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Neighbour Resolving Partition of a Graph

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Abstract

Let $G = (V,E)$ be a simple, finite and connected graph. A partition $\Pi = \{V_1, V_2, \ldots V_k\}$ is called a resolving partition if the code of a vertex $u \in V(G)$ with respect to Π, denoted by $C_\Pi(u)$ and defined as $C_\Pi(u) = (d(u,V_1), \ldots, d(u, V_K))$ is distinct for distinct vertices. In [3], a new type of resolving partition is introduced in which only adjacent vertices are to be resolved. In this paper, this type of partition is given the name neighbour resolving partition. Neighbour resolving partition dimension is derived for various classes of graphs and some bounds for neighbour resolving partition dimension are derived

Keywords: *Resolving set, Neighbour Resolving Dimension, Neighbour Resolving Partition, Neighbour Resolving Partition Dimension.*

1. Introduction

Chartrand et al. introduced metric colouring of G [3]. In a metric colouring, the vertex set V(G) is partitioned into subsets, not necessarily independent and the resolution is demanded only for adjacent vertices. The minimum cardinality of such a adjacent vertices resolving partition is called the metric chromatic number of G and is denoted by $\mu(G)$. In this paper, the above type of resolving partition is name as neighbour resolving partition and the minimum cardinality is denoted by the same symbol $\mu(G)$. The value of $\mu(G)$ is determined for several classes of graphs and some bounds are obtained.

2.Neighbour Resolving Partition Dimension

Definition 2.1. Let $G = (V, E)$ be a simple, finite and connected graph. Let $\Pi = \{V_1, V_2, \ldots V_k\}$ be a partition of $V(G)$.

Π is called a resolving partition if $C_\Pi(u) = (d(u, V_1), \ldots, d(u, V_k))$ is distinct for distinct vertices u. If this condition is relaxed and only adjacent vertices need to be resolved by Π, then Π is called a neighbour resolving partition of G. The minimum cardinality of neighbour resolving partition is called neighbour resolving partition dimension of G and is denoted by $\mu(G)$.

Observation 2.2. Since any resolving partition is a neighbour resolving partition,

$\mu(G) \leq pd(G)$.

Observation 2.3. For any connected graph G,

$2 \leq \mu(G) \leq n$.

3. Neighbour Resolving Partition of Certain Graphs

Theorem 3.1. $\mu(K_n) = n$.

Proof. Let $\Pi = \{V_1, V_2, \ldots, V_k\}$ be a minimum neighbour resolving partition of K_n.

Suppose, $|V_i| \geq 2$. Let $u, v \in V_i$. Then, $d(u, V_j) = d(v, V_j)$ for every $j \neq i$, $1 \leq j \leq k$. u and v are adjacent and they are not resolved by Π. Therefore, $|V_i| = 1$ for every i. Therefore, $|\Pi| = n$. Therefore, $\mu(K_n) = n$.

Observation 3.2. Let Π be a neighbour resolving partition of $V(G)$. Let u, v be adjacent vertices in $V(G)$. If $d(u,w) = d(v,w)$ for all vertices $w \in V(G) - \{u,v\}$, then u and v belong to different classes of Π.

Theorem 3.3. $\mu(K_{1, n}) = 2$ where $K_{1,n}$ is a star graph.

Proof. Let u be the center vertex and $v_1, v_2, \ldots, v_n$ be the pendant vertices. Then,

$\Pi = \{\{u\}, \{v_1, v_2, \ldots, v_n\}\}$ is a minimum neighbour resolving partition of $K_{1,n}$. Therefore, $\mu(K_{1, n}) = 2$.

Theorem 3.4. Let $G = \mu(K_{m, n})$ be a complete bipartite graph. Then, $\mu(K_{m, n}) = 2$.

Proof. If V_1, V_2 are the partite sets of $K_{m,n}$, then $\Pi = \{V_1,V_2\}$ is a minimum neighbour resolving partition of $K_{m,n}$.

Theorem 3.5. Let P_n be a path. Then, $\mu(P_n) = 2$.

Proof. Let $V(P_n) = \{u_1,u_2,\dots,u_n\}$. Let $V_1 = \{u_1,u_3,\dots,u_n \text{ (or) } u_{n-1}\}$ according as n is odd or n is even. Let $V_2 = V(P_n) - V_1$. Then $\Pi = \{V_1, V_2\}$ is a neighbour resolving partition of P_n.

Theorem 3.6. Let C_n be a cycle. Then,

$$\mu(C_n) = \begin{cases} 3 \text{ if } n \text{ is odd} \\ 2 \text{ if } n \text{ is even} \end{cases}$$

Proof. Let $V(C_n) = \{u_1, u_2, \dots, u_n\}$.
Case (i): n is odd.
Let $\Pi = \{V_1, V_2, V_3\}$ where $V_1 = \{u_1, u_3, \dots, u_{n-2}\}$, $V_2 = \{u_2, u_4, \dots, u_{n-1}\}$, $V_3 = \{u_n\}$. Then, Π is a neighbour resolving partition of C_n. Therefore, $\mu(C_n) \leq 3$ if n is odd.
Suppose, $\mu(C_n) \leq 2$. Clearly, $\mu(C_n) \geq 2$. Therefore, $\mu(C_n) = 2$. Let $\Pi = \{V_1, V_2\}$ be a neighbour resolving partition of C_n. Suppose, $u_1 \in V_1$. Therefore, $C_\Pi(u_1) = (0, 1)$. Let $u_n \in V_1$. Then u_1 and u_n are adjacent and have the same code. Therefore, $u_n \in V_2$. Since $n \geq 3$, $u_{n-1} \in V_1$ and $u_{n-2} \in V_2$, $u_{n-3} \in V_1$ etc., since n is odd, $u_1 \in V_2$, a contradiction. Similar proof holds if $u_1 \in V_2$. Therefore, $\mu(C_n) = 3$.
Case (ii): n is even.
Let $\Pi = \{V_1, V_2\}$ where $V_1 = \{u_1, u_3, \dots, u_{n-1}\}$ and $V_2 = \{u_2, u_4,\dots,u_n\}$. Clearly, Π is a neighbour resolving partition of C_n. Therefore, $\mu(G) \leq 2$. Clearly, $\mu(C_n) \geq 2$.
Therefore, $\mu(G) = 2$.

Observation 3.7. $\mu(W_n) = 3$ where W_n is a Wheel graph.

Proof. Let u be the center and $\{u_1, u_2, \dots, u_n\}$ be the vertices on the cycle.
Case(i): n is odd. (say $n = 2k + 1$).
Let $\Pi = \{\{u, u_1, u_2, u_{2k+1}\}, \{u_3, u_5,\dots, u_{2k-1}\}, \{u_4, u_6,\dots,u_{2k}\}\}$. $C_\Pi(u) = (0, 1, 1)$, $C_\Pi(u_1) = (0, 2, 2)$, $C_\Pi(u_2) = (0, 1, 2)$, $C_\Pi(u_{2k+1}) = (0, 2, 1)$. The vertices in second and third elements of Π are non-adjacent. Therefore, Π is a neighbour resolving partition of G. Therefore, $\mu(G) \leq 3$. But G is not bipartite. Therefore, $\mu(G) = 3$.
Case(ii): n is even. (say $n = 2k$).
Let $\Pi = \{\{u,u_1,u_2,u_{2k}\},\{u_3,u_5,\dots,u_{2k-1}\}, \{u_4,u_6,\dots,u_{2k-2}\}\}$. $C_\Pi(u) = (0,1,1)$, $C_\Pi(u_1) = (0,2,2)$, $C_\Pi(u_2) = (0,1,2)$, $C_\Pi(u_{2k}) = (0,1,2)$. Therefore, Π is a neighbour resolving partition of G. Therefore, $\mu(G) \leq 3$. But G is not bipartite. Therefore, $\mu(G) = 3$.

Definition 3.8. A **fan** is defined to be a graph which is got by joining every vertex of P_{n-1}, ($n \geq 3$), with a new vertex and is denoted by $\boldsymbol{F_n}$.

Observation 3.9. $\mu(F_n) = 3$ where F_n is the fan graph.

Proof. Let u be the center and $\{u_1, u_2,\dots,u_n\}$ be the vertices on the path.
Case (i): n is odd. (say $n = 2k + 1$).
Let $\Pi = \{\{u, u_1, u_2, u_{2k+1}\}, \{u_3, u_5,\dots,u_{2k-1}\}, \{u_4, u_6,\dots,u_{2k}\}\}$. $C_\Pi(u) = (0,1,1)$, $C_\Pi(u_1) = (0, 2,2)$, $C_\Pi(u_2) = (0,1,2)$, $C_\Pi(u_{2k+1}) = (0,2,1)$. The vertices in second and third elements of Π are non-adjacent. Therefore, Π is a neighbour resolving partition of G. Therefore, $\mu(G) \leq 3$. But G is not bipartite. Therefore, $\mu(G) = 3$.
Case (ii): n is even. (say $n = 2k$).
Let $\Pi = \{\{u, u_1, u_2,u_{2k}\}, \{u_3, u_5,\dots, u_{2k-1}\}, \{u_4, u_6,\dots, u_{2k-2}\}\}$. $C_\Pi(u) = (0, 1, 1)$, $C_\Pi(u_1) = (0,2, 2)$, $C_\Pi(u_2) = (0,1,2)$, $C_\Pi(u_{2k}) = (0, 1, 2)$. Therefore, Π is a neighbour resolving partition of G. Therefore, $\mu(G) \leq 3$. But G is not bipartite. Therefore, $\mu(G) = 3$.

Definition 3.10. A **complete multipartite** is one that is simple and in which each vertex in one partitioned set is joined to every vertex that is not in the same subset. A complete k-partite graph with cardinalities of partitioned sets a_1, a_2, a_3, …, a_k denoted by $\boldsymbol{K_{a1, a2, \dots, ak}}$,

Theorem 3.11. Let $G = K_{a1, a2, \dots, an}$, $n \geq 2$, where G is the complete k- partite graph with cardinalities of partitioned sets a_1, a_2, …,a_n. Then, $\mu(K_{a1, a2, \dots, an}) = n$.

Proof. Let $G = K_{a1, a2, \dots, an}$. Let $\{V_1, V_2, \dots,V_n\}$ be the partite sets of $K_{a1, a2, \dots, an}$. Then $\Pi = \{V_1, V_2,\dots,V_n\}$ is a neighbour resolving partition of G. Therefore, $\mu(G) \leq n$.
Suppose, $\mu(G) < n$. Let $\Pi_1 = \{W_1, W_2, \dots, W_{n-1}\}$ be a neighbour resolving partition of G. Let $u_i \in V_i$, $1 \leq i \leq n$. By pigeon hole principle, there exists W_i which contains at least two of the vertices from $\{u_1, u_2, \dots, u_n\}$. Without loss of generality, let $u_1, u_2 \in W_1$. If for any W_i ($2 \leq i \leq n-1$), W_i contains vertices from V_j ($3 \leq j \leq n$), then u_1 and u_2 being adjacent, will not be resolved by W_i. If W_i contains vertices from V_1, V_2, then also u_1 and u_2 will not be resolved. Therefore, for some W_i, either $V_1 = W_i$ or $V_2 = W_i$. Suppose, $V_1 = W_1$. Then u_2, u_3, ……, u_n belong to $W_2,W_3,\dots,W_{n-1}$. Therefore, there exist some W_j ($2 \leq j \leq n - 1$) containing two of the vertices from $\{u_2, u_3, \dots, u_n\}$ (say u_2, u_3). Arguing as before, there exist some W_j such that $V_2 = W_j$ or $V_3 = W_j$. Without loss of generality, let $V_2 = W_2$. Proceeding in this way, we get that $V_1 = W_1$, $V_2 = W_2$, …, $V_{n-2} = W_{n-2}$. Therefore, $W_{n-1} = V_{n-1} \cup V_n$. Let $x \in V_{n-1}$ and

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$y \in V_n$. Then x and y are adjacent, and the codes of x and y are (1, 1, 1, …, 0). That is, x and y are not resolved. Therefore, $\mu(G) \geq n$. Therefore, $\mu(G) = n$.

Definition 3.12. The **multi star graph** $K_m(a_1, a_2, \ldots, a_m)$ is the graph formed by joining $a_i \geq 1$, $1 \leq i \leq m$ pendant vertices to each vertex x_i of a complete graph K_m with $V(K_m) = \{x_1, x_2, \ldots, x_m\}$.

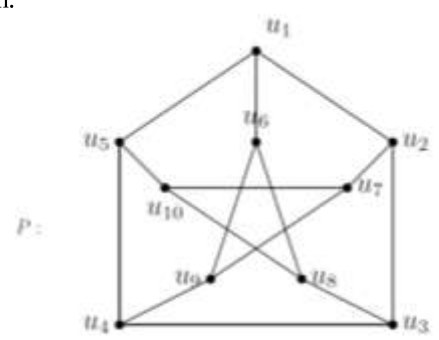
Definition 3.13. A **double star** *is a graph* obtained by taking two stars and joining the centers with an edge. If the stars are $K_{1,r}$ and $K_{1,s}$, then the double star obtained by joining the centers of $K_{1,r}$ and $K_{1,s}$, is denoted by $D_{r,s}$.

Theorem 3.14. $\mu(K_m(a_1, a_2, \ldots\ldots, a_m)) = m$, *where* $K_m(a_1, a_2, \ldots, a_m)$ *is the multi star graph*.

Proof. If $V(K_m) = \{v_1, v_2, \ldots, v_m\}$, $u_{i1}, u_{i2}, \ldots\ldots, u_{iri}$ are the pendant vertices at v_i, then $\Pi = \{\{v_1, u_{11}, u_{12}, \ldots, u_1r_1\}, \ldots, \{v_m, u_{m1}, u_{m2}, \ldots, u_{m\ rm}\}\}$ is a neighbour resolving partition of $K_m(a_1, a_2, \ldots, a_m)$. Therefore, $\mu(K_m(a_1, a_2, \ldots, a_m))$ is m.

Observation 3.14.1. $\mu(D_{r,s}) = 2$, where $D_{r,s}$ is the double star graph.

Theorem 3.15. $\mu(P) = 3$, where P is the Petersen graph.



Proof. Let $\Pi = \{\{u_1, u_3, u_9, u_{10}\}, \{u_2, u_4, u_6\}, \{u_5, u_7, u_8\}\}$. Then Π is a neighbour resolving partition of P. Therefore, $\mu(P) \leq 3$.

Suppose, $\mu(P) = 2$. Let $\Pi = \{V_1, V_2\}$ be μ-partition of P. Since $\beta_o(P) = 4$, either V_1 or V_2 contains adjacent vertices. Let V_1 contain adjacent vertices. Suppose $\{u_1, u_6\} \subset V_1$, $u_2 \in V_2$. Then u_8 and u_9 do not belong to V_2, since otherwise u_1, u_6 will have the same code with respect to $\{V_1, V_2\}$. Therefore, $u_8, u_9 \in V_1$. If $u_3 \in V_1$, then u_1 and u_3 have a common neighbour $u_2 \in V_2$ and hence u_1, u_3 cannot be resolved. Therefore, $u_3 \in V_2$. Since u_8 adjacent with u_3, u_4 cannot belong to V_2. Therefore, $u_4 \in V_1$. If $u_5 \in V_2$, then u_5 and u_3 have a common adjacent point in V_1. Therefore, u_5 and u_3 will not be resolved. Therefore, $u_5 \in V_1$. If $u_{10} \in V_2$, then u_1 and u_5 are adjacent and have neighbours in V_2. Therefore, u_1 and u_5 cannot be resolved. Therefore, $u_{10} \in V_1$. If $u_7 \in V_2$, then u_4 and u_9 will have the same code. If $u_7 \in V_1$, then u_2 and u_3 will not be resolved. Therefore, $\mu(P) \geq 3$. Thus $\mu(P) = 3$.

Theorem 3.16. [3] Let G be a connected bipartite graph. Then, $\mu(G) = 2$ if and only if G is bipartite.

Corollary 3.16.1. $\mu(T) = 2$, $\mu(C_{2n}) = 2$, $\mu(P_n) = 2$ and $\mu(D_{r,s}) = 2$.

Definition 3.17. Let $G = (V, E)$ be a connected graph. A subset S of $V(G)$ is called a **Neighbour Resolving Set** if $CS(u) = \{d(u, v_1), d(u, v_2), \ldots, d(u, v_k)\}$ where $S = \{v_1, v_2, \ldots, v_k\}$ is distinct for adjacent vertices. The minimum cardinality of a neighbour resolving set of G is called the **Neighbour Resolving dimension** of G and is denoted by ***ndim(G).***

Theorem 3.18. Let G be a non-trivial connected graph. $\mu(G) \leq ndim(G) + 1$.

Proof. Let $ndim(G) = k$. Let $W = \{w_1, w_2, \ldots, w_k\}$ be a n-basis of G. Let $S_i = W_i, (1 \leq i \leq k)$ and let $S_{k+1} = V(G) - W$. Let $\Pi = \{S_1, S_2, \ldots, S_k, S_{k+1}\}$. Let u, $v \in V(G)$ and u and v are adjacent.

Case (i): $u, v \in W$.

Then, for some i, j, $i \leq k$, $j \leq k$, $\{u\} = S_i$ and $\{v\} = S_j$. Therefore, Π resolves u and v.

Case (ii): $u \in W$ and $v \in V(G) - W$.

Then $\{u\} = S_i$ for some i, $1 \leq i \leq k$, $v \in S_{k+1}$.

$C_\Pi(u)$ has zero at the i^{th} place and $C_\Pi(v)$ has a positive value at the i^{th} place. Therefore, Π resolves u and v.

Case (iii): $u, v \in V(G) - W$.

Therefore, $u, v \in S_{k+1}$. Therefore, $u, v \in W$. Since W is neighbour resolving, there exists $w_i \in W$ such that $d(u, w_i) \neq d(v, w_i)$. That is, $d(u, \{w_i\}) \neq d(v, \{w_i\})$. That is, $d(u, S_i) \neq d(v, S_i)$. That is, Π resolves u and v. Therefore, Π is a neighbour resolving partition of G. Therefore, $\mu(G) \leq |\Pi| = k + 1 = ndim(G) + 1$.

Observation 3.19. From Theorem 3.1 [2], $pd(G) \leq n - d + 1$, where d is a diameter of the graph G. Since $\mu(G) \leq pd(G)$ we get that, $\mu(G) \leq n - d + 1$. When $G = K_n$, $\mu(G) = n$, $d = 1$. Therefore, $\mu(G) = n - d + 1$.

Corollary 3.19.1. Let G be a connected graph of order n, $(n \geq 2)$. Let $\mu(G) = n - 1$. Then diameter of G is 2.

Proof. $\mu(G) \leq n - d + 1$. Therefore, $n - 1 \leq n - d + 1$. Therefore, $d \leq 2$. If $d = 1$, then diameter of G is 1. That is, G is complete. Therefore, $\mu(G) = n$, a contradiction. Therefore, $d = 2$.

Definition 3.20. A subset S of V in a graph G is said to be independent if no two vertices in S are adjacent. The maximum number of vertices in an independent set of a graph G is called the **independence number of G** and is denoted by $\beta_o(G)$

Observation 3.21. $\mu(G) \leq n - \beta_o(G) + 1$.

Proof. Let S be a maximum independent set of G. Let $\Pi = \{S, \{v_1\}, \{v_2\}, \ldots\ldots, \{v_k\}\}$ where $S_1 = V - S = \{v_1, v_2, \ldots, v_k\}$. Let $x, y \in V(G)$. If x and y are adjacent, then x or $y \notin S$. Therefore, x or $y \in S_1$.

Case (i): $x, y \in S_1$.

Therefore, x and y are resolved by Π. Therefore, Π is a neighbour resolving partition of G. Therefore, $\mu(G) \leq |\Pi| = |V - S| + 1 = n - \beta_o(G) + 1$.

Case(ii): $x \in S$ and $y \in S_1$.

Then Π resolves x and y since at the position {y}, code of y with respect to Π is 0 and code of x with respect to Π is 1. Therefore, $\mu(G) \leq |\Pi| = n - \beta_o(G) + 1$.

Observation 3.22. $\mu(G) \leq \min\{n - d + 1, n - \beta_o(G) + 1\}$, where d is the diameter of G.

Observation 3.23. Let G be a connected graph of order n, $(n \geq 3)$ with diameter d.

Let $g(n, d)$ be the least positive integer k such that $(d + 1)^k \geq n - \beta_o(G)$, where $\beta_o(G)$ is the independence number of G. Then $g(n, d) \leq \mu(G)$.

Proof. Let $\mu(G) = k$. Let $\Pi = \{V_1, V_2, \ldots, V_k\}$ be a npd-partition of $V(G)$. Since adjacent vertices have representation with respect to Π as a k – vector whose elements are in the range 0 to d, there are $(d+1)^k$ possible representations. Since only adjacent vertices are to be resolved, at most $n - \beta_o(G)$ vertices are to be resolved. Therefore, $(d + 1)^k \geq n - \beta_o(G)$. $\mu(G) = k$ and $g(n, d) \leq k$. Therefore, $g(n, d) \leq \mu(G)$.

Theorem 3.24. [3] Let G be a connected graph of order $n \geq 3$. Then $\mu(G) = n - 1$ if and only if G is either $K_n - e$ or $K_1 + (K_1 \cup K_{n-2})$.

Definition 3.25. A graph G obtained from a wheel by subdividing each pair of adjacent vertices on the cycle exactly once is called the Gear Graph.

Theorem 3.26. $\mu(G_{2n}) = 2$, where G_{2n} is the Gear graph.

Proof. A Gear graph G_{2n} has order $2n + 1$ and size $3n$. Every vertex on the cycle of G_{2n} has degree 2 or 3. The central vertex has degree n. Since G_{2n} is bipartite, $\mu(G_{2n}) = 2$.

Definition 3.27. The graph H_{n+1} obtained from a wheel W_{n+1} with cycle C_n having a pendant edge attached at each vertex of the cycle is called the Helm Graph.

Theorem 3.28. $\mu(H_{n+1}) = 3$, where H_{n+1} is the Helm Graph.

Proof. Let H_{n+1} be a Helm graph.

Case (i): n is even.

Then the cycle surrounding the wheel has n vertices. Let $\Pi = \{\{u, u_1', \ldots, u_n'\}, \{u_1, u_3, \ldots, u_{n-1}\}, \{u_2, u_4, \ldots, u_n\}\}$. Then Π is a neighbour resolving partition of H_{n+1}. Therefore, $\mu(H_{n+1}) \leq 3$. Since H_{n+1} is not bipartite, $\mu(H_{n+1}) \geq 3$. Therefore, $\mu(H_{n+1}) = 3$.

Case(ii): n is odd.

Let u be the center of the wheel, $u_1, u_2, \ldots, u_{2k+1}$ be the vertices on the cycle surrounding the wheel. Let u_i' be the pendant vertex attached with $u_i, 1 \leq i \leq 2k + 1$.

Let $\Pi = \{\{u, u_1, u_2, u_{2k+1}, u_1', \ldots, u_{2k+1}'\}, \{u_3, u_5, \ldots, u_{2k-1}\}, \{u_4, u_6, \ldots u_{2k}\}\}$. Then Π is a neighbour resolving partition of H_{n+1}. Therefore, $\mu(H_{n+1}) \leq 3$. Since H_{n+1} is not bipartite, $\mu(H_{n+1}) \geq 3$.

Definition 3.29. A graph $\boldsymbol{f_n}$ which is a collection of n-triangles with a common vertex is called the **Friendship Graph.**

Theorem 3.30. $\mu(f_n) = 3$, where f_n is the friendship graph.

Proof. Let u be the central vertex of f_n and $u, u_1', u_2, u_2', \ldots\ldots, u_n, u_n'$ be the edges of the n-triangles with a common vertex u. Let $\Pi = \{\{u\}, \{u_1, u_2, \ldots, u_n\}, \{u_1', u_2', \ldots, u_n'\}\}$. Then Π is a neighbour resolving partition of f_n. Therefore, $\mu(f_n) \leq 3$. But f_n is not bipartite. Therefore, $\mu(f_n) \geq 3$. Therefore, $\mu(f_n) = 3$.

Definition 3.31. Let $G=(V, E)$ be a simple graph. The **Myceilskian of G** is the graph $\mu(G)$ with vertex set equal to the disjoint union $V \cup V' \cup \{u\}$ where $V' = \{x' / x \in V\}$ and the edge set $E \cup \{xy', y'x / xy \in E\} \cup \{y'v / y' \in V'\}$. The vertex x' is called the twin of the vertex x and the vertex v the root of $\mu(G)$.

Theorem 3.32. Let G be a connected graph. $\mu(G_1) \leq \mu(G) + 1$, where G is the Mycielskian of G.

Proof. Let $V(G) = \{u_1, u_2, \ldots\ldots, u_n\}$. Let $V(G_1) = \{u_1, u_2, \ldots, u_n, u_1', u_2', \ldots, u_n', v\}$, where u_i' is adjacent with neighbour of u_i in $V(G)$ $(1 \leq i \leq n)$ and

v is adjacent with $\{u_1^{'}, u_2^{'}, ..., u_n^{'}\}$. Let $\Pi = \{V_1, V_2, ..., V_k\}$ be a neighbour resolving partition of $V(G)$. Let $\Pi_1 = \{\{v\} \cup V_1, V_2, ..., V_k, \{u_1^{'}, u_2^{'}, ..., u_n^{'}\}\}$. Then Π_1 is a neighbour resolving partition of $V(G_1)$. Therefore, $\mu(G_1) \leq \mu(G) + 1$

4. Conclusion

The study of metric chromatic number was initiated by Chartrand et al. In this paper, neighbour resolving partition dimensions for some classes of graphs are found out. Further study can be done on neighbour resolving partition of a graph like determining neighbour resolving partition number for different types of product graphs, finding graphs for which neighbour resolving partition dimension of G and Mycielskian of G are equal, characterization of graphs with neighbour resolving partition dimension being equal to n, n– 1, n– 2, 1 and 2.

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