(\gamma-1)}{2}u^2\right) + \frac{B_x}{\rho}\left(B_x u_x + B_y u_y + B_z u_z\right),$$

$$v_5 = \frac{(3-2\gamma)}{2}u_x^2 + \frac{1}{2}u_y^2 + \frac{1}{2}u_z^2 - \frac{B_x^2}{\rho} + \frac{B^2}{\rho} + \frac{\gamma}{(\gamma-1)}\frac{p}{\rho},$$

$$v_6 = -(\gamma-1)u_x u_y - \frac{B_x B_y}{\rho}, \; v_7 = -(\gamma-1)u_x u_z - \frac{B_x B_z}{\rho},$$

$$v_8 = (2-\gamma)u_x B_y - B_x u_y,$$

$$v_9 = (2-\gamma)u_x B_z - B_x u_z, v_{10} = \gamma u_x.$$

The eigenvalues of M are:

$$\left.\begin{aligned} &\lambda_1 = u_x + c_1^*, \lambda_2 = u_x - c_1^*, \lambda_3 = u_x + c_2^*, \lambda_4 = u_x - c_2^*, \\ &\lambda_4 = u_x - c_2^*, \lambda_5 = u_x + c_3^*, \lambda_6 = u_x - c_3^*, \lambda_7 = u_x \end{aligned}\right\}, \quad (11)$$

$$\lambda_5 = u_x + c_3^*, \; \lambda_6 = u_x - c_3^*, \; \lambda_7 = u_x,$$

where $c_1^* = \frac{B_x}{\sqrt{\rho}}$; and $c_i^{*2} = \frac{c^2 \pm \sqrt{c^4 - 4a^2 b_x^2}}{2}$, $i = 2, 3$; the magnetoacoustic wave speeds; regular acoustic wave speed $a = \sqrt{\gamma p/\rho}$, Alfven wave speed ; $b_x^2 = B_x^2/\rho$, $b_y^2 = B_y^2/\rho$ and $b_z^2 = B_z^2/\rho$.

Right eigenvectors of M correspond to (11) are:

$$r_1 = \begin{pmatrix} 1, u_x + c_1^*, u_y - \frac{B_x B_y c_1^*}{\rho\left(c_1^{*2} - b_x^2\right)}, u_z - \frac{B_x B_z c_1^*}{\rho\left(c_1^{*2} - b_x^2\right)}, \\ \frac{B_y c_1^{*2}}{\rho\left(c_2^{*2} - b_x^2\right)}, \frac{B_z c_1^{*2}}{\rho\left(c_2^{*2} - b_x^2\right)}, \frac{u^2}{2} + \frac{c_1^{*2}}{\gamma-1} + c_1^* u_x \\ -\frac{c_1^* B_x\left(B_y u_y + B_z u_z\right)}{\rho\left(c_1^{*2} - b_x^2\right)} + \frac{(\gamma-2)\left(c_1^{*2} - a^2\right)}{(\gamma-1)} \end{pmatrix}, \quad (12)$$

$$r_2 = \begin{pmatrix} 1, u_x - c_1^*, u_y + \frac{B_x B_y c_1^*}{\rho\left(c_1^{*2} - b_x^2\right)}, u_z + \frac{B_x B_z c_1^*}{\rho\left(c_1^{*2} - b_x^2\right)}, \\ \frac{B_y c_1^{*2}}{\rho\left(c_1^{*2} - b_x^2\right)}, \frac{B_z c_1^{*2}}{\rho\left(c_1^{*2} - b_x^2\right)}, \frac{u^2}{2} + \frac{c_1^{*2}}{2} - c_1^* u_x \\ +\frac{c_1^* B_x\left(B_y u_y + B_z u_z\right)}{\rho\left(c_1^{*2} - b_x^2\right)} + \frac{(\gamma-2)\left(c_1^{*2} - a^2\right)}{(\gamma-1)} \end{pmatrix}, \quad (13)$$

$$r_3 = \begin{pmatrix} 1, u_x + c_2^*, u_y - \frac{B_x B_y c_2^*}{\rho\left(c_2^{*2} - b_x^2\right)}, u_z - \frac{B_x B_z c_2^*}{\rho\left(c_2^{*2} - b_x^2\right)}, \\ \frac{B_y c_2^{*2}}{\rho\left(c_2^{*2} - b_x^2\right)}, \frac{B_z c_2^{*2}}{\rho\left(c_2^{*2} - b_x^2\right)}, \frac{u^2}{2} + \frac{c_2^{*2}}{2} + c_2^* u_x \\ -\frac{c_2^* B_x\left(B_y u_y + B_z u_z\right)}{\rho\left(c_2^{*2} - b_x^2\right)} + \frac{(\gamma-2)\left(c_2^{*2} - a^2\right)}{(\gamma-1)} \end{pmatrix}, \quad (14)$$

$$r_4 = \begin{pmatrix} 1, u_x - c_2^*, u_y + \frac{B_x B_y c_2^*}{\rho\left(c_2^{*2} - b_x^2\right)}, u_z + \frac{B_x B_z c_2^*}{\rho\left(c_2^{*2} - b_x^2\right)}, \\ \frac{B_y c_2^{*2}}{\rho\left(c_2^{*2} - b_x^2\right)}, \frac{B_z c_2^{*2}}{\rho\left(c_2^{*2} - b_x^2\right)}, \frac{u^2}{2} + \frac{c_2^{*2}}{2} - c_2^* u_x \\ +\frac{c_2^* B_x\left(B_y u_y + B_z u_z\right)}{\rho\left(c_2^{*2} - b_x^2\right)} + \frac{(\gamma-2)\left(c_2^{*2} - a^2\right)}{(\gamma-1)} \end{pmatrix}, \quad (15)$$

$$r_5 = \begin{pmatrix} 1, u_x + c_3^*, u_y - \frac{B_x B_y c_3^*}{\rho\left(c_3^{*2} - b_x^2\right)}, u_z - \frac{B_x B_z c_3^*}{\rho\left(c_3^{*2} - b_x^2\right)}, \\ \frac{B_y c_3^{*2}}{\rho\left(c_3^{*2} - b_x^2\right)}, \frac{B_z c_3^{*2}}{\rho\left(c_3^{*2} - b_x^2\right)}, \frac{u^2}{2} + \frac{c_1^{*2}}{\gamma-1} + c_3^* u_x \\ -\frac{c_3^* B_x\left(B_y u_y + B_z u_z\right)}{\rho\left(c_3^{*2} - b_x^2\right)} + \frac{(\gamma-2)\left(c_3^{*2} - a^2\right)}{(\gamma-1)} \end{pmatrix}, \quad (16)$$

$$r_6 = \begin{pmatrix} 1, u_x - c_3^*, u_y + \dfrac{B_x B_y c_3^*}{\rho\left(c_1^{*2} - b_x^2\right)}, u_z + \dfrac{B_x B_z c_3^*}{\rho\left(c_3^{*2} - b_x^2\right)}, \\ \dfrac{B_y c_3^{*2}}{\rho\left(c_3^{*2} - b_x^2\right)}, \dfrac{B_z c_3^{*2}}{\rho\left(c_3^{*2} - b_x^2\right)}, \dfrac{u^2}{2} + \dfrac{c_3^{*2}}{2} - c_3^* u_x \\ + \dfrac{c_3^* B_x\left(B_y u_y + B_z u_z\right)}{\rho\left(c_3^{*2} - b_x^2\right)} + \dfrac{(\gamma - 2)\left(c_3^{*2} - a^2\right)}{(\gamma - 1)} \end{pmatrix}, \quad (17)$$

$$r_7 = \left(1,\ u_x,\ u_y, u_z,\ 0, 0,\ (u_x^2 + u_y^2 + u_z^2)/2\right). \quad (18)$$

The matrix

$$A = P^{-1}Q = \begin{bmatrix} u_x & \rho & 0 & 0 & 0 & 0 & 0 \\ 0 & u_x & 0 & 0 & \dfrac{1}{\rho} & \dfrac{B_y}{\rho} & \dfrac{B_z}{\rho} \\ 0 & 0 & u_x & 0 & 0 & -\dfrac{B_x}{\rho} & 0 \\ 0 & 0 & 0 & u_x & 0 & 0 & -\dfrac{B_x}{\rho} \\ 0 & \gamma p & 0 & 0 & u_x & 0 & 0 \\ 0 & B_y & -B_x & 0 & 0 & u_x & 0 \\ 0 & B_z & 0 & -B_x & 0 & 0 & u_x \end{bmatrix}. \quad (19)$$

The matrix A and M has same eigenvalues. To determine eigenvalues of A is more simpler than M With the help of the matrix M flow quantities can be determind by analyzing its eigenvectors.

4. Determination of Matrix $\tilde{A}$

Various arithmetic averaging is applied on the vector of conservative variables as well as flux function $F(U)$, and some degree of freedom is allowed to make choices for the physical variables involved. The application of arithmetic averaging give rise to some identities for the components of ΔU and $\Delta F(U)$ respectively which enable in constructing cases for the transformed jacobian matrices. Some component of ΔU is found in terms of $\vec{u}$,

where

$$\vec{u} = (\rho, u_x, u_y, u_z, p, B_y, B_z)^T, \quad \text{and}$$

$$U = \left(\rho, \rho u_x, \rho u_y, \rho u_z, B_y, B_z, E\right)^T.$$

Using Δ operator defined as $\Delta(q) = (q)_R - (q)_L$ and averaging operator denoted as $\bar{q} = \dfrac{(q)_R + (q)_L}{2}$ below [8-10],with q is any component of U, $F(U)$ and $\vec{u}$.

Forth components of ΔU is:

$$\Delta\left(\rho u_z\right) = \bar{\rho}\Delta u_z + \bar{u}_z\Delta\rho, \quad (20)$$

and fifth and sixth component of ΔU is defined by $\Delta B_y = \Delta B_y$, and $\Delta B_z = \Delta B_z$, (21)

where

$$\bar{\rho} = \frac{1}{2}\left(\rho_L + \rho_R\right), \quad (22)$$

where $\bar{u}_z = \dfrac{1}{2}\left(u_{zL} + u_{zR}\right)$. (23)

$$\left.\begin{aligned} \Delta E = {} & \frac{1}{2}\Delta\left(\rho u_x^2\right) + \frac{1}{2}\Delta\left(\rho u_y^2\right) + \frac{1}{2}\Delta\left(\rho u_z^2\right) \\ & + \frac{1}{\gamma - 1}\Delta p + \frac{1}{2}\Delta\left(B_x^2\right) + \frac{1}{2}\Delta\left(B_y^2\right) + \frac{1}{2}\Delta\left(B_z^2\right) \end{aligned}\right\}, \quad (24)$$

Two alternatives can be obtained for the third term $\Delta\left(\rho u_z^2\right)$ of ΔE, the first expression for this is:

$$\Delta\left(\rho u_z^2\right) = \overline{u_z^2}\Delta\rho + 2\bar{\rho}\bar{u}_z\Delta u_z, \quad (25)$$

where $\overline{u_z^2} = \dfrac{1}{2}\left(u_{zL}^2 + u_{zR}^2\right)$, (26)

the second expression for the term $\Delta\left(\rho u_z^2\right)$ is:

$$\Delta\left(\rho u_z^2\right) = \bar{u}_z^2\Delta\rho + 2\tilde{u}_z\bar{\rho}\Delta u_z, \quad (27)$$

where $\overline{\rho u_z} = \dfrac{1}{2}\left(\rho_L u_{zL} + \rho_R u_{zR}\right)$, (28)

$$\hat{u}_z = \frac{\overline{\rho u_z}}{\bar{\rho}} \quad (29)$$

and $\tilde{u}_z = \dfrac{1}{2}\left(\bar{u}_z + \hat{u}_z\right)$. (30)

Combined expression for equations (25) and (27) is:

$$\Delta\left(\rho u_z^2\right) = \alpha^*\Delta\rho + 2\beta^*\Delta u_z, \quad (31)$$

where

$$\alpha^* = \overline{u_z^2}, \beta^* = \overline{\rho}\,\overline{u}_z, \quad (32)$$

$$\alpha^* = \overline{u_z^2}, \beta^* = \tilde{u}_z\bar{\rho}. \quad (33)$$

Fifth, sixth and seventh term of seventh component of ΔU is written respectively as:

$$\Delta B_x^2 = \Delta\left(B_x B_x\right) = 2\bar{B}_x\Delta B_x, \quad (34)$$

similarly

$$\Delta B_y^2 = \Delta\left(B_y B_y\right) = 2\bar{B}_y\Delta B_y, \quad (35)$$

and

$$\Delta\left(B_z^2\right) = \Delta\left(B_z . B_z\right) = 2\bar{B}_z\Delta B_z. \quad (36)$$

Thus the expression for ΔE in the equation (24) is written as

$$\Delta E = \begin{pmatrix} \dfrac{1}{\gamma - 1}\Delta p + \dfrac{1}{2}\left(\overline{u_x^2} + \mu + \alpha^*\right)\Delta\rho + \bar{\rho}\bar{u}_x\Delta u_x + \omega\Delta u_y \\ + 2\beta^*\Delta u_z + \bar{B}_x\Delta B_x + \bar{B}_y\Delta B_y + \bar{B}_z\Delta B_z. \end{pmatrix} \quad (37)$$

Using the identities obtained after Applying arithmetic averaging matrices P and P^{-1} in equations(7) and (9) transformed to $\tilde{P}$ and $\tilde{P}^{-1}$ respectively, as:

$$\tilde{P}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{u}_x & \bar{\rho} & 0 & 0 & 0 & 0 & 0 \\ \bar{u}_y & 0 & \bar{\rho} & 0 & 0 & 0 & 0 \\ \bar{u}_z & 0 & 0 & \bar{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{2}\overline{u_x^2}+\frac{1}{2}\mu+\frac{1}{2}\overline{u_z^2} & \bar{\rho}\bar{u}_x & \bar{\rho}\bar{u}_y & \bar{\rho}\bar{u}_z & \frac{1}{\gamma-1} & \bar{B}_y & \bar{B}_z \end{bmatrix}, \quad (38)$$

$$\tilde{P}^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\bar{u}_x}{\bar{\rho}} & \frac{1}{\bar{\rho}} & 0 & 0 & 0 & 0 & 0 \\ -\frac{\bar{u}_y}{\bar{\rho}} & 0 & \frac{1}{\bar{\rho}} & 0 & 0 & 0 & 0 \\ -\frac{\bar{u}_z}{\bar{\rho}} & 0 & 0 & \frac{1}{\bar{\rho}} & 0 & 0 & 0 \\ (\gamma-1)\left(\bar{u}_x^2+\frac{\omega\bar{u}_y}{\bar{\rho}}+\frac{\beta^*\bar{u}_z}{\bar{\rho}}-\frac{1}{2}\alpha^*\right)-\frac{1}{2}\overline{u_x^2}-\frac{1}{2}\mu & -(\gamma-1)\bar{u}_x & -(\gamma-1)\frac{\omega}{\bar{\rho}} & -(\gamma-1)\bar{u}_z & -(\gamma-1)\bar{B}_y & -(\gamma-1)\bar{B}_z & (\gamma-1) \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \quad (39)$$

where the symbols involved ω,μ reffered to [10] Glaister.

Writing the vector $F(U)$ in the form of $\Delta F(U)$

$$\Delta F(U)=\Delta\begin{pmatrix} \rho u_x, \rho u_x^2+\mathrm{p}^*, \rho u_x u_y - B_x B_y, \rho u_x u_z \\ -\mathrm{B}_x\mathrm{B}_z, B_y u_x - B_x u_y, u_x B_z - u_z B_x (E+p*)u_x \\ -B_x\left(B_x u_x + B_y u_y + B_z u_z\right) \end{pmatrix}$$

second term of the third component of $\Delta F(U)$ is express as

$$\Delta\left(B_x B_y\right)=\bar{B}_x\Delta B_y+\bar{B}_y\Delta B_x. \quad (40)$$

Three alternatives could be obtained for the first term of forth component of ΔF as follows.

$$\Delta\left(\rho u_x u_z\right)=\overline{\rho u_x}\Delta u_z+\bar{\rho}\bar{u}_z\Delta u_x+\bar{u}_z\bar{u}_x\Delta\rho, \quad (41)$$

$$\Delta\left(\rho u_x u_z\right)=\overline{\rho u_z}\Delta u_x+\bar{\rho}\bar{u}_x\Delta u_z+\bar{u}_z\bar{u}_x\Delta\rho, \quad (42)$$

$$\Delta\left(\rho u_x u_z\right)=\overline{u_x u_z}\Delta\rho+\bar{\rho}\bar{u}_x\Delta u_z+\bar{\rho}\bar{u}_z\Delta u_x, \quad (43)$$

where $\hat{u}_z$ and $\overline{\rho u_z}$ is denoted as

$$\hat{u}_z=\frac{\overline{\rho u_z}}{\bar{\rho}} \quad (44)$$

and $\overline{\rho u_z}=\frac{1}{2}\left(\rho_L u_{zL}+\rho_R u_{zR}\right)$, (45)

using the identities in (44) and (45), equations (41) and (42) respectively, takes the form as

$$\Delta\left(\rho u_x u_z\right)=\bar{\rho}\hat{u}_x\Delta u_z+\bar{\rho}\bar{u}_z\Delta u_x+\bar{u}_z\bar{u}_x\Delta\rho, \quad (46)$$

$$\Delta\left(\rho u_x u_z\right)=\bar{\rho}\hat{u}_z\Delta u_x+\bar{\rho}\bar{u}_x\Delta u_z+\bar{u}_z\bar{u}_x\Delta\rho, \quad (47)$$

Either (46), (47) can be use or a general linear combination of these, or a general linear combination of (41), (42), and (43). In its place a arithmetic mean of (46) and (47) can be made andusing (30), we get

$$\Delta\left(\rho u_x u_z\right)=\bar{\rho}\tilde{u}_x\Delta u_z+\tilde{u}_z\bar{\rho}\Delta u_x+\bar{u}_x\bar{u}_z\Delta\rho. \quad (48)$$

Now in particular, seeking the case $u_x=u_z$ in $\Delta\left(\rho u_x u_z\right)$ which gives $\Delta\left(\rho u_z^2\right)$, in the equations, (42) and (43) respectively, we get

$$\Delta\left(\rho u_z^2\right)=\overline{u_z^2}\Delta\rho+2\bar{\rho}\bar{u}_z\Delta u_x, \quad (49)$$

and $\Delta\left(\rho u_z^2\right)=\bar{u}_z^2\Delta\rho+2\bar{\rho}\tilde{u}_z\Delta u_z$. (50)

Combined expression for (43), (46), (47) and (48) is $\Delta\left(\rho u_x u_z\right)=\xi^*\Delta\rho+\sigma^*\Delta u_x+\tau^*\Delta u_z$, (51)

where either $\xi^*=\bar{u}_x\bar{u}_z$, $\sigma^*=\bar{\rho}\bar{u}_z$ and $\tau^*=\bar{\rho}\hat{u}_x$, (52)

or

$$\xi^*=\bar{u}_x\bar{u}_z,\ \sigma^*=\bar{\rho}\hat{u}_z \text{ and } \tau^*=\bar{\rho}\bar{u}_x, \quad (53)$$

or

$$\xi^*=\overline{u_x u_z},\ \sigma^*=\bar{\rho}\bar{u}_z \text{ and } \tau^*=\bar{\rho}\bar{u}_x, \quad (54)$$

or

$$\xi^*=\bar{u}_x\bar{u}_z,\ \sigma^*=\bar{\rho}\tilde{u}_z \text{ and } \tau^*=\bar{\rho}\tilde{u}_x, \quad (55)$$

making use of equations (40) and (51) the third component of ΔF becomes

$$\Delta\left(\rho u_x u_z-B_x B_z\right)=\begin{pmatrix} \xi^*\Delta\rho+\sigma^*\Delta u_x+\tau^*\Delta u_z \\ -\bar{B}_x\Delta B_z-\bar{B}_z\Delta B_x \end{pmatrix}. \quad (56)$$

Fifth and sixth components of ΔF can be written as

$$\Delta\left(u_x B_y-B_x u_y\right)=\begin{pmatrix} \bar{u}_x\Delta B_y+\bar{B}_y\Delta u_x \\ -\bar{B}_x\Delta u_y-\bar{u}_y\Delta B_x \end{pmatrix}, \quad (57)$$

$$\Delta\left(u_x B_z-B_x u_z\right)=\begin{pmatrix} \bar{u}_x\Delta B_z+\bar{B}_z\Delta u_x \\ -\bar{B}_x\Delta u_z-\bar{u}_z\Delta B_x \end{pmatrix}, \quad (58)$$

using the identities described above the seventh component of ΔF becomes:

$$\left.\begin{aligned} &\left(\left(E+p^*\right)u_x-B_x\left(B_x u_x+B_y u_y+B_z u_z\right)\right)=\frac{\gamma}{\gamma-1}\Delta\left(pu_x\right) \\ &+\frac{1}{2}\Delta\left(\rho u_x^3\right)+\frac{1}{2}\Delta\left(\rho u_x u_y^2\right)+\frac{1}{2}\Delta\left(\rho u_x u_z^2\right)+\Delta\left(u_x B_y^2\right) \\ &+\Delta\left(u_x B_z^2\right)-\Delta\left(B_x B_y u_y\right)-\Delta\left(B_x B_z u_z\right) \end{aligned}\right\}. \quad (59)$$

Expression is made for $\Delta\left(\rho u_x u_z^2\right)$

$$\Delta\left(\rho u_x u_z^2\right)=2\bar{\rho}\bar{u}_z\hat{u}_x\Delta u_z+\overline{u_z^2}\bar{\rho}\Delta u_x+\overline{u_z^2}\bar{u}_x\Delta\rho, \quad (60)$$

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or

$$\Delta\left(\rho u_x u_z^2\right) = 2\beta^* \hat{u}_x \Delta u_z + \overline{u_z^2}\bar{\rho}\Delta u_x + \alpha^* \bar{u}_x \Delta\rho, \qquad (61)$$

combined expression for (60) and (61) is

$$\Delta\left(\rho u_x u_z^2\right) = \theta^* \Delta\rho + \eta^* \Delta u_x + 2\kappa^* \Delta u_z, \qquad (62)$$

where $\kappa^* = \bar{\rho}\bar{u}_z \hat{u}_x$, $\eta^* = \overline{u_z^2}\bar{\rho}$, $\theta^* = \overline{u_z^2}\bar{u}_x$, (63)

and $\kappa^* = 2\beta^* \bar{u}_x$, $\eta^* = \overline{u_z^2 \rho}$, $\theta^* = \alpha^* \bar{u}_x$, (64)

where β^* and α^* is defined earlier by the expression for $\Delta\left(\rho u_z^2\right)$.

There are two choices for the fifth term $\Delta\left(u_x B_y^2\right)$,of seventh component of ΔF among of them, the first expression is:

$$\Delta\left(u_x B_y^2\right) = \overline{B_y^2}\Delta u_x + 2\bar{u}_x \Delta B_y, \qquad (65)$$

where $\overline{B_y^2} = \frac{1}{2}\left(B_{yL}^2 + B_{yR}^2\right)$, (66)

and the second expression for $\Delta\left(u_x B_y^2\right)$ is

$$\Delta\left(u_x B_y^2\right) = \Delta\left(u_x B_y B_y\right) = \overline{u_x B_y}\Delta B_y + \bar{B}_y^2 \Delta u_x + \bar{B}_y \bar{u}_x \Delta B_y, \qquad (67)$$

with $\hat{u}_x = \frac{\overline{B_y u_x}}{\bar{B}_y}$ (68)

and $\overline{B_y u_x} = \frac{1}{2}\left(B_{yL} u_{xL} + B_{yR} u_{xR}\right)$, (69)

using (68), equation (67) takes the form

$$\Delta\left(u_x B_y^2\right) = \hat{u}_x \bar{B}_y \Delta B_y + \bar{B}_y^2 \Delta u_x + \bar{B}_y \bar{u}_x \Delta B_y$$
$$= \bar{B}_y^2 \Delta u_x + 2\hat{u}_x \bar{B}_y \Delta B_y, \qquad (70)$$

combined expression for (67) and (70) as below

$$\Delta\left(u_x B_y^2\right) = \phi \Delta u_x + 2\varphi \Delta B_y, \qquad (71)$$

where $\phi = \overline{B_y^2}$, $\varphi = \bar{u}_x \bar{B}_y$, (72)

$$\phi = \overline{B_y^2}, \ \varphi = \hat{u}_x \bar{B}_y. \qquad (73)$$

A similar expressions as $\Delta\left(u_x B_y^2\right)$ is also made for

$$\Delta\left(u_x B_z^2\right) = \overline{B_z^2}\Delta u_x + 2\bar{u}_x \bar{B}_z \Delta B_z, \qquad (74)$$

$$\Delta\left(u_x B_z^2\right) = \bar{B}_z^2 \Delta u_x + 2\hat{u}_x \bar{B}_z \Delta B_z, \qquad (75)$$

combined expression for (74) and (75) is

$$\Delta\left(u_x B_z^2\right) = \phi^* \Delta u_x + 2\varphi^* \Delta B_z, \qquad (76)$$

where $\phi^* = \overline{B_z^2}$, $\varphi^* = \bar{u}_x \bar{B}_z$, (77)

$$\phi^* = \overline{B_z^2}, \varphi^* = \hat{u}_x \bar{B}_z. \qquad (78)$$

As $\hat{u}_x = \frac{\overline{B_z u_x}}{\bar{B}_z}$ (79)

and

$$\overline{B_z u_x} = \frac{1}{2}\left(B_{zL} u_{xL} + B_{zR} u_{xR}\right). \ (80)$$

Expression for seventh term for $\Delta\left(B_x B_y u_y\right)$ for the seventh component of ΔF

$$\Delta\left(B_x B_y u_y\right) = \overline{B_x B_y}\Delta u_y + \bar{u}_y \bar{B}_x \Delta B_y + \bar{u}_y \bar{B}_y \Delta B_x, \qquad (81)$$

or

$$\Delta\left(B_x B_y u_y\right) = \bar{B}_x \bar{B}_y \Delta u_y + \bar{u}_y \bar{B}_x \Delta B_y + \overline{u_y B_y}\Delta B_x, \qquad (82)$$

or

$$\Delta\left(B_x B_y u_y\right) = \bar{B}_x \bar{B}_y \Delta u_y + \overline{u_y B_x}\Delta B_y + \bar{B}_y \bar{u}_y \Delta B_x, \qquad (83)$$

the combined expression of (80), (81) and (82) is given by (84) as below

$$\Delta\left(B_x B_y u_y\right) = \zeta \Delta u_y + \chi \Delta B_y + \psi \Delta B_x, \qquad (84)$$

where $\zeta = \overline{B_x B_y}$, $\chi = \bar{u}_y \bar{B}_x$ and $\psi = u_y B_y$ (85)

or

$$\zeta = \bar{B}_x \bar{B}_y, \chi = \bar{B}_x \bar{u}_y \text{ and } \psi = \overline{B_y u_y} \qquad (86)$$

or

$$\zeta = \bar{B}_y \bar{B}_x, \ \chi = \overline{B_x u_y} \text{ and } \psi = \bar{B}_y \bar{u}_y. \qquad (87)$$

$\hat{u}_x = \frac{\overline{B_y u_x}}{\bar{B}_y}$ (68)

A similar expression as $\Delta\left(B_x B_y u_y\right)$ is made for $\Delta\left(B_x B_z u_z\right)$ in the seventh component of ΔF

$$\Delta\left(B_x B_z u_z\right) = \overline{B_z u_z}\Delta B_x + \bar{u}_z \bar{B}_x \Delta B_z + \bar{B}_x \bar{B}_z \Delta u_x, \qquad (88)$$

$$\Delta\left(B_x B_z u_z\right) = \overline{B_z B_x}\Delta u_z + \bar{u}_z \bar{B}_x \Delta B_z + \bar{B}_z \bar{u}_z \Delta B_x, \qquad (89)$$

$$\Delta\left(B_x B_z u_z\right) = \overline{B_x u_z}\Delta B_z + \bar{u}_z \bar{B}_z \Delta B_x + \bar{B}_x \bar{B}_z \Delta u_z, \qquad (90)$$

combined expression for equations (88), (89) and (90) is

$$\Delta\left(B_x B_z u_z\right) = \chi^* \Delta B_z + \psi^* \Delta B_x + \zeta^* \Delta u_z, \qquad (91)$$

$\zeta^* = \overline{B_x B_z}$, $\chi^* = \bar{u}_z \bar{B}_x$, $\psi^* = \bar{u}_z \bar{B}_z$, (92) or

$$\zeta^* = \bar{B}_x \bar{B}_z, \ \chi^* = \bar{B}_x \bar{u}_z, \ \psi^* = \overline{u_z B_z}, \qquad (93)$$

or

$$\zeta^* = \bar{B}_x \bar{B}_z, \chi^* = \overline{B_x u_z}, \psi^* = \bar{u}_z \bar{B}_z. \qquad (94)$$

Summary of results for fourth component of ΔF

$$\begin{pmatrix} \left(E+p^*\right)u_x \\ -B_x\left(B_x u_x + B_y u_y + B_z u_z\right) \end{pmatrix} = \begin{pmatrix} \frac{\gamma}{\gamma-1}\bar{u}_x \Delta p \\ +\begin{pmatrix} \frac{\gamma}{\gamma-1}\bar{p}+\frac{3}{2}\varepsilon \\ +\frac{\eta}{2}+\frac{\eta^*}{2}+\phi^*+\phi^{**} \end{pmatrix}\Delta u_x \\ +\left(\frac{\delta}{2}+\frac{\theta}{2}+\frac{\theta^*}{2}\right)\Delta\rho + \left(\kappa-\zeta\right)\Delta u_y \\ +\left(\kappa^*-\zeta^*\right)\Delta u_z - \left(\psi+\psi^*\right)\Delta B_x. \\ +\left(2\varphi^*-\chi\right)\Delta B_y + \left(2\varphi^{**}-\chi^*\right)\Delta B_z \end{pmatrix}.(95)$$

Using equations (88) or (89) and (90) or any other linear combinations of these equations and rest of some symbols $\alpha,\beta,\mu,\omega,\xi,\sigma,\tau,\delta,\varepsilon,\theta,\eta,\kappa$ are reffered to [10] Glaister, Q takes the form

$$\tilde{Q} = \begin{bmatrix} \bar{u}_x & \rho & 0 & 0 & 0 & 0 & 0 \\ \alpha & 2\beta & 0 & 0 & 1 & \bar{B}_y & \bar{B}_z \\ \xi & \sigma & \tau & 0 & 0 & -\bar{B}_x & 0 \\ \xi^* & \sigma^* & 0 & \tau^* & 0 & 0 & -\bar{B}_x \\ 0 & \bar{B}_y & -\bar{B}_x & 0 & 0 & \bar{u}_x & 0 \\ 0 & \bar{B}_z & 0 & -\bar{B}_x & 0 & 0 & \bar{u}_x \\ \left(\frac{\delta}{2}+\frac{\theta}{2}+\frac{\theta^*}{2}\right) & \begin{pmatrix} \frac{3}{2}\varepsilon+\frac{\eta}{2}+\frac{1}{2}\eta^* \\ +\frac{\gamma p}{\gamma-1}+\overline{B_y^2}+\overline{B_z^2} \end{pmatrix} & \left(\kappa-\zeta\right) & \left(\kappa^*-\zeta^*\right) & \frac{\gamma}{\gamma-1}\bar{u}_x & 2\bar{B}_y\bar{u}_x-\chi & 2\bar{B}_z\bar{u}_x-\chi^* \end{bmatrix},(96)$$

where α^*, β^* are defined by the equations(32) and (33); ξ^*, σ^* and τ^* are given by equations (52)-(55) ; θ^*,η^* and κ^* are given by the equations(63) and (64).

There are two choices for α^* and β^* defined by the equations (32) and (33); Four choices for ξ^*, σ^* and τ^* are given by equations (52)-(55); Two choices for θ^*,η^* and κ^* given by the equations (63) and (64); Three choices for ζ, χ and ψ; ζ^*, χ^* and ψ^* are given by (85)-(87) and (92)-(94) respectively.Two alternatives for ϕ,φ in (72),(73); ϕ^*,φ^* defined by the equations (77), (78) respectively. Here we are seeking for that alternative of these symbols which gives a simplest possible form of eigenvalues.The matrix $A = P^{-1}Q$ can be redifined and takes the form as $\tilde{A} = \tilde{P}^{-1}\tilde{Q}$.

$$\tilde{A} = \begin{bmatrix} \bar{u}_x & \bar{\rho} & 0 & 0 & 0 & 0 & 0 \\ \frac{\alpha-\bar{u}_x^2}{\bar{\rho}} & \frac{2\beta}{\bar{\rho}}-\bar{u}_x & 0 & 0 & \frac{1}{\bar{\rho}} & \frac{\bar{B}_y}{\bar{\rho}} & \frac{\bar{B}_z}{\bar{\rho}} \\ \frac{\xi-\bar{u}_x\bar{u}_y}{\bar{\rho}} & \frac{\sigma}{\bar{\rho}}-\bar{u}_y & \frac{\tau}{\bar{\rho}} & 0 & 0 & -\frac{\bar{B}_x}{\bar{\rho}} & 0 \\ \frac{\xi^*-\bar{u}_x\bar{u}_z}{\bar{\rho}} & \frac{\sigma^*}{\bar{\rho}}-\bar{u}_z & 0 & \frac{\tau^*}{\bar{\rho}} & 0 & 0 & -\frac{\bar{B}_x}{\bar{\rho}} \\ \upsilon_1 & \upsilon_2 & \upsilon_3 & \upsilon_4 & \upsilon_5 & \upsilon_6 & \upsilon_7 \\ 0 & \bar{B}_y & -\bar{B}_x & 0 & 0 & \bar{u}_x & 0 \\ 0 & \bar{B}_z & 0 & -\bar{B}_x & 0 & 0 & \bar{u}_x \end{bmatrix},(97)$$

where

$$\upsilon_1 = \left(\gamma-1\right)\begin{bmatrix} \bar{u}_x^3 - \frac{1}{2}\bar{u}_x\overline{u_x^2} - \frac{1}{2}\mu\bar{u}_x - \frac{1}{2}\bar{u}_x\alpha^* + \frac{\omega}{\bar{\rho}}\left(\bar{u}_x\bar{u}_y-\xi\right) \\ +\frac{\beta^*}{\bar{\rho}}\left(\bar{u}_x\bar{u}_z-\xi^*\right)+\frac{\delta}{2}+\frac{\theta}{2}+\frac{\theta^*}{2}-\alpha\bar{u}_x \end{bmatrix},$$

$$\upsilon_2 = \left(\gamma-1\right)\left[\bar{\rho}\bar{u}_x^2 - \frac{1}{2}\bar{\rho}\overline{u_x^2} - \frac{1}{2}\mu\bar{\rho} - \frac{1}{2}\alpha^*\bar{\rho} + \omega\left(\bar{u}_y-\frac{\sigma}{\bar{\rho}}\right) + \beta^*\left(\bar{u}_z-\frac{\sigma^*}{\bar{\rho}}\right) - 2\beta\bar{u}_x + \frac{3}{2}\varepsilon + \frac{\eta}{2} + \frac{\eta^*}{2}\right] + \gamma\bar{p},$$

$$\upsilon_3 = \left(\gamma-1\right)\left(-\tau^*\bar{u}_z + \bar{B}_x\bar{B}_z + \kappa^* - \zeta^*\right),$$

$$\upsilon_4 = \left(\gamma-1\right)\left(-\tau\bar{u}_y + \bar{B}_x\bar{B}_y + \kappa - \zeta\right),$$

$$\upsilon_5 = \bar{u}_x,$$

$$\upsilon_6 = \left(\gamma-1\right)\left(\bar{u}_y\bar{B}_x - \chi\right),$$

$$\upsilon_7 = \left(\gamma-1\right)\left(\bar{u}_z\bar{B}_x - \chi^*\right).$$

The resultant matrix $\tilde{A}$ using the identities takes the form as follows.

$$\tilde{A} = \begin{bmatrix} \bar{u}_x & \bar{\rho} & 0 & 0 & 0 & 0 & 0 \\ \frac{\alpha-\bar{u}_x^2}{\bar{\rho}} & \frac{2\beta}{\bar{\rho}}-\bar{u}_x & 0 & 0 & \frac{1}{\bar{\rho}} & \frac{\bar{B}_y}{\bar{\rho}} & \frac{\bar{B}_z}{\bar{\rho}} \\ 0 & 0 & \frac{\tau}{\bar{\rho}} & 0 & 0 & -\frac{\bar{B}_x}{\bar{\rho}} & 0 \\ 0 & 0 & 0 & \bar{u}_x & 0 & 0 & -\frac{\bar{B}_x}{\bar{\rho}} \\ 0 & \upsilon_2 & \upsilon_3 & 0 & \bar{u}_x & 0 & 0 \\ 0 & \bar{B}_y & -\bar{B}_x & 0 & 0 & \bar{u}_x & 0 \\ 0 & \bar{B}_z & 0 & -\bar{B}_x & 0 & 0 & \bar{u}_x \end{bmatrix}.(98)$$

Here three cases are described using resultant matrix $\tilde{A}$ in (98) and analyzed behaviour of eigenvectors of the jacobian matrices so constructed using identities obtained after applying arithmetic averaging on the flux function and the vector of conservative variables.

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CASE 1:- When $\alpha = \overline{u_x^2}$, $\beta = \bar{\rho}\bar{u}_x$, and $\tau = \bar{\rho}\bar{u}_x$.

$$\tilde{A} = \begin{bmatrix} \bar{u}_x & \bar{\rho} & 0 & 0 & 0 & 0 & 0 \\ \dfrac{\overline{u_x^2} - \bar{u}_x^2}{\bar{\rho}} & \bar{u}_x & 0 & 0 & \dfrac{1}{\bar{\rho}} & \dfrac{\bar{B}_y}{\bar{\rho}} & \dfrac{\bar{B}_z}{\bar{\rho}} \\ 0 & 0 & \bar{u}_x & 0 & 0 & -\dfrac{\bar{B}_x}{\bar{\rho}} & 0 \\ 0 & 0 & 0 & \bar{u}_x & 0 & 0 & -\dfrac{\bar{B}_x}{\bar{\rho}} \\ 0 & \gamma \bar{p} & 0 & 0 & \bar{u}_x & 0 & 0 \\ 0 & \bar{B}_y & -\bar{B}_x & 0 & 0 & \bar{u}_x & 0 \\ 0 & \bar{B}_z & 0 & -\bar{B}_x & 0 & 0 & \bar{u}_x \end{bmatrix}, (99)$$

we get the matrix $\tilde{A}$ whose eigenvalues are:

$$\left.\begin{array}{l} \lambda_1 = \bar{u}_x + c_{f_1}, \lambda_2 = \bar{u}_x - c_{f_1}, \lambda_3 = \bar{u}_x + c_{f_2}, \\ \lambda_4 = \bar{u}_x - c_{f_2}, \lambda_5 = \bar{u}_x + c_{f_3}, \lambda_7 = \bar{u}_x, \end{array}\right\}, \quad (100)$$

$$\lambda_6 = \bar{u}_x - c_{f_3},$$

where

$$c_{f_1} = \frac{B_x}{\sqrt{\rho}}, \quad (101)$$

$$c_{f_2}^2 = \frac{\bar{c}^2 + \frac{1}{4}\left(\Delta u_x^2\right) + \sqrt{\left(\bar{c}^2 + \frac{1}{4}\left(\Delta u_x\right)^2\right)^2 - 4b_x^2\left(\tilde{a}^2 + \frac{1}{4}\left(\Delta u_x\right)^2\right)}}{2}, \quad (102)$$

$$c_{f_3}^2 = \frac{\bar{c}^2 + \frac{1}{4}\left(\Delta u_x^2\right) - \sqrt{\left(\bar{c}^2 + \frac{1}{4}\left(\Delta u_x\right)^2\right)^2 - 4b_x^2\left(\tilde{a}^2 + \frac{1}{4}\left(\Delta u_x\right)^2\right)}}{2}, \quad (103)$$

where

$$\bar{c}^2 = \tilde{a}^2 + \tilde{b}^2 = \frac{\gamma \bar{p}}{\bar{\rho}} + \frac{\bar{B}_x^2}{\bar{\rho}} + \frac{\bar{B}_y^2}{\bar{\rho}} + \frac{\bar{B}_z^2}{\bar{\rho}}. \quad (104)$$

We have the identity

$$\overline{ab} - \bar{a}\bar{b} = \frac{1}{2}\left(a_L b_L + a_R b_R\right) - \frac{1}{2}\left(a_L + a_R\right)\frac{1}{2}\left(b_L + b_R\right)$$

$$= \frac{1}{4}\left(a_R - a_L\right)\left(b_R - b_L\right) = \frac{1}{4}\Delta a \Delta b, \quad (105)$$

using $a = b = u_x$ in (105) we get

$$\overline{u_x^2} - \bar{u}_x^2 = \frac{1}{4}\left(\Delta u_x\right)^2. \quad (106)$$

Eigenvalues of this matrix in (99) are:

$$r_1 = \begin{pmatrix} 1, -\dfrac{c_{f_1}}{\bar{\rho}}, \dfrac{\bar{B}_x\bar{B}_y c_{f_1}}{\bar{\rho}^2\left(c_{f_1}^2 - b_x^2\right)}, \dfrac{\bar{B}_x\bar{B}_z c_{f_1}}{\bar{\rho}^2\left(c_{f_1}^2 - b_x^2\right)}, c_{f_1}^2 - \dfrac{1}{4}\left(\Delta u\right)^2 \\ -\dfrac{c_{f_1}^2 b_y^2}{\left(c_{f_1}^2 - b_x^2\right)} - \dfrac{c_{f_1}^2 b_z^2}{\left(c_{f_1}^2 - b_x^2\right)}, \dfrac{c_{f_1}^2 \bar{B}_y}{\bar{\rho}\left(c_{f_1}^2 - b_x^2\right)}, \dfrac{c_{f_1}^2 \bar{B}_z}{\bar{\rho}\left(c_{f_1}^2 - b_x^2\right)} \end{pmatrix}, \quad (107)$$

$$r_2 = \begin{pmatrix} 1, +\dfrac{c_{f_1}}{\bar{\rho}}, \dfrac{-\bar{B}_x\bar{B}_y c_{f_1}}{\bar{\rho}^2\left(c_{f_1}^2 - b_x^2\right)}, \dfrac{-\bar{B}_x\bar{B}_z c_{f_1}}{\bar{\rho}^2\left(c_{f_1}^2 - b_x^2\right)}, c_{f_1}^2 - \dfrac{1}{4}\left(\Delta u\right)^2 \\ -\dfrac{c_{f_1}^2 b_y^2}{\left(c_{f_1}^2 - b_x^2\right)} - \dfrac{c_{f_1}^2 b_z^2}{\left(c_{f_1}^2 - b_x^2\right)}, \dfrac{c_{f_1}^2 \bar{B}_y}{\bar{\rho}\left(c_{f_1}^2 - b_x^2\right)}, \dfrac{c_{f_1}^2 \bar{B}_z}{\bar{\rho}\left(c_{f_1}^2 - b_x^2\right)} \end{pmatrix}, \quad (108)$$

$$r_3 = \begin{pmatrix} 1, -\dfrac{c_{f_2}}{\bar{\rho}}, \dfrac{\bar{B}_x\bar{B}_y c_{f_2}}{\bar{\rho}^2\left(c_{f_2}^2 - b_x^2\right)}, \dfrac{\bar{B}_x\bar{B}_z c_{f_2}}{\bar{\rho}^2\left(c_{f_2}^2 - b_x^2\right)}, c_{f_2}^2 - \dfrac{1}{4}\left(\Delta u\right)^2 \\ -\dfrac{c_{f_2}^2 b_y^2}{\left(c_{f_2}^2 - b_x^2\right)} - \dfrac{c_{f_2}^2 b_z^2}{\left(c_{f_2}^2 - b_x^2\right)}, \dfrac{c_{f_2}^2 \bar{B}_y}{\bar{\rho}\left(c_{f_2}^2 - b_x^2\right)}, \dfrac{c_{f_2}^2 \bar{B}_z}{\bar{\rho}\left(c_{f_2}^2 - b_x^2\right)} \end{pmatrix}, (109)$$

$$r_4 = \begin{pmatrix} 1, +\dfrac{c_{f_2}}{\bar{\rho}}, \dfrac{-\bar{B}_x\bar{B}_y c_{f_2}}{\bar{\rho}^2\left(c_{f_2}^2 - b_x^2\right)}, \dfrac{-\bar{B}_x\bar{B}_z c_{f_2}}{\bar{\rho}^2\left(c_{f_2}^2 - b_x^2\right)}, c_{f_2}^2 \\ -\dfrac{1}{4}\left(\Delta u\right)^2 - \dfrac{c_{f_2}^2 b_y^2}{\left(c_{f_2}^2 - b_x^2\right)} - \dfrac{c_{f_2}^2 b_z^2}{\left(c_{f_2}^2 - b_x^2\right)}, \dfrac{c_{f_2}^2 \bar{B}_y}{\bar{\rho}\left(c_{f_2}^2 - b_x^2\right)}, \dfrac{c_{f_2}^2 \bar{B}_z}{\bar{\rho}\left(c_{f_2}^2 - b_x^2\right)} \end{pmatrix} (110)$$

$$r_5 = \begin{pmatrix} 1, -\dfrac{c_{f_3}}{\bar{\rho}}, \dfrac{\bar{B}_x\bar{B}_y c_{f_3}}{\bar{\rho}^2\left(c_{f_3}^2 - b_x^2\right)}, \dfrac{\bar{B}_x\bar{B}_z c_{f_3}}{\bar{\rho}^2\left(c_{f_3}^2 - b_x^2\right)}, c_{f_3}^2 - \dfrac{1}{4}\left(\Delta u\right)^2 \\ -\dfrac{c_{f_3}^2 b_y^2}{\left(c_{f_3}^2 - b_x^2\right)} - \dfrac{c_{f_3}^2 b_z^2}{\left(c_{f_3}^2 - b_x^2\right)}, \dfrac{c_{f_3}^2 \bar{B}_y}{\bar{\rho}\left(c_{f_3}^2 - b_x^2\right)}, \dfrac{c_{f_3}^2 \bar{B}_z}{\bar{\rho}\left(c_{f_3}^2 - b_x^2\right)} \end{pmatrix}, \quad (111)$$

$$r_6 = \begin{pmatrix} 1, \dfrac{c_{f_3}}{\bar{\rho}}, \dfrac{-\bar{B}_x\bar{B}_y c_{f_3}}{\bar{\rho}^2\left(c_{f_3}^2 - b_x^2\right)}, \dfrac{-\bar{B}_x\bar{B}_z c_{f_3}}{\bar{\rho}^2\left(c_{f_3}^2 - b_x^2\right)}, c_{f_3}^2 - \dfrac{1}{4}\left(\Delta u\right)^2 \\ -\dfrac{c_{f_3}^2 b_y^2}{\left(c_{f_3}^2 - b_x^2\right)} - \dfrac{c_{f_3}^2 b_z^2}{\left(c_{f_3}^2 - b_x^2\right)}, \dfrac{c_{f_3}^2 \bar{B}_y}{\bar{\rho}\left(c_{f_3}^2 - b_x^2\right)}, \dfrac{c_{f_3}^2 \bar{B}_z}{\bar{\rho}\left(c_{f_3}^2 - b_x^2\right)} \end{pmatrix}, \quad (112)$$

$$r_7 = \left(1, 0, 0, 0, \bar{u}_x^2 - \overline{u_x^2}, 0, 0\right). (113)$$

CASE 2:- When $\alpha = \bar{u}_x^2$, $\beta = \bar{\rho}\tilde{u}_x$ and $\tau = \bar{\rho}\hat{u}_x$.

$$\tilde{A} = \begin{bmatrix} \bar{u}_x & \bar{\rho} & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{u}_x & 0 & 0 & \dfrac{1}{\bar{\rho}} & \dfrac{\bar{B}_y}{\bar{\rho}} & \dfrac{\bar{B}_z}{\bar{\rho}} \\ 0 & 0 & \bar{u}_x & 0 & 0 & -\dfrac{\bar{B}_x}{\bar{\rho}} & 0 \\ 0 & 0 & 0 & \bar{u}_x & 0 & 0 & -\dfrac{\bar{B}_x}{\bar{\rho}} \\ 0 & \gamma \bar{p} & 0 & 0 & \bar{u}_x & 0 & 0 \\ 0 & \bar{B}_y & -\bar{B}_x & 0 & 0 & \bar{u}_x & 0 \\ 0 & \bar{B}_z & 0 & -\bar{B}_x & 0 & 0 & \bar{u}_x \end{bmatrix}, (114)$$

the eigenvalues matrix $\tilde{A}$ are:

$$\left.\begin{array}{l} \lambda_1 = \tilde{u}_x + c_{g_1}, \lambda_2 = \tilde{u}_x - c_{g_1}, \lambda_4 = \tilde{u}_x - c_{g_2}, \\ \lambda_5 = \tilde{u}_x + c_{g_3}, \lambda_6 = \tilde{u}_x - c_{g_3}, \lambda_7 = \bar{u}_x, \end{array}\right\} \quad (115)$$

$$\lambda_3 = \tilde{u}_x + c_{g_2},$$

where

$$c_{g_1} = \frac{B_x}{\sqrt{\rho}}, \quad (116)$$

where

$$c_{g_2}^2 = c_{G_1}^2 + \frac{1}{4}\left(\hat{u}_x - \bar{u}_x\right)^2 ; c_{G_1}^2 = \frac{\bar{c}^2 + \sqrt{\bar{c}^4 - 4\tilde{a}^2 b_x^2}}{2}, \quad (117)$$

$$c_{g_3}^2 = c_{G_2}^2 + \frac{1}{4}\left(\hat{u}_x - \bar{u}_x\right)^2 ; c_{G_2}^2 = \frac{\bar{c}^2 - \sqrt{\bar{c}^4 - 4\tilde{a}^2 b_x^2}}{2}, \quad (118)$$

expressions in the equations (117) and (118) may also be expressed a

$$c_{g_1}^2 = c_{G_1}^2 + \frac{\left(\Delta\rho\Delta u_x\right)^2}{64\bar{\rho}^2} ; c_{g2}^2 = c_{G2}^2 + \frac{\left(\Delta\rho\Delta u_x\right)^2}{64\bar{\rho}^2}. \quad (119)$$

The right eigenvectors of this matrix are:

$$\tilde{r}_1 = \begin{pmatrix} 1, -\dfrac{c_{g_1}}{\bar{\rho}}, \dfrac{\bar{B}_x\bar{B}_y c_{g_1}}{\bar{\rho}^2\left(c_{g_1}^2 - b_x^2\right)}, \dfrac{\bar{B}_x\bar{B}_z c_{g_1}}{\bar{\rho}^2\left(c_{g_1}^2 - b_x^2\right)}, \tilde{a}^2, \\ \dfrac{\bar{B}_y c_{g_1}^2}{\bar{\rho}\left(c_{g_1}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{g_1}^2}{\bar{\rho}\left(c_{g_1}^2 - b_x^2\right)} \end{pmatrix}, \quad (120)$$

$$\tilde{r}_2 = \begin{pmatrix} 1, -\dfrac{c_{g_2}}{\bar{\rho}}, \dfrac{\bar{B}_x\bar{B}_y c_{g_2}}{\bar{\rho}^2\left(c_{g_2}^2 - b_x^2\right)}, \dfrac{\bar{B}_x\bar{B}_z c_{g_2}}{\bar{\rho}^2\left(c_{g_2}^2 - b_x^2\right)}, \tilde{a}^2, \\ \dfrac{\bar{B}_y c_{g_2}^2}{\bar{\rho}\left(c_{g_2}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{g_2}^2}{\bar{\rho}\left(c_{g_2}^2 - b_x^2\right)} \end{pmatrix}, \quad (121)$$

(121)

$$\tilde{r}_3 = \begin{pmatrix} 1, \dfrac{c_{g_1}}{\bar{\rho}}, \dfrac{-\bar{B}_x\bar{B}_y c_{g_1}}{\bar{\rho}^2\left(c_{g_1}^2 - b_x^2\right)}, \dfrac{-\bar{B}_x\bar{B}_z c_{g_1}}{\bar{\rho}^2\left(c_{g_1}^2 - b_x^2\right)}, \tilde{a}^2, \\ \dfrac{\bar{B}_y c_{g_1}^2}{\bar{\rho}\left(c_{g_1}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{g_1}^2}{\bar{\rho}\left(c_{g_1}^2 - b_x^2\right)} \end{pmatrix}, \quad (122)$$

(122)

$$\tilde{r}_4 = \begin{pmatrix} 1, \dfrac{c_{g_2}}{\bar{\rho}}, \dfrac{-\bar{B}_x\bar{B}_y c_{g_2}}{\bar{\rho}^2\left(c_{g_2}^2 - b_x^2\right)}, \dfrac{-\bar{B}_x\bar{B}_z c_{g_2}}{\bar{\rho}^2\left(c_{g_2}^2 - b_x^2\right)}, \tilde{a}^2, \\ \dfrac{\bar{B}_y c_{g_2}^2}{\bar{\rho}\left(c_{g_2}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{g_2}^2}{\bar{\rho}\left(c_{g_2}^2 - b_x^2\right)} \end{pmatrix}, \quad (123)$$

$$\tilde{r}_5 = \begin{pmatrix} 1, -\dfrac{c_{g_3}}{\bar{\rho}}, \dfrac{\bar{B}_x\bar{B}_y c_{g_3}}{\bar{\rho}^2\left(c_{g_3}^2 - b_x^2\right)}, \dfrac{\bar{B}_x\bar{B}_z c_{g_3}}{\bar{\rho}^2\left(c_{g_3}^2 - b_x^2\right)}, \tilde{a}^2, \\ \dfrac{\bar{B}_y c_{g_3}^2}{\bar{\rho}\left(c_{g_3}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{g_3}^2}{\bar{\rho}\left(c_{g_3}^2 - b_x^2\right)} \end{pmatrix}, \quad (124)$$

$$\tilde{r}_6 = \begin{pmatrix} 1, \dfrac{c_{g_3}}{\bar{\rho}}, \dfrac{-\bar{B}_x\bar{B}_y c_{g_3}}{\bar{\rho}^2\left(c_{g_3}^2 - b_x^2\right)}, \dfrac{-\bar{B}_x\bar{B}_z c_{g_3}}{\bar{\rho}^2\left(c_{g_3}^2 - b_x^2\right)}, \tilde{a}^2, \\ \dfrac{\bar{B}_y c_{g_3}^2}{\bar{\rho}\left(c_{g_3}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{g_3}^2}{\bar{\rho}\left(c_{g_3}^2 - b_x^2\right)} \end{pmatrix}, \quad (125)$$

$$\tilde{r}_7 = \left(1, 0, 0, \tilde{a}^2, 0, 0, 0\right). \quad (126)$$

CASE 3:-When $\alpha = \bar{u}_x^2$, $\beta = \bar{\rho}\bar{u}_x$, $\mu = \overline{u_y^2}$

$$\tilde{A} = \begin{bmatrix} \bar{u}_x & \bar{\rho} & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{u}_x & 0 & 0 & \dfrac{1}{\bar{\rho}} & \dfrac{\bar{B}_y}{\bar{\rho}} & \dfrac{\bar{B}_z}{\bar{\rho}} \\ 0 & 0 & \bar{u}_x & 0 & 0 & -\dfrac{\bar{B}_x}{\bar{\rho}} & 0 \\ 0 & 0 & 0 & \bar{u}_x & 0 & 0 & -\dfrac{\bar{B}_x}{\bar{\rho}} \\ 0 & \aleph_1 & \aleph_2 & 0 & \bar{u}_x & 0 & 0 \\ 0 & \bar{B}_y & -\bar{B}_x & 0 & 0 & \bar{u}_x & 0 \\ 0 & \bar{B}_z & 0 & -\bar{B}_x & 0 & 0 & \bar{u}_x \end{bmatrix}, \quad (127)$$

where

$$\aleph_1 = (\gamma - 1)\bar{\rho}\bar{u}_x\left(\hat{u} - \bar{u}_x\right) + \gamma\bar{p},$$

$$\aleph_2 = (\gamma - 1)\bar{\rho}\left(\hat{u}_x - \bar{u}_x\right)\bar{u}_y,$$

the eigenvalues matrix $\tilde{A}$ in (127) are:

$$\left.\begin{aligned} &\lambda_1 = \bar{u}_x + c_{h_1}, \lambda_2 = \bar{u}_x - c_{h_1}, \lambda_3 = \bar{u}_x + c_{h_2}, \\ &\lambda_4 = \bar{u}_x - c_{h_2}, \lambda_5 = \bar{u}_x + c_{h_3}, \lambda_6 = \bar{u}_x - c_{h_3}, \lambda_7 = \bar{u}_x, \end{aligned}\right\} \quad (128)$$

$$c_{h_1} = \frac{B_x}{\sqrt{\rho}}, \quad (129)$$

$$c_{h_2}^2 = \frac{\bar{c}^2 + c_b^2 + \sqrt{\left(\bar{c}^2 + c_b^2\right)^2 - 4\left(a^2 b_x^2 + \dfrac{c_b^2}{\bar{u}_x}\dfrac{\bar{B}_x}{\bar{\rho}}\left(\bar{u}_x\bar{B}_x + \bar{u}_y\bar{B}_y\right)\right)}}{2}, \quad (130)$$

$$c_{h_3}^2 = \frac{\bar{c}^2 + \bar{c}_b^2 - \sqrt{\left(\bar{c}^2 + \bar{c}_b^2\right)^2 - 4\left(a^2 b_x^2 + \dfrac{\bar{c}_b^2}{\bar{u}_x}\dfrac{\bar{B}_x}{\bar{\rho}}\left(\bar{u}_x\bar{B}_x + \bar{u}_y\bar{B}_y\right)\right)}}{2}, \quad (131)$$

$$\tilde{r}_2 = \begin{pmatrix} 1, \dfrac{c_{h_1}}{\bar{\rho}}, \dfrac{-\bar{B}_x\bar{B}_y c_{h_1}}{\bar{\rho}^2\left(c_{h1}^2 - b_x^2\right)}, \dfrac{-\bar{B}_x\bar{B}_z c_{h_1}}{\bar{\rho}^2\left(c_{h1}^2 - b_x^2\right)}, c_{h_1}^2 \\ -\dfrac{b_y^2 c_{h_1}^2}{\left(c_{h_1}^2 - b_x^2\right)} - \dfrac{b_z^2 c_{h_1}^2}{\left(c_{h_1}^2 - b_x^2\right)}, \dfrac{\bar{B}_y c_{h_1}^2}{\bar{\rho}\left(c_{h1}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{h_1}^2}{\bar{\rho}\left(c_{h1}^2 - b_x^2\right)} \end{pmatrix}, \quad (134)$$

$$\tilde{r}_3 = \begin{pmatrix} 1, \dfrac{-c_{h_2}}{\bar{\rho}}, \dfrac{\bar{B}_x\bar{B}_y c_{h_2}}{\bar{\rho}^2\left(c_{h_2}^2 - b_x^2\right)}, \dfrac{\bar{B}_x\bar{B}_z c_{h_2}}{\bar{\rho}^2\left(c_{h_2}^2 - b_x^2\right)}, c_{h_2}^2 \\ -\dfrac{b_y^2 c_{h_2}^2}{\left(c_{h_2}^2 - b_x^2\right)} - \dfrac{b_z^2 c_{h_2}^2}{\left(c_{h_2}^2 - b_x^2\right)}, \dfrac{\bar{B}_y c_{h_2}^2}{\bar{\rho}\left(c_{h_2}^2 - b_x^2\right)}, \dfrac{\bar{B}_z c_{h_2}^2}{\bar{\rho}\left(c_{h_2}^2 - b_x^2\right)} \end{pmatrix}, \quad (135)$$

$$\tilde{r}_4 = \begin{pmatrix} 1, \frac{c_{h_2}}{\bar\rho}, \frac{-\bar B_x \bar B_y c_{h_2}}{\bar\rho^2\left(c_{h_2}^2 - b_x^2\right)}, \frac{-\bar B_x \bar B_z c_{h_2}}{\bar\rho^2\left(c_{h_2}^2-b_x^2\right)}, \\ c_{h_2}^2 - \frac{b_y^2 c_{h_2}^2}{\left(c_{h_2}^2-b_x^2\right)} - \frac{b_z^2 c_{h_2}^2}{\left(c_{h_2}^2-b_x^2\right)}, \frac{\bar B_y c_{h_2}^2}{\bar\rho\left(c_{h_2}^2-b_x^2\right)}, \frac{\bar B_z c_{h_2}^2}{\bar\rho\left(c_{h_2}^2-b_x^2\right)} \end{pmatrix}, \quad (136)$$

$$\tilde{r}_5 = \begin{pmatrix} 1, \frac{-c_{h_3}}{\bar\rho}, \frac{\bar B_x \bar B_y c_{h_3}}{\bar\rho^2\left(c_{h_3}^2 - b_x^2\right)}, \frac{\bar B_x \bar B_z c_{h_3}}{\bar\rho^2\left(c_{h_3}^2-b_x^2\right)}, c_{h_3}^2 \\ - \frac{b_y^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)} - \frac{b_z^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_y c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_z c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)} \end{pmatrix}, \quad (137)$$

$$\tilde{r}_6 = \begin{pmatrix} 1, \frac{c_{h_3}}{\bar\rho}, \frac{-\bar B_x \bar B_y c_{h_3}}{\bar\rho^2\left(c_{h_3}^2 - b_x^2\right)}, \frac{-\bar B_x \bar B_z c_{h_3}}{\bar\rho^2\left(c_{h_3}^2-b_x^2\right)}, c_{h_3}^2 \\ - \frac{b_y^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)} - \frac{b_z^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_y c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_z c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)} \end{pmatrix}, \quad (138)$$

$$\tilde{r}_7 = \left(1,0,0,0,0,0,0\right). \quad (139)$$

$$\tilde{r}_4 = \begin{pmatrix} 1, \frac{c_{h_2}}{\bar\rho}, \frac{-\bar B_x \bar B_y c_{h_2}}{\bar\rho^2\left(c_{h_2}^2 - b_x^2\right)}, \frac{-\bar B_x \bar B_z c_{h_2}}{\bar\rho^2\left(c_{h_2}^2-b_x^2\right)}, \\ c_{h_2}^2 - \frac{b_y^2 c_{h_2}^2}{\left(c_{h_2}^2-b_x^2\right)} - \frac{b_z^2 c_{h_2}^2}{\left(c_{h_2}^2-b_x^2\right)}, \frac{\bar B_y c_{h_2}^2}{\bar\rho\left(c_{h_2}^2-b_x^2\right)}, \frac{\bar B_z c_{h_2}^2}{\bar\rho\left(c_{h_2}^2-b_x^2\right)} \end{pmatrix}, \quad (136)$$

$$\tilde{r}_5 = \begin{pmatrix} 1, \frac{-c_{h_3}}{\bar\rho}, \frac{\bar B_x \bar B_y c_{h_3}}{\bar\rho^2\left(c_{h_3}^2 - b_x^2\right)}, \frac{\bar B_x \bar B_z c_{h_3}}{\bar\rho^2\left(c_{h_3}^2-b_x^2\right)}, c_{h_3}^2 \\ - \frac{b_y^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)} - \frac{b_z^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_y c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_z c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)} \end{pmatrix}, \quad (137)$$

$$\tilde{r}_6 = \begin{pmatrix} 1, \frac{c_{h_3}}{\bar\rho}, \frac{-\bar B_x \bar B_y c_{h_3}}{\bar\rho^2\left(c_{h_3}^2 - b_x^2\right)}, \frac{-\bar B_x \bar B_z c_{h_3}}{\bar\rho^2\left(c_{h_3}^2-b_x^2\right)}, c_{h_3}^2 \\ - \frac{b_y^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)} - \frac{b_z^2 c_{h_3}^2}{\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_y c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)}, \frac{\bar B_z c_{h_3}^2}{\bar\rho\left(c_{h_3}^2-b_x^2\right)} \end{pmatrix}, \quad (138)$$

$$\tilde{r}_7 = \left(1,0,0,0,0,0,0\right). \quad (139)$$

5. Discusson and Conclusion

In this paper we analyze a Riemann solver in an ideal magnetogasdynamics equations using arithmetic averaging. We have 192 alternatives for the symbols used in place of physical variables; it is not possible to make 192 Jacobian matrices and to find their eigenvalues and corresponding eigenvectors, therefore we made three Jacobian matrices out of all alternative choices using averaging procedure and hence obtained its eigenvalues and their eigenvectors.

Further, discuss the flux and behaviour characteristic wave's speed of Jacobian matrices. Fast waves c_{f_2}; c_{g_2} increase and slow wave c_{f_3}; c_{g_3} decrease from waves c_2^*; c_3^*, due to $\overline{u_x^2} > \bar{u}^2$ and $\frac{1}{4}\left(\hat{u}_x - \bar{u}_x\right)^2$, respectively. Thus, characteristic-speeds λ_3, λ_5 increase in the shock region and the characteristic speeds λ_4 and λ_6 decrease in rarefaction region.

The fast wave c_{h_2} increases due to an additional term $\frac{\gamma-1}{4}\bar{u}_x \frac{(\Delta u_x)(\Delta\rho)}{\bar\rho}$ of (128) in compared to (11). Thus, characteristic speed λ_3 increases in the shock region and the characteristic speed λ_4 decreases in rarefaction region in (128). Thus, characteristic speed λ_3 decreases in the shock region and forth characteristic speed λ_4 increases in rarefaction region. The magnetic field interaction does not effect on characteristic speed λ_5 in all cases.

The formula for the flux across $x = 0$, when the equations in one dimensional conservation form.

$$F^*(U_L, U_R) = \frac{F_L + F_R}{2} - \frac{1}{2}\sum_{k=1}^{5} a_k \left|\lambda_k\right| A.r_k,$$

where a_k is the wave strength

Many numerical methods are available to determine the solution of flow variables as finite difference methods, finite volume methods, Lax-Friedriche's Scheme, finite element methods, Pseudo-spectral methods, compact scheme and total diminishing type's schemes to determine the flow variables of conservation form (1). In this study, Riemann solvers for magnetogasdynamics (MGD) analyze by arithmetic averaging.The Riemann solver has a seven-wave structure, where six waves are used in MGD, and the seventh wave is free from the magnetic field effect in each case. Each component of flux and the vector of the conservative variables are expressed by the average procedure. Determine the eigenvalues and their corresponding eigenvectors of the Jacobian matrices and compare its behaviour of fast wave slow wave and contact discontinuity.

Reference

[1] Lax P.D, Hyperbolic systems of conservation laws II, *Commun. Pure Appl. Math. X*, 537–566,(1957).

ISSN 2455-6378

[2] Jeffrey andTaniuti T, Non-linear Wave PropagationAcademic Press, *New YorkLondon*, (1964).

[3] Glimm J., G. Marshall G.andPlohr B., A generalized Riemann problem for quasi one-dimensional gas flows, *Adv. Appl. Math.*, 51–30,(1984).

[4] Courant R. and Friedrichs K.O., Supersonic Flow and Shock Waves,*Interscience.NewYork*,(1999).

[5] Chorin A.J., Random choice solutions of hyperbolic systems*J. Comput. Phys*, 22, 517–533,(1976).

[6] Toro E.F., Riemann Solvers and Numerical Methods for Fluid Dynamics, *2nd ed., Springer, Berlin*, (1997).

[7] Toro E.F., A fast Riemann solver with constant co-volume applied to the random choice method, *Int. J. Numer. Methods Fluids*, 9(4), 1145–1164,(1989).

[8] GlaisterP., An analysis of arithmetic averaging in approximate Riemann solvers with an application to steady, supercritical flows, *Computers & Mathematics with Applications*, 28(4), 77-85,(1994).

[9] Glaister P., An analysis of averaging procedures in a Riemann solver for compressible flows of a real gas, *Computers & Mathematics with Applications*, 33(3),(1997).

[10] Glaister, An analysis of averaging Procedures in a Riemann Solver for the two dimensional Euler equations, *Computers Math. Applic.*, 35(8), 66-77,(1998).

[11] Powell K.G., An approximate Riemann solver for magnetohydrodynamics Upwind and high resolution schemes[Hussaini et al., (eds)], Springer, Berlin, Heideberg ",pp 570-583,(1997).

[12] Woodward P.R.andDai.W., A Riemann solver for ideal magnetohydrodynamics, *Journal of Computational Physics*, 111 (2),(1994).

[13] Andrew L. Zachary,Andrea Malagoli and Phillip Colella, A higher-order Godunov method for the equations of ideal magnetohydrodynamics. *SIAM J. Sci. Comput*., 15(2), 263–284,(1992).

[14] Yujin Liu and Wenhua Sun, Riemann problem and wave interactions in Magnetogasdynamics, *J.Math. Anal. Appl.*,397,454-466,(2013).

[15] Shengtai Li, An HLLC Riemann Solver for Magnetohydrodynamics, *J. Computational Physics*, 203 (1) 344-257,(2003).

[16] Roe P. L. and Balsara D. S., Notes on the eigen system of magnetohydrodynamics ,*SIAM J. APPL. MATH*., 56(1), 57-67,(1996).

[17] Shen Chun, "The limits of Riemann solutions to the isentropic magnetogasdynamics", *Applied Mathematics Letters* , 24, 1124–1129,(2011).

[18] Singh R. and Singh L., Solution of the Riemann Problem in magnetogasdynamics *I. J. Nonlinear linear Mechanics*,67, 326-330,(2014).

[19] Raja Sekhar T. and Sharma V.D., Riemann problem and elementary wave interactions in isentropic magnetogasdynamics. *Nonlinear Anal. Real World Appl*., 11(2), 619–636,(2010).

[20] Raja Sekhar T. and Sharma V.D., Solution to the Riemann problem in a one dimensional magnetogasdynamics flow,*International Journal of Computer Mathematics*, 89(2), 200–216,(2012).