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α regularity and α normality axioms in a space

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Abstract

In this paper α regularity and a normality axioms are introduced in a space defined by A . D. Alexandroff and some of their properties are investigated. Also α bi compact and α Housdorff spaces are defined and a few results are expressed.

Key words: *α open sets, α bi compact set, α regular space, α normal space.*

AMS Subject Classification: 54 A 05.

1 Introduction:

Topological spaces have been generalized in several ways. For example Mashhour et. al. [6] omitted the intersection condition and then Das and Samanta [3] investigated a space without any structural conditions. Perhaps the first to introduce such a generalisation was Alexandroff [1], who weakened the union requirements of a topological space. In this paper α bi compact sets and α regularity axioms are introduced in a space defined by A.D.Alexandroff and some of their properties are investigated.

2. Preliminaries:

Definition 1 [1]. A set X is called a space if in it is chosen a system of subsets F satisfying the following axioms

(i) The intersection of a countable number of sets from F is a set in F.

(ii) The union of a finite number of sets from F is a set in F.

(iii) The void set is a set in F.

(iv) The whole set X is a set in F.

Sets of F are called closed sets. Their complementary sets are called open. It is clear that instead of closed sets in the definition of a space, one may put open sets with subject to the conditions of countable summability , finite intersectability and the condition that X and the void set should be open. The collection of such open sets will sometimes be denoted by τ and the space by (X, τ). In general τ is not a topology. By a space we shall always mean an Alexandroff space.

Definition 2[1] . With every M ⊂ X we associate its closure cl(M), the intersection of all closed sets containing M and scl(M), the intersection of all semi closed sets containing M.

Note that cl(M) and scl(M) is not necessarily closed and semi closed respectively in a space.

Definition 3[7]. Two sets A, B in X are said weakly *α* separated if there are two *α* open sets U,V such that A ⊂ U,B ⊂ V and A ∩ V=B ∩ U=Φ.

Definition 4[7]. A set N, a subset of X is said to be a *α* neighborhood of a point x of X if and only if there exist a *α* open set O containing x such that O ⊂ N.

Definition 5[7]. The *α* interior of a set A in a space X is define as the union of all *α* open sets contained in A and is denoted by *α* -int (A).

Definition 6[7]. A subset A of a space (X,τ) is said to be α open if A⊂int(cl(intA)). Complement of α open set is called α closed set.

3. α Regular and α Normal space:

Definition 7. A space (X, *α* is said to be α regular space if for any x∈X and any *α* closed set F such that x does not belongs to F ,there exist α open sets U,V such that x∈U,F ⊂ V and U ∩ V=Φ.

Definition 8 . A space (X,τ) is said to be α normal space if for any two disjoint *α* closed sets F and T ,there exist α open sets U,V such that F ⊂ U,T ⊂ V and U ∩ V=Φ.

Definition 9. A space (X,τ) is said to be α R0-space if for each α open set G and x∈G implies α cl{x} ⊂ G.

Definition 10. A space (X,τ) is said to be α R1-space if for x,y∈X such that x does not belong to *α* cl {y}, there exist α open sets U,V such that x∊ U ,y ∊ V and U ∩ V=Φ.

Definition 11. A space (X,τ) is said to be α Housdorff space if for x,y∊X such that x ≠y, there exist α open sets U,V such that x∊ U ,y ∊ V and U ∩ V=Φ.

We know that a topological space (X,τ) is α regular if and only if for any point x∊X and any α open set U containing x there is a α open set V such that x∊V ⊂ α cl(V) ⊂ U.

But this is not true in a space.

Example1. Let X=R-Q and τ ={X, ,G_i} where G_i runs over all countable subsets of R-Q .Then (X,τ) is a space but not a topological space .Clearly in this space for any x ∊X, α cl{x}={x}.Thus for x ∊X and α open set U such that x∊U then x∊{x} ⊂ α cl{x} ⊂ U . But (X,τ) is not α regular.

Theorem 1. A space X is α regular if and only if for any x∊X and any α open set U containing x ,there is a α open set V and a closed set F such that x ∊V ⊂ F ⊂ U .

The proof is simple and omitted.

The similar characterization of α normal space in a space also does not hold.

Theorem 2 . A space X is α normal if and only if for any closed set F and any α open set U containing F, there is α open set V and α closed set F' such that F ⊂ V ⊂ F' ⊂ U.

Theorem 3. Every α regular space is α R1.
The proofs are straight forward and so omitted.

4. α bi compact and locally α bi compact space:

Definition 10. A space (X,τ) is said to be α bi compact if every α open cover of it has a finite sub cover

Definition 12. A space X is said to be locally α bi compact if for every point x∊X there is α open neighborhood U of x such that α cl(U) is α bi compact.

Theorem 4. If a space (X,τ) is α R1 and α bi compact ,then it is both α regular and α normal.

Proof. Let x∊X and F be a α closed set such that x does not belongs to F. Let y∊F. Then x does not belong to α cl{y}. Since (X,τ) is α R1 ,there are α open sets U y , V y such that x∊ Uy, y∊ Vy and Uy ∩ Vy =Φ. Now [Vy : y∊F} along with X-F forms an α open cover of X. Since X is α bi compact , there are finite number of points y1,……yn in F such that F ⊂ U{ V $_{yi.}$:i=1,2,..n}. V= U{ V $_{yi}$:i=1,2,..n}, U= ∩{ U $_{yi}$:i= 1,2,..n}

Then x∊U, F ⊂ V and U ∩ V=Φ. So X is α regular. To show that (X,τ) is α normal , let F_1,F_2 are two disjoint α closed sets in X. Let x∊ F_1, then x does not belongs to F_2. .Since X is α regular there are α open sets U'$_x$, V'$_x$ such that x∊ U'$_x$, F ⊂ V'$_x$ and U'$_x$, ∩ V'$_x$ =Φ. Then {U' $_x$: x∊F_1} along with X-F_1 forms an α open cover of X. Then proceeding as above we can find α open sets U',V' such that F_1 ⊂ U',F_2 ⊂ V' and U' ∩ V'= Φ.

Note 1. A α Housdorff α bi compact space is both α regular and α normal.

Definition 13 . A space X is said to be locally α bi compact if for every point x∊ X, there is a α open neighborhood U of x such that α cl {U} is α bi compact.

Remark 1. It is well known that a α Housdorff locally α bicompact topological space is α regular . But this is not true in a space.

Example 2. Consider the above example , then X is α Housdorff . Since for any for α∊X , {α} is an α open set that is α open set containing α such that {α} = α cl{α} is α bi compact , so X is locally α bi compact .But X is not α regular.

Remark 2 .In a α Housdorff topological space α compact subsets are α closed. But this is not true in a space.

Example 3. Consider the above example 1 . (X,τ) is α Housdorff . Let α∊X, then {α} is α bi compact but not α closed.

Theorem 5. In a α Housdorff space α bi compact sets are α closed if and only if arbitrary union of α open sets whose complement is α bi compact is α open.

Proof. Let (X,τ) is a α Housdorff space in which α bi compact sets are α closed . Let {Gi} be a collection of α open sets and G= U Gi such that X-G is α bi compact . Then X-G is α closed so G is α open.

Conversely let the condition be hold. Let A be a α bicompact set. We shall show that A is semi closed .Let x∊X-A. Then x does not belongs to A .Let y∊A. Then x ≠ y. Since (X,τ) is α Housdorff , there exist α open sets Uy ,Vy such that x∊ Uy, y∊ Vy and Uy ∩ Vy =Φ.Now {Vy : y∊A} is a α open cover of A . Since A is α bi compact there exist finite number of points y_1,……yn in A such that A ⊂ U{ V $_{yi.}$:i=1,2,..n}=V x(say). Let U x = ∩{ U $_{yi}$:i= 1,2,..n}. Then x∊ Ux , A ⊂ V x and U x ∩ V x =Φ .Now Ux ⊂ X-V x ⊂ X-A. Thus U Ux ⊂ X-A ⊂

U{{x}: xϵX-A} ⊂ U{Ux: xϵX-A} , which implies that X-A= U{Ux: xϵX-A} . So X-A is the union of α open sets whose complement A is α bicompact. So X-A is α open. So A is α closed.

Theorem 6 . In a α R_1 topological space (X,τ) ,if A is α compact then U{ α -cl({a}):aϵA} is α closed.

Proof . Let (X,τ) be a α R_1 topological space .Let A be α i compact .Let x does not belongs to U{ α -cl ({a}):aϵA} .Then x does not belongs to α -cl({a}),for all aϵA .Since (X,τ) is α R_1 there exist α open sets Ua,Va such that xϵ Ua, aϵ Va and Ua ∩ Va=Φ.Now {Va: aϵA} is an α open cover of A . Since A is α compact there exist finite number of points a_1,….,.an€A such that A ⊂ U{ V $_{ai}$:i=1,2,..n}=V., Let U= ∩{ U $_{ai}$:i= 1,2,..n}. Then xϵU, A ⊂ V and U ∩ V=Φ.Now aϵA ⊂ V ⊂ X-U , where X-U is α closed .So α -cl({a}) ⊂ X-U, for all a belongs to A .Hence U{ α -cl({a}):aϵA} ⊂ X-U . This implies that U ∩ U{ α cl({a}):aϵA} =Φ.Thus x is not a α limiting point of U{τ-cl({a}):aϵA} and this shows that U{s-cl({a}):aϵA} is α closed.

Remark 3. The result of above theorem is not true in a space . If we take A={a} the previous example then result will follow.

Theorem 7. In a α R1 space (X,τ) ,if A is α bi compact then U{ α cl({a}):aϵA} is α closed iff union of an arbitrary number of α open sets whose complement is the α closure of a α bicompact set is α open.

Proof .Let G be an arbitrary union of α open sets such that X-G= α -cl(A), where A is α bi compact .Let A*= U{ α -cl({a}):aϵA} .Then A* is α closed. Also since A ⊂ A*, α cl(A) ⊂ A*.Clearly A* ⊂ α - cl(A).Hence X-G= α -cl(A)=A*.Since A* is α closed ,G is α open.

Conversely let the condition be hold .Let A be a α bi compact set .Let y does not belongs to A*,so that y does not belongs to α - cl{a}: for all aϵA.

Since (X,τ) is α R1 there exist α open sets Ua , Va such that yϵ Ua , aϵ Va and Ua ∩ Va =Φ.Now {Va : aϵA} is an α open cover of A. Since A is α bi compact there exist finite number of points a1,…..anϵA such that A ⊂ U{ V $_{ai}$:i=1,2,..n}=Vy, Let Uy = ∩{ U $_{ai}$:i= 1,2,..n}. Then yϵ Uy , A ⊂ Vy and Uy ∩ Vy =Φ. Now A ⊂ Vy ⊂ X-Uy, where X - Uy is α closed .So α cl (A) ⊂ X-Uy and y does not belongs to X-Uy .Therefore y ∉ α cl (A). So α cl (A) ⊂ A*. Again clearly A* ⊂ α -cl (A).Hence α cl (A)= A*.So X-A*=X- α cl (A)= U{X-F:F is a α closed set containing A} .Thus X - A* is arbitrary union of α open sets whose complement A*= α - cl (A) is the closure of a α bi compact set A . Hence X - A* is α open and so A* is α closed.

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