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An optimal replenishment of fuzzy inventory model for time dependent deteriorating item with fuzzy planning horizon

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Abstract

This paper deals with an inventory model for single deteriorating item during its seasonal time where lifetime of an item has an upper limit. Deterioration rate increases with time and depends on the duration of lifetime left. Demand of the item is price dependent and unit cost of item is time dependent. Unit cost is a decreasing function at the beginning of the season and an increasing function at the end of the season and is constant during the remaining part of the season. So, the inventory model is formulated to maximize the average proceeds out of the system from the imprecise planning horizon. As the optimization of fuzzy objective is not well defined, optimistic/pessimistic return of the objective function (using possibility/necessity measure of the fuzzy event) is optimized. A fuzzy simulation process is proposed to evaluate this optimistic/pessimistic return. A genetic algorithm (GA) is developed based on entropy theory where region of the search space gradually decreases to a small neighbourhood of the optima. This is named as region reducing genetic algorithm (RRGA) and is used to solve this model when planning horizon is crisp. As simulation based region reducing genetic algorithm, called fuzzy simulation based region reducing genetic algorithm (FSRRGA) is developed to solve the fuzzy objective value. The model is illustrated with some numerical examples and some sensitivity analyses have been performed.

Keywords: *Fuzzy planning horizon, Seasonal product, Inventory, Time dependent deterioration, Region Reducing Genetic Algorithm (RRGA), Fuzzy simulation.*

1. Introduction

In general, planning horizon of many seasonal items fluctuate to some extend. As for example, in India, winter starts with November and ends with February. But its duration is not always fixed. A little variability can be easily noticed over the years. Thus, planning horizon of seasonal products such as fruits, potato, onion, cabbage, cauliflower, food grains, etc. is a fuzzy variable instead of a fixed deterministic constant. For the seasonable item, it is normally observed that price of the item decreases with time at the beginning of the production season due to availability in the market and reaches to a minimum value. Price of the item remains constant at this minimum value during the major part of the season due to sufficient availability of the item in the market and towards the end of the season due to scarcity, cost again increases gradually and reaches its off season value. This price remains stable during the remaining part of the year. A considerable number of research works have been done for seasonal products by several researchers Zhou *et al*. (2004), Chen and Chang (2007), Panda *et al*. (2008), Banerjee and Sharma (2010A, 2010B), Skouri and Konstantaras (2013), Tayal *et. al*. (2015), Krommyda *et. al* .(2017) etc. Recently, Mohanty *et. al* (2018) developed an trade credit inventory modeling of deteriorating items over random planning horizon due to fluaction of season.

In most of these research works, it is assumed that price of the item decreases with time or demand increases with time. But the above mentioned real life phenomenon of a seasonal product is overlooked by the researchers. Another shortcoming of these research work is the assumption that the duration of the season of such products as crisp value. Although, the duration of the season for an item is finite but it varies from year to year due to environmental changes. So, it is worthwhile to assume this duration as a fuzzy parameter. Occurrence of fuzzy seasonal time leads to optimization problem with fuzzy objective function. In the last two decades extensive research work has been done on inventory control problems in fuzzy environment (Lee *et al*. (1991), Lam and Wong (1996), Roy and Maiti (2000), Mondal and Maiti (2002), Kao and Hsu (2002), Bera *et al*. (2012), Bera and Maiti (2012), Maiti *et al*.

(2014), De and Sana (2015), Garai *et. al*. (2016), Bera and Jana (2017), De and Mahata (2017) etc. These problems considered different inventory parameters as fuzzy numbers which render fuzzy objective function. As optimization in fuzzy environment is not well defined some of these researcher transform the fuzzy parameters as equivalent crisp number or crisp interval and then the objective function is transformed to an equivalent crisp number/interval (Maiti and Maiti (2007), Bera *et al*.(2012)). Some of the researchers (Mondal and Maiti, (2002)) set the fuzzy objective as fuzzy goal whose membership function as a linear/non-linear fuzzy number and try to optimize this membership function using Bellman Zadeh's principle (Bellman and Zadeh, 1970). Maiti and Maiti (2006) propose a technique where instead of objective function pessimistic return of the fuzzy objective is optimized. They use necessity measure on fuzzy event to determine this pessimistic return and propose fuzzy simulation process to find this return function. Maiti (2008, 2011) proposes a technique where possibility/necessity measure of objective function (fuzzy profit) on fuzzy goal is optimized to find optimal decision. Recently, Manna *et.al*.(2016), Garai *et. al*. (2016) and others developed inventory models using possibility and necessity constraints for a given level of optimistic/pessimistic sense. All these studies transform the fuzzy objective of the problem to an equivalent crisp objective and solution of the reduced problem is taken as approximate solution of the fuzzy problem. But there exist always some error in such approximation. In present day competitive market, an erroneous inventory decision may invite a huge loss in business. So modeling of present day inventory control problems should be very realistic and a methodology is required which can deal with fuzzy objective function directly without reducing it to crisp form.

Most of the seasonal products have finite lifetime and are deteriorating in nature (Mahata and Goswami, (2010)). Rate of deterioration increases with time and actually depends on the length of lifetime left. Rate of deterioration becomes 100% when age of product covers the lifetime. In the literature, there are several investigations for deteriorating items such as Jaber *et al*. (2009); Yadav *et al.* (2011); Sana (2011), Skouri and Konstantaras (2013), Chaudhury *et. al*. (2015), Tayal *et. al*. (2015, Dutta and Kumar (2015), Karmakar and Chaudhury(2014), Kumar and Rajput (2015), , Mohanty *et. al* (2018), Rastogi et. al. (2018) and others. Most of the inventory articles are developed with constant deterioration. But the deterioration mentioned earlier, deterioration increases with time as stress of units on others causes damage. According to the author’s best knowledge, very few articles have been published incorporating time varying deterioration (Sarkar (2011)). However, Janssen *et. al*. (2016) presented a review article on deteriorating items including this publication from 2012 to 2015.

Use of soft computing techniques for inventory control problems is a well established phenomenon. Several authors use Genetic Algorithm (GA) in different forms to find marketing decisions for their problems. Pal *et al*. (2009) uses GA to solve an EPQ model with price discounted promotional demand in an imprecise planning horizon. Roy *et al*. (2009) used a GA with varying population size to solve a production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon. Bera and Maiti (2012) used GA to solve multi-item inventory model incorporating discount. Maiti *et al*. (2009) used GA to solve inventory model with stochastic lead time and price dependent demand incorporating advance payment. Mondal *et al*. (2002) uses a dominance based GA to solve a production-recycling model with variable demand, demand-dependent fuzzy return rate. Combining the features of GA and PSO a hybrid algorithm PSGA is used by Guchhait *et al*. (2014) to solve an inventory model of a deteriorating item with price and credit linked fuzzy demand. All these soft computing techniques are not capable to deal with fuzzy objective directly.

From the above discussion it is clear that there are some lacunas in fuzzy inventory models of deteriorating items, especially for seasonal products. In this research work an attempt has been made to reduce these lacunas. The aim of this research work is fourfold:

The aim of this research work is fourfold:

- Firstly to model price of a seasonal product as a function $f_1(t)$ of time which decreases monotonically for a duration H_1 at the beginning of the season and reaches a minimum value $f_1(H_1)$. The price remains at this value $f_1(H_1)$ during a period H_2. Then it again follows an increasing function $f_2(t)$ and after a period H_3 it reaches the off season value, i.e., $f_1(0)=f_2(H_1+H_2+H_3)$.
- Secondly to model the season length $(H_1+H_2+H_3)$ as imprecise parameter.
- Thirdly for such a realistic inventory model, rate of deterioration as increasing function of time which actually depends on the lifetime of the item.

- At length to introduce an approach which can deal with fuzzy optimization problem, without reducing the objective function to any deterministic form.

Here, inventory model for a deteriorating seasonal product is developed whose demand depends upon the unit cost of the product. Unit cost of the product is time dependent. During the beginning of the period as availability of the item gradually increases, unit cost decreases monotonically with time and reaches a constant value when availability of the item becomes stable. Unit cost remains constant until the items availability again decreases towards the end of the season. Then as availability decreases, unit cost gradually increases and reaches its value as it was at the beginning of the season and then the season ends. Here exponential increasing and decreasing rate of unit cost function is considered. Rate of deterioration θ of the item increases with time and is of the form $\theta = [1/(1+R-t)]$, where R is the lifetime of the product, t is the time passed after the arrival of the units in the inventory.

Clearly as $t \to R,\ \theta \to 1$, i.e., when t=R, all units in the inventory will be spoiled. It is assumed that time horizon of the season is fuzzy in nature. In fact three parts in which unit cost function can be divided are considered as fuzzy number. The model is formulated to maximize the total proceeds out of the system which is fuzzy in nature. As the optimization of fuzzy objective is not well defined, optimistic/pessimistic return of the objective function (using possibility/necessity measure of the fuzzy event) is optimized. A fuzzy simulation process is proposed to evaluate this optimistic/pessimistic return. A genetic algorithm (GA) is developed based on entropy theory where region of the search space gradually decreases to a small neighbourhood of the optima. This is named as region reducing genetic algorithm (RRGA) and simulation based region reducing genetic algorithm, called fuzzy simulation based region reducing genetic algorithm (FSRRGA) is developed to solve the fuzzy objective value. The models are illustrated with some numerical examples and some sensitivity analyses have been presented.

2. Definitions and Preliminaries

2.1 Possibility/Necessity in fuzzy environment

Any fuzzy number $\tilde{a}$ of $\Re$ (where $\Re$ represents set of real numbers) with membership function $\mu_{\tilde{a}}: \Re \to [0,1]$ is called a fuzzy number. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$ respectively. Then according to Zadeh(1978), Dubois and Prade (1983) and Liu andIwamura(1998a,1998b):

$$pos(\tilde{a} * \tilde{b}) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re,\ x * y\} \quad (1)$$

where abbreviation pos represents possibility and * is any one of the relations $<, >, =, \leq, \geq$. Analogously, if $\tilde{b}$ is a crisp number,say,b,then

$$pos(\tilde{a} * b) = \sup\{\mu_{\tilde{a}}(x),\quad x \in \Re,\ x * b\} \quad (2)$$

The necessity measure of an event $\tilde{a} * \tilde{b}$ is a dual of the possibility measure. The grade of an event is the grade of impossibility of the opposite event and is defined as:

$$nes(\tilde{a} * \tilde{b}) = 1 - pos(\overline{\tilde{a} * \tilde{b}}) \quad (3)$$

where the abbreviation nes represents the necessity measure and $\overline{\tilde{a} * \tilde{b}}$ represents the complement of the event $\tilde{a} * \tilde{b}$.

If $\tilde{a}, \tilde{b} \in \Re$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$ where $f: \Re \times \Re \to \Re$ is binary operation then, the extension principle by Zadeh(1978), the membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is given by

$$\mu_{\tilde{c}}(z) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)),\ x,\ y \in \Re \text{ and } z = f(x, y), \forall z \in \Re\} \quad (4)$$

2.2 Triangular Fuzzy Number (TFN)

A TFN $\tilde{a} = (a_1, a_2, a_3)$ (cf. Fig-1) has three parameters a_1, a_2, a_3 where $a_1 < a_2 < a_3$ and is characterized by the membership function $\mu_{\tilde{a}}(x)$, is given by

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Fig-1: Membership function of a triangular fuzzy number

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, a_2\le x\le a_3\\ 0, \quad otherwise\end{cases}$$

(5)

4. Optimization of fuzzy objective using possibility/necessity measure

A general single-objective unconstrained mathematical programming problem is of the following form:

$$\text{Max} \quad f(x,\xi)$$
$$\text{subject to } x\in X \qquad (6)$$

where x is a decision vector, ξ is a vector of crisp parameters, $f(x, \xi)$ is the return function, X is the search space. In the above problem when ξ is a fuzzy vector $\tilde{\xi}$, then return function $f(x, \tilde{\xi})$ becomes imprecise in nature. In that case the statement maximize $f(x, \tilde{\xi})$ is not well defined. In that situation one can maximize the optimistic (pessimistic) return z corresponding to the objective function using possibility (necessity) measure of the fuzzy event $\{\tilde{\xi} \mid f(x, \tilde{\xi}) \ge z\}$ as suggested by Liu and Iwamura (1998a, 1998b), Maiti and Maiti (2006). So when ξ is a fuzzy vector one can convert the above problem (6) to the following equivalent possibility/necessity constrained programming problem (analogous to the chance constrained programming problem).

$$Max \quad z$$
$$subject\ to \quad pos/nes\{\tilde{\xi} \mid f(x,\tilde{\xi})\ge z\}\ge\beta$$
$$x\in X$$

(7)

where β is the predetermined confidence level for fuzzy objective, pos{.} nes{.}denotes the possibility (necessity) of the event in {.}. Here the objective value z should be the maximum that the objective function $f(x, \tilde{\xi})$ achieves with at least possibility (necessity) β, in optimistic (pessimistic) sense.

4.1 Fuzzy simulation

The basic technique to deal problem (7) is to convert the possibility/necessity constraint to its deterministic equivalent. However, the procedure is usually very hard and successful in some particular cases (Maiti and Maiti, 2006). Liu and Iwamura (1998a,1998b) proposed fuzzy simulation process to determine optimum value of z for the problem (7) under possibility measure of the event $\{\tilde{\xi} \mid f(x, \tilde{\xi}) \ge z\}$. Following Liu and Iwamura (1998b) two algorithms are developed to determine z in (7) and are presented below.

Algorithm 1 Algorithm to determine z, for problem (6) under possibility measure of the event$\{\tilde{\xi} \mid f(x, \tilde{\xi}) \ge z\}$

1. Set $z = -\infty$.
2. Generate ξ_0 uniformly from the β cut set of fuzzy vector $\tilde{\xi}$.
3. If $z < f(x, \xi_0)$ then set $z = f(x, \xi_0)$.
4. Repeat Steps 2 and 3, N times, where N is a sufficiently large positive integer.
5. Return z.
6 End algorithm.

We know that $nes\{\tilde{\xi} \mid f(x, \tilde{\xi}) \ge z\} \ge \beta \Rightarrow pos\{\tilde{\xi} \mid f(x, \tilde{\xi}) < z\} < 1-\beta$. Now roughly find a point ξ_0 from fuzzy vector ξ, which approximately minimizes f. Let this value be z_0 and ε be a positive number. Set $z = z_0 - \varepsilon$ and if $pos\{\tilde{\xi} \mid f(x, \tilde{\xi}) < z\} < 1-\beta$ then increase z with ε. Again check $pos\{\tilde{\xi} \mid f(x, \tilde{\xi}) < z\} < 1-\beta$ and it continues until $pos\{\tilde{\xi} \mid f(x, \tilde{\xi}) < z\} \ge 1-\beta$. At this stage decrease value of ε and again tries to improve z.

When ε becomes sufficiently small then we stop and final value of z is taken as value of z. Using this criterion, Algorithm 2 is developed.

Algorithm 2 Algorithm to determine z, for problem (6) under necessity measure of the event $\{\tilde{\xi} \mid f(x, \tilde{\xi}) \geq z\}$

1. Set $z = z_0 - \varepsilon$, F = $z_0 - \varepsilon$, $F_0 = z_0 - \varepsilon$.
2. Generate ε_0 uniformly from the $1 - \beta$ cut set of fuzzy vector $\tilde{\xi}$.
3. If $f(x, \tilde{\xi}_0) < z$.
4. then go to Step 10.
5. End If
6. Repeat Step 2 to Step 5 N times
7. Set $F = z$.
8. Set $z = z + \varepsilon$.
9. Go to Step 2.
10. If($z = F$) //In this case optimum value of $z < z_0 - \varepsilon$
11. Set $z = z_0 - \varepsilon$, $F = F - \varepsilon$, $F_0 = F_0 - \varepsilon$.
12. Go to Step 2
13. End If
14. If ($\varepsilon < tol$)
15. go to Step 20
16. End If
17. $\varepsilon = \varepsilon/N$
18. $z = F + \varepsilon$
19. Go to Step 2.
20. Output F.

5. Fuzzy simulation-based region reducing genetic algorithm

GAs are exhaustive search algorithms based on the mechanics of natural selection and genesis (crossover, mutation, etc.) and have been developed by Holland, his colleagues and students at the University of Michigan (Goldberg (1989)). Because of its generality and other advantages over conventional optimization methods it has been successfully applied to different decision making problems (Zegordi *et al*.(2010), Simon *et al*.(2011), Das et al., (2012), Maiti *et. al.* (2014) and others). Generally a GA starts with a single population (Goldberg (1989), Michalewicz (1992)), randomly generated in the search space. Consequently they are easily trapped into local optima of the objective function. This difficulty is mainly due to the premature loss of diversity of the population during the search. To overcome this difficulty, Bessaou and Siarry (2001) propose a GA where initially more than one population of solutions are generated. Genetic operations are done on every population a finite number of times to find a promising zone of optimum solution. Finally a population of solutions is generated in this zone and genetic operations are performed on this population a finite number of times to get a final solution. Again the convergence towards the global optima of a GA, operating with a constant probability of crossover p_c, is ensured if the probability of mutation $p_m(k)$ follows a given decreasing law, in function of the generation number k (Davis and Principe, 1991). Following Bessaou and Siarry (2001) a GA is developed using them entropy generated from information theory, where promising zone is gradually reduces to a small neighbourhood of the optimal solution. In the algorithm any possibility constraint on objective function is checked via fuzzy simulation technique. This algorithm is named as FSRRGA and is used to solve our models. The algorithm is given below:

Algorithm 3 FSRRGA algorithm

1. Initialize probability of crossover p_c and probability of mutation p_m.
2. Set iteration counter T = 0.
3. Generate M sub-populations of solutions, each of order N (i.e., each sub-population contains N solutions), from search space of optimization problem under consideration, such that the diversity among the solutions of each population is maintained. Diversity is maintained using the entropy originating from information theory [cf., § 5.1-(b)]. Solutions for each of the population are generated randomly from the search space in such a way that the constraints of the problem are satisfied. Possibilistic constraints are checked using the algorithms of Section 4.1. Let P_1, P_2, … ,P_M be these populations.
4. Evaluate fitness of each solution of every population.
5. Repeat
   A. Do for each sub-populations P_i.
      a. Select N solutions from P_i for mating pool using Roulette-wheel selection process (Michalewicz, 1992) (These N solutions may not be distinct. Solution with higher fitness value may be selected more than once). Let this set be P_i^1.
      b. Select solutions from P_i^1, for crossover and mutation depending on p_c and p_m respectively.
      c. Make crossover on selected solutions for crossover.
      d. Make mutation on selected solutions for mutation.
      e. Evaluate fitness of the child solutions.

f.Replace the parent solutions with the child solutions.
g. Replace P_i with P_i^1
B. End Do
C. Reduce probability of mutation p_m.
6.Until number of generations < Maxgen1, where Maxgen1 represents the maximum number ofgenerations to be made on initial populations.
7.Select optimum solutions from each sub-populations and S* be the best among these solutions.
8. Select a neighbourhood V(T) of S*
9. Repeat
a. Generate a population of solutions of size N in V(T). Let it be P.
b. Evaluate fitness of each solutions.
c. Initialize probability of mutation p_m.
d. Repeat
(i)Select N solutions from P for mating pool using Roulette-wheel selection process. Let this set be P^1.
(ii)Select solutions from P^1 for crossover and mutation depending on p_c and p_m respectively.
(iii)Make crossover on selected solutions for crossover.
(iv) Make mutation on selected solutions for mutation.
(v) Evaluate fitness of the child solutions.
(vi) Replace the parent solutions with the child solutions.
(vii) Replace P with P^1.
(viii) Reduce probability of mutation p_m.
e. Until number of generations < Maxgen2, where Maxgen2 represents the maximum number of generations to be made on this population.
f. Update S* by the best solution found.
g. Reduce the neighbourhood V(T).
h. Increment T by 1.
10. Until T < Maxgen3, where Maxgen3 represents the maximum number of times for which the search space to be reduced.
11. Output S^*.

5.1 FSRRGA procedures for the proposed model

a. Representation A '*K*-dimensional real vector' $X_{li} = (x_{li1}, x_{li2}, ... x_{liK})$ is used to represent i^{th} solution in l^{th} population, where $x_{li1}, x_{li2}, ... x_{liK}$ represent different decision variables of the problem such that constraints of the problem are satisfied.

b. **Initialization** At this step *M* sub-populations, each of size *N* are randomly generated in the search space in such a way that diversity among the solutions of each of the populations is maintained and the constraints of the problem are satisfied. Possibility constraints are checked using the algorithms of Section 4.1. Let $X_{l1}, X_{l2}, ... X_{lN}$, are the solutions of l^{th} population P_l, $l = 1, 2, ...M$. Diversity can be maintained using the entropy originating from information theory. Entropy of j^{th} variable for the l^{th} population P_l can be obtained by the formula:

$$E_j(P_l) = \sum_{i=1}^{N} \sum_{k=i+1}^{N} -p_{ik} \log(p_{ik})$$

where p_{ik} represents the probability that the value of j^{th} variable of i^{th} solution (x_{lij}) is different from the one of the j^{th} variable of the k^{th} solution (x_{lkj}) and is determined by the formula:

$$p_{ik} = 1 - \frac{|X_{lij} - X_{lkj}|}{U_j - L_j}$$

Where $[L_j, U_j]$ is the variation domain of the j^{th} variable. The average entropy $E(P_l)$ of the l^{th} subpopulation P_l is taken as the average of the entropies of the different variables fo the population, i.e.,

$$E(P_l) = \frac{1}{k} \sum_{j=1}^{K} E_j(P_l)$$

It is clear that if P_l is made-up of same solutions, then $E(P_l)$ vanishes and more varied the solutions, higher the value of $E(P_l)$ and the better is its quality. So to maintain diversity, every time a new solution is randomly generated for P_l from the search space, the entropy between this one and the previously generated individuals for P_l is calculated. If this value is higher than a fixed threshold E_0, fixed from the beginning, the current chromosome is accepted. This process is repeated until *N* solutions are generated. Following the same procedure all the sub-populations P_l,
$l = 1, 2, ..M$ are generated. This solution sets are taken as initial sub-populations.

c. **Fitness value** Value of the objective function due to the solution X_{ij} (j^{th} solution in i^{th} population), is taken as fitness of X_{ij}. Let it be $f(X_{ij})$. Objective function is calculated using Algorithm 2 of Section 4.1.

d. **Selection process for mating pool** The following steps are followed for this purpose:
1. For each population P_i, find total fitness of

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the population $F_i = \sum_{j=1}^{N} f(X_{ij})$

2. Calculate the probability of selection pr_{ij} of each solution X_{ij} by the formula
$pr_{ij} = f(X_{ij})/F_i$.
3. Calculate the cumulative probability qr_{ij} for each solution X_{ij} by the formula

$$qr_{ij} = \sum_{k=0}^{j} pr_{ik}$$

4. Generate a random number 'r' from the range [0, 1].
5. If $r < qr_{i1}$ then select X_{i1} otherwise select $X_{ij} (2 \le j \le N)$ where $qr_{ij-1} \le r < qr_{ij}$.
6. Repeat Step 4 and 5 N times to select N solutions for mating pool. Clearly one solution may be selected more than once.
7. Selected solution set is denoted by P_i^1 in the proposed FSRRGA algorithm.

e. **Crossover**

1. **Selection for crossover** For each solution of P_i^1 generate a random number r from the range [0, 1]. If $r < p_c$ then the solution is taken for crossover, where p_c is the probability of crossover.

2. **Crossover process** Crossover taken place on the selected solutions. For each pair of coupled solutions Y_1, Y_2 a random number c is generated from the range [0, 1] and Y_1, Y_2 are replaced by their offspring's Y_{11} and Y_{21} respectively where $Y_{11} = cY_1 + (1 - c)Y_2$, $Y_{21} = cY_2 + (1 - c)Y_1$.

f. **Mutation**

1. **Selection for mutation** For each solution of P_i^1 generate a random number r from the range [0, 1]. If $r < p_m$ then the solution is taken for mutation, where p_m is the probability of mutation.

2. **Mutation process** To mutate a solution $X_{li} = (x_{li1}, x_{li2}, \ldots, x_{lik})$ select a random integer r in the range [1, k]. Then replace x_{ijr} by randomly generated value within the boundary of r^{th} component of X_{ij}.

g. **Reduction process of p_m** Let $p_m(0)$ is the initial value of p_m. $p_m(T)$ is calculated by the formula $p_m(T) = p_m(0)\exp(-T/\alpha)$, where α is calculated so that the final value of p_m is small enough (10^{-3} in our case).

So, $\alpha = Maxgen1/\log\left[\dfrac{p_m(0)}{10^{-3}}\right]$ for the population P_i, i=1,2,….M and

$\alpha = Maxgen2/\log\left[\dfrac{p_m(0)}{10^{-3}}\right]$ for the population P(T) in the promising zone.

h. **Reduction process of neighbourhood** V(0) is the initial neighbourhood of S*. V(T) is calculated by the formula $V(T) = V(0)\exp(-T/\alpha)$, where α is calculated so that the final neighbourhood is small enough(10^{-2} in our case). So $\alpha = Maxgen3/\log\left[\dfrac{V(0)}{10^{-2}}\right]$

6. Assumptions and notations for the proposed model

The following notations and assumptions are used in developing the model.

6.1 Notations

c_h holding cost per unit/unit time.

H time horizon.

$p(t)$ purchase cost per unit.

$s(t)$ selling price per unit.

$\theta(t)$ deterioration rate

c_0 ordering cost.

$Q(T_i)$ order quantity at $t=T_i$.

$q(t)$ inventory level at time t.

Z total profit from the planning horizon H.

$D(t)$ Demand per unit time.

n_1, n_2, n_3 number of replenishment made during $(0,H_1)$,(H_1,H_1+H_2), $(H_1+H_2, H_1+H_2+H_3)$ respectively.

m_1, m_2, m_3 mark up of purchasing cost during $(0,H_1)$,(H_1,H_1+H_2),$(H_1+H_2,H_1+H_2+H_3)$ respectively.

R maximum lifetime of the product.

t_1 first cycle length over the time interval $(0, H_1)$.

$t_1^{/}$ initial cycle length over the time interval $(H_1+H_2, H_1+H_2+H_3)$.

T_i Total time elapses upto and including i^{th} cycle ($i=1,2,\ldots\ldots n_1+n_2+n_3$)

6.2 Assumptions

(i) Inventory system involves only one item.
(ii) Time horizon(H) is finite and $H=H_1+H_2+H_3$.
(iii) Shortages are not allowed.
(iv) Unit cost, i.e., purchase price p(t) is a function of t and is of the form

$$p(t)=\begin{cases} be^{-ct} & for\ 0\le t\le H_1 \\ be^{-cH_1} & for\ H_1\le t\le H_1+H_2 \\ Ae^{\frac{cH_1(t-H_1-H_2)}{H_3}} & for\ H_1+H_2\le t\le H_1+H_2+H_3 \end{cases}$$

where A= be^{-cH_1}

(v) Selling price *s(t)* is mark-up *m* of *p(t)* and *m* takes the values m_1, m_2 and m_3 during $(0,H_1)$, (H_1, H_1+H_2) and $(H_1+H_2, H_1+H_2+H_3)$ i.e. $s(t)=m[=m_1, m_2, m_3]\ p(t)$.

(vi)Demand is a function of selling price *s(t)* and is of the form

$$D(t)=\frac{D_0}{[s(t)]^{\gamma}}=\frac{D_1}{[p(t)]^{\gamma}}\ where\ D_1=\frac{D_0}{m^{\gamma}},\quad D_0>0$$

(vii) The lead time is zero.

(viii) Deterioration rate $\theta(t)$ is a function of time where $\theta(t)=\frac{1}{1+R+T_{j-1}-t}$ where R is the maximum lifetime of the product. This form of deterioration comes from the fact that as $(t-T_{j-1})\to R$, $\theta(t)\to 1$ i.e. rate of deterioration tends to 100%.

(ix) T_i is the total time that elapses up to and including the *i*-th cycle ($i=1,2,\ldots,n_1+n_2+n_3$) where $n_1+n_2+n_3$ denotes the total number of replenishment to be made during the interval $(0, H_1+H_2+H_3)$ and $T_0=0$.

(x) n_1 is the number of replenishment to be made during $(0,H_1)$ at $t=T_0, T_1,\ldots, T_{n_1-1}$ So, there are n_1 cycles in this duration. As purchase cost decreases during this session, so demand increases. Hence, successive cycle length must decrease. Here, α is the rate of reduction of successive cycle length and t_1 is the first cycle length. So, i-th cycle length $t_i=t_1-(i-1)\alpha$.

$$T_i=\sum_{j=1}^{i}t_j=it_1-\alpha\frac{i(i-1)}{2},\ i=1,2,\ldots,n_1.Clearly, T_{n_1}=H_1$$

$$Thus,\ n_1t_1-\alpha\frac{n_1(n_1-1)}{2}=H_1$$

$$\Rightarrow \alpha=\frac{2(n_1t_1-H_1)}{n_1(n_1-1)}$$

(8)

Here, t_1 is decision variable.

(xi) n_2 be the number of replenishment to be made during (H_1, H_1+H_2). Since purchase cost is constant, demand is also constant during this interval. So, all the sub-cycle length in this interval is assumed as constant. Replenishment are done at

$$t=T_{n_1},T_{n_1+1},\ldots,T_{n_1+n_2-1}\ where\ T_{n_1+j}=T_{n_1}+(j-1)\frac{H_2}{n_2},\ j=1,2,\ldots,n_2$$

(xii) n_3 is the number of replenishment to be made during $(H_1+H_2, H_1+H_2+H_3)$. During this interval, purchase cost increases, as a result demand decreases. So, the duration of placing of order gradually increases. Here, β be the rate of increase of cycle length. Let t_1' be the initial cycle length. Then *i*-th cycle length $t_i'=t_1'+(i-1)\beta$.

Thus, $t'_{n_3}=t_1'+(n_3-1)\beta$. Orders are made at $t=T_{n_1+n_2},T_{n_1+n_2+1},\ldots,T_{n_1+n_2+n_3-1}$ Where

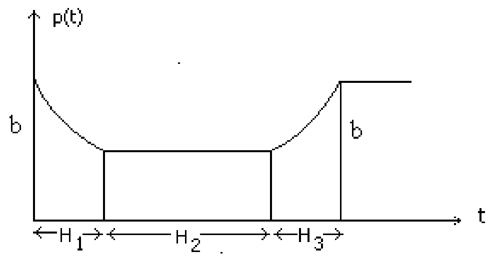
$$T_{n_1+n_2+i}=T_{n_1+n_2}+\sum_{j=1}^{i}t'_j=H_1+H_2+it_1'+\beta\frac{i(i-1)}{2}$$

$$Clearly,\ T_{n_1+n_2+n_3}=H_1+H_2+H_3$$

$$H_1+H_2+n_3t'_1+n_3(n_3-1)\beta/2=H_1+H_2+H_3$$

$$\Rightarrow \beta=\frac{2(H_3-n_3t'_1)}{n_3(n_3-1)} \qquad (9)$$

A wavy bar (~) is used with this symbol to represent corresponding fuzzy numbers when required.



Pictorial representation of p(t)
Fig-2

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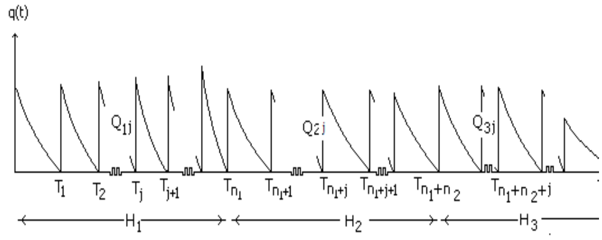

Inventory situation of the model
Fig-3

7. Model development and analysis

In the development of the model, it is assumed that at the beginning of every *j*-th cycle *[T*$_{j-1}$*, T*$_j$*]*, an amount *Q1*$_j$ units of item is ordered. As lead time negligible, replenishment of an item occurs as soon as order is made. Item is sold during the cycle and inventory level reaches zero at time t=T_j. Then order for next cycle is made. Here, selling price is a markup of initial purchase cost for each cycle. The inventory situation and the purchase cost are shown in Fig-2 and Fig-3.

7.1 Formulation of the model in crisp environment

This part is formulated in three phases.

7.1.1 Formulation for first phase (i.e., $0 \le t \le H_1$)

Duration of *j*-th ($1 \le j \le n_1$) cycle is *[T*$_{j-1}$, *T*$_j$*]* where $T_{j-1} = jt_1 - \alpha\, j(j-1)/2$ at the beginning of the cycle inventory level is Q1$_j$. So, the governing differential equation of the model in the presence of deterioration of the item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} + \theta(t)\, q = -D_j \qquad (10)$$

where $D_j = \dfrac{D_1}{\left(m_1 b e^{-cT_{j-1}}\right)^{\gamma}}$ and $\theta(t) = \dfrac{1}{1+R+T_{j-1}-t}$

Solving the above differential equation using the initial condition at t=T$_j$, q(t)=0, we get

$$q(t) = (1+R+T_{j-1}-t)D_j \log\left(\frac{1+R+T_{j-1}-t}{1+R+T_{j-1}-T_j}\right)$$

(11)

When $t = T_{j-1}$,

$$Q1_j = q(T_{j-1}) = (1+R)D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right) \qquad (12)$$

So, the holding cost for jth ($1 \le j \le n_1$) cycle, $H1_j$ is given by $H1_j = c_h \int_{T_{j-1}}^{T_j} q(t)\,dt$

$$= c_h D_j \left[\begin{array}{l} \frac{1}{4}\left\{(1+R+T_{j-1}-T_j)^2 - (1+R)^2\right\} \\ + \frac{(1+R)^2}{2} \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right) \end{array} \right]$$

Thus, the total holding cost during (0, H$_1$), HOC1, is given by $HOC1 = \sum_{j=1}^{n_1} H1_j$ (13)

Total purchase cost during (0, H$_1$), PC1, is given by

$$PC1 = \sum_{j=1}^{n_1}\left[Q1_j\, p(T_{j-1})\right]$$

$$= \sum_{j=1}^{n_1}\left[(1+R)D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right) p(T_{j-1})\right]$$

(14)

where $p(T_{j-1}) = be^{-cT_{j-1}}$

Total ordering cost during *(0 ,H*$_1$*)*, OC1, is given by

$$OC1 = \sum_{j=1}^{n_1}\left[c_{o1} + c_{o2} Q1_j\right] \qquad (15)$$

where Q1$_j$ is given by (12)

Selling price for j-th ($1 \le j \le n_1$) cycle SP1$_j$, is given by $SP1_j = m_1 p(T_{j-1}) \int_{T_{j-1}}^{T_j} D_j\, dt$

$$= m_1 p(T_{j-1}) D_j (T_j - T_{j-1})$$

Total selling price during *(0, H*$_1$*)*, SP1, is given by

$$SP1 = \sum_{j=1}^{n_1} SP1_j \qquad (16)$$

7.1.2. Formulation of second phase (i.e., $H_1 \le t \le H_1 + H_2$)

In the second phase, the purchase price of an item remains constant. So, the demand of customer is taken as constant. During of *j*-th ($n_1 \le j \le n_1 + n_2$) cycle is *[T*$_{j-1}$, *T*$_j$*]*. The governing differential equation of the model of deteriorating item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} + \theta(t)\,q = -D_j \qquad (17)$$

where $D_j = \dfrac{D_1}{\left(m_2 b e^{-cH_1}\right)^{\gamma}}$ and

$$\theta(t) = \frac{1}{1+R+T_{j-1}-t}$$

Solving the above differential equation using the initial condition t=T_j, q(t)=0, we get

$$q(t) = (1+R+T_{j-1}-t)D_j \log\left(\frac{1+R+T_{j-1}-t}{1+R+T_{j-1}-T_j}\right) \qquad (18)$$

When $t = T_{j-1}$,

$$Q2_j = q(T_{j-1}) = (1+R)D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right)$$

(19)

So, the holding cost for j-th ($n_1 \le j \le n_1+n_2$) cycle, $H2_j$, is given by $H2_j = c_h \int_{T_{j-1}}^{T_j} q(t)\,dt$

$$= c_h D_j \left[\frac{1}{4}\left\{ \left(1+R+T_{j-1}-T_j\right)^2 - (1+R)^2 \right\} + \frac{(1+R)^2}{2} \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right) \right]$$

Thus, the total holding cost during (H_1, H_1+H_2), *HOC2* is given by $HOC2 = \sum_{j=n_1+1}^{n_1+n_2} H2_j$ (20)

Total purchase cost during *(H_1, H_1+H_2), PC2*, is given by

$$PC2 = \sum_{j=n_1+1}^{n_1+n_2} \left[Q2_j \, p(T_{j-1}) \right]$$

$$= \sum_{j=n_1+1}^{n_1+n_2} \left[(1+R) D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right) p(T_{j-1}) \right]$$

(21)

where $p(T_{j-1}) = b e^{-cH_1}$

Total ordering cost during (H_1, H_1+H_2), *OC2*, is given by $OC2 = \sum_{j=n_1+1}^{n_1+n_2} \left[c_{o1} + c_{o2} Q2_j \right]$ (22)

where $Q2_j$ is given by (19)

Selling price for j-th ($n_1 \le j \le n_1+n_2$) cycle *$SP2_j$*, is given by $SP2_j = m_2 p(T_{j-1}) \int_{T_{j-1}}^{T_j} D_j\,dt$

$$= m_2 p(T_{j-1}) D_j (T_j - T_{j-1})$$

Total selling price during *(H_1, H_1+H_2), SP2*, is given by $SP2 = \sum_{j=n_1+1}^{n_1+n_2} SP2_j$ (23)

7.1.3. Formulation of third phase (i.e., $H_1+H_2 \le t \le H_1+H_2+H_3$)

In the second phase, duration of *j*-th ($n_1+n_2 \le j \le n_1+n_2+n_3$) cycle is $[T_{j-1}, T_j]$ where

$$T_j = H_1 + H_2 + (j-n_1-n_2)t_1' + (j-n_1-n_2)(j-n_1-n_2-1)\beta/2$$

and at the beginning of cycle inventory level is *Q3j* . So, instantaneous state q(t) of deteriorating item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} + \theta(t)\,q = -D_j$$

(24)

where $D_j = \dfrac{D_1}{\left(m_3 A e^{\frac{cH_1}{H_3}\left(T_{j-1}-H_1-H_2\right)} \right)^{\gamma}}$,

$\theta(t) = \dfrac{1}{1+R+T_{j-1}-t}$ and $A = b e^{-H_1}$

Solving the above differential equation using the initial condition t=T_j, q(t)=0, we get

$$q(t) = (1+R+T_{j-1}-t)D_j \log\left(\frac{1+R+T_{j-1}-t}{1+R+T_{j-1}-T_j}\right)$$

(25)

When $t = T_{j-1}$,

$$Q3_j = q(T_{j-1}) = (1+R)D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right)$$
(26)

So,the holding cost for j-th ($n_1+n_2 \le j \le n_1+n_2+n_3$) cycle, $H3_j$, is given by

$$H3_j = c_h \int_{T_{j-1}}^{T_j} q(t)\,dt$$

$$= c_h D_j \left[\begin{array}{l} \frac{1}{4}\left\{\left(1+R+T_{j-1}-T_j\right)^2 - (1+R)^2\right\} \\ + \frac{(1+R)^2}{2}\log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right)\end{array}\right]$$

Thus, the total holding cost during ($H_1+H_2 \le t \le H_1+H_2+H_3$), HOC3,

is given by $HOC3 = \sum_{j=n_1+1}^{n_1+n_2} H3_j$ (27)

Total purchase cost during ($H_1+H_2 \le t \le H_1+H_2+H_3$), PC3, is given by

$$PC3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[Q3_j\, p(T_{j-1})\right]$$

$$= \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[(1+R)D_j \log\left(\frac{1+R}{1+R+T_{j-1}-T_j}\right) p(T_{j-1})\right]$$
(28)

where $p(T_{j-1}) = Ae^{\frac{cH_1}{H_3}(T_{j-1}-H_1-H_2)}$, $A = be^{-H_1}$

Total ordering cost during ($H_1+H_2 \le t \le H_1+H_2+H_3$), *OC3*, is given by

$$OC3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[c_{o1} + c_{o2} Q3_j\right] \quad (29)$$

where $Q3_j$ is given by (26)

Selling price for j-th ($n_1+n_2 \le j \le n_1+n_2+n_3$)

cycle , $SP3_j$ is given by $SP3_j = m_3 p(T_{j-1}) \int_{T_{j-1}}^{T_j} D_j\, dt$

$$= m_3 p(T_{j-1}) D_j (T_j - T_{j-1})$$

Total selling price during ($H_1+H_2 \le t \le H_1+H_2+H_3$), *SP3*, is given by

$$SP3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} SP3_j \quad (30)$$

Thus, total profit Z, for this model over the planning horizon ($H_1+H_2+H_3$), is given by

Z=(SP1+SP2+SP3)-(PC1+PC2+PC3)-(HOC1+HOC2+HOC3)-(OC1+OC2+OC3)(31)

8. Mathematical model in crisp environment

According to the above discussion, as lifetime of the product is R, so, no cycle should exceed R which implies $t_1 \le R, H_2/n_2 \le R, t'_{n_3} \le R.$ Therefore, the problem reduces to determine the decision variables $t_1, t'_1, m_1, m_2, m_3, n_1, n_2$ *and* n_3 . The problem becomes

Maximize Z

subject to $t_1 \le R, H_2/n_2 \le R, t'_{n_3} \le R.$ (32)

This constrained optimization problem is solved using proposed RRGA for crisp objective function.

9. Mathematical model in fuzzy environment

As discussed in introduction section, in real life phase intervals H_1, H_2 and H_3 are imprecise in nature i.e $\tilde{H}_1, \tilde{H}_2$ and $\tilde{H}_3$ respectively, then the profit function Z reduces to the fuzzy number $\tilde{Z}$ whose membership function is a function of the decision variables t_1, t_1', m_1, m_2, m_3, n_1, n_2 and n_3. Also the last cycle length t'_{n3} becomes imprecise $\tilde{t}'_{n3}$. So, the problem reduces to fuzzy optimization problem

Maximize $\tilde{Z}$ (33)

subject to $t_1 \le R, \tilde{H}_2/n_2 \le R, \tilde{t}'_{n3} \le R$

If $\tilde{H}_1, \tilde{H}_2$ and $\tilde{H}_3$ are considered as TFNs (H_{11}, H_{12}, H_{13}), (H_{21}, H_{22}, H_{23}) and (H_{31}, H_{32}, H_{33}) respectively, then $\tilde{Z}$ becomes a TFN (Z_1, Z_2, Z_3), where Z_i=value of Z for $H_1=H_{1i}$, $H_2=H_{2i}$, $H_3=H_{3i}$, i=1,2,3. In this case $\tilde{t}'_{n3}$ also becomes a TFN (t'_{n31}, t'_{n32}, t'_{n33}) .So it is an obvious assumption that fuzzy constraints should necessarily hold. The problem reduces to

Maximize $\tilde{Z}$

subject to $t_1 \le R, H_{23}/n_2 \le R, t'_{n33} \le R$ (34)

Since optimization of fuzzy number is not well defined one can optimize the optimistic (pessimistic) return of the fuzzy objective $\tilde{Z}$ with some degree of possibility (necessity) $\alpha_1\,(\alpha_2)$ as described in §2.1. Accordingly, in optimistic sense the problem reduces to

Maximize z_1

subject to $pos\{Z \geq z_1\} \geq \alpha_1$

and $t_1 \leq R, H_{23}/n_2 \leq R, t'_{n33} \leq R$ (35)

and in pessimistic sense the problem reduces to

Maximize z_1

subject to $nes\{Z \geq z_1\} \geq \alpha_2$
i.e., $pos\{Z \leq z_1\} < 1-\alpha_2$

and $t_1 \leq R, H_{23}/n_2 \leq R, t'_{n33} \leq R$ (36)

This constraint optimization problem is solved using proposed FSRRGA.

10. Numerical Experiments

10.1 Results obtained for crisp environment

To illustrate the model following hypothetical set of data is used. This data set is taken for items like rice, potato, wheat, onion, cabbage, cauliflower, etc, whose demand exists in the market throughout the year. When new crops come in the market, then its price gradually decreases during some weeks (say H_1) and reaches a lowest level. This minimum price prevails for few weeks (say H_2). Then again it gradually increases during few weeks (say H_3) and reaches its normal value. This normal price prevails remaining part of the year. For an item like potato, values of H_1, H_2 and H_3 are about 5 weeks, 15 weeks, 7 weeks in the state of West Bengal, India. Normal price of the item throughout the year is about \$3 for a 10 kg bag. Lowest price of it in the season is about \$2 for a 10 kg bag. Keeping this real situation, following data set is fixed to illustrate the model in crisp environment. In the data set 10 kg of the item is considered as one unit item, one week is considered as unit time and costs are represented in \$.

b=10, c=0.2, H_1=5(weeks), H_2=(15 weeks), H_3=7(weeks), D_0=1500, γ=2.5, c_h=0.5, c_{01}=10, c_{02}=0.5, R=3.

For the above parametric values, results are obtained via RRGA and presented in Table-1.

Table-1Results for crisp model via RRGA

n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit(\$)
3	13	4	2.432	2.380	2.577	2.051	1.408	280.981

For above parametric values, results are obtained for different values of γ and presented in Table-2. It is observed that as γ increases, profit decreases due to decrease of demand which agrees with reality. It is also found that as γ increases for same values of n_1, n_2 and n_3, t_1 increases but t'_1decreases. Moreover, m_1, m_2 and m_3 also decrease with increase of γ. It happens because as γ increases demand decreases in each cycle and demand is minimum when purchase cost is maximum. According to assumption, purchase cost is maximum in first and last cycle of the whole planning horizon. As demand decreases length of first and last cycle increases as a result t_1 increases and t'_1 decreases. Again as demand decreases due to increase of γ to keep the demand high markup of selling price m_1, m_2 and m_3 also decreases. From the table-2, it has been seen that the parameter γ is highly sensible. The observation is more practical and hence realistic one.

Table-2 Results for crisp model due to different γ via RRGA

γ	n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit(\$)
2.40	4	14	4	2.372	2.400	2.641	1.573	1.434	407.980
2.42	4	14	4	2.358	2.386	2.628	1.567	1.432	379.800
2.44	4	14	4	2.345	2.372	2.614	1.562	1.427	353.961
2.45	4	14	4	2.331	2.359	2.602	1.558	1.423	342.311
2.46	3	13	4	2.463	2.402	2.620	2.058	1.422	320.955
2.48	3	13	4	2.448	2.388	2.588	2.052	1.416	299.105
2.50	3	13	4	2.434	2.373	2.573	2.047	1.412	278.648
2.52	3	13	4	2.421	2.360	2.560	2.041	1.409	258.864

For the above parametric values, results are obtained for different values of R and presented in Table-3. It is observed that as R increases profit increases. It happens because increase of R, i.e., increase of lifetime of the product, decreases rate of deterioration which in turn increases profit.

Table-3 Results for crisp model due to different R

R	n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit(\$)
2.70	3	13	4	2.487	2.411	2.653	2.029	1.430	267.983
2.80	3	13	4	2.472	2.398	2.629	2.036	1.423	272.680
2.90	3	13	4	2.454	2.387	2.606	2.043	1.417	277.804
3.00	3	13	4	2.436	2.375	2.581	2.049	1.412	281.379
3.10	3	13	4	2.416	2.365	2.569	2.054	1.404	286.675
3.20	3	13	4	2.389	2.354	2.549	2.061	1.397	289.172
3.30	3	13	4	2.377	2.343	2.521	2.065	1.391	294.784
3.40	3	13	4	2.365	2.335	2.497	2.071	1.384	296.226

10.2 Results obtained for fuzzy environment

To illustrate the proposed inventory models, following input data are considered. In this case also hypothetical data set is used and source of this data has been discussed for crisp model. For crisp model it was considered that unit price of the item decreases during the period H_1=5 weeks, but in reality it is about 5 weeks which is fuzzy in nature. Due to this reason here H_1 is considered as TFN (4.75, 5, 5.2). Following the same argument other parameters are fixed and the data set are presented below. In the data set fuzzy numbers are considered as TFN types.

b=10, c=0.2, $\tilde{H}_1$=(4.75, 5, 5.2), $\tilde{H}_2$=(14.5,15, 15.4), $\tilde{H}_3$=(6.8, 7, 7.3), D_0=1500, γ=2.5, c_h=0.5, c_{01}=10, $\alpha_1 = 0.9, \alpha_2 = 0.1$, c_{02}=0.5, R=3.

For the above parametric values, results are obtained via FSRRGA in optimistic and pessimistic sense and presented in Table-4 and 5.

Table-4

Table-4 Results for fuzzy model via FSRRGA in optimistic sense

n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit ($)
3	13	4	2.422	2.370	2.577	2.051	1.408	311.242

Table-5

Table-5 Results obtained for fuzzy model via FSRRGA in pessimistic sense

n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit ($)
4	13	4	2.430	2.380	2.587	2.156	1.439	245.644

From the Tables 6 and 7, it is observed that as the degree of acceptability (α_1) for optimistic sense increases, the profit decreases and the increase of degree of acceptability (α_2) for pessimistic sense brings down, the profit also decreases. All these observations agree with reality.

Table-6 Sensitivity analysis in optimistic sense

α_1	n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit ($)
0.92	3	13	4	2.422	2.370	2.577	2.051	1.408	310.560
0.94	3	13	4	2.422	2.370	2.577	2.051	1.408	309.614
0.96	3	13	4	2.422	2.370	2.577	2.051	1.408	308.664
0.98	3	13	4	2.422	2.370	2.577	2.051	1.408	307.714
1.00	3	13	4	2.422	2.370	2.577	2.051	1.408	306.774

Table-7 Sensitivity analysis in pessimistic sense

α_2	n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit ($)
0.12	4	13	4	2.430	2.380	2.587	2.156	1.439	244.804
0.14	4	13	4	2.430	2.380	2.587	2.156	1.439	243.974
0.16	4	13	4	2.430	2.380	2.587	2.156	1.439	243.144
0.18	4	13	4	2.430	2.380	2.587	2.156	1.439	242.304
0.20	4	13	4	2.430	2.380	2.587	2.156	1.439	241.494

For the above parametric values, results are obtained for different values of γ and presented in Table-8 . In this case also same trend of result is obtained as found in crisp model.

Table-8

Table-8 Results for fuzzy model due to different γ via FSRRGA

γ	n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Profit ($)
2.40	4	14	4	2.307	2.405	2.653	1.569	1.421	446.593
2.42	4	14	4	2.294	2.391	2.637	1.571	1.417	416.973
2.44	4	14	4	2.281	2.378	2.620	1.576	1.415	388.757
2.46	3	13	4	2.458	2.406	2.607	2.040	1.413	359.851
2.48	3	13	4	2.444	2.394	2.591	2.044	1.409	334.977
2.50	3	13	4	2.431	2.380	2.577	2.050	1.407	311.285
2.52	3	13	4	2.416	2.368	2.564	2.058	1.405	288.713

For the above parametric values, results are obtained for different values of R and presented in Table-6. As expected in this case also same trend of result is obtained as in crisp model, i.e., profit increases with increase of R, which agrees in reality.

Table-9 Results due to different R for fuzzy model via FSRRGA

R	n_1	n_2	n_3	m_1	m_2	m_3	t_1	t'_1	Z($)
2.90	3	13	4	2.494	2.392	2.652	2.046	1.411	306.583
3.00	3	13	4	2.431	2.380	2.577	2.050	1.407	311.285
3.10	3	13	4	2.412	2.369	2.555	2.055	1.402	315.735
3.20	3	13	4	2.394	2.359	2.537	2.059	1.394	319.966
3.30	3	13	4	2.379	2.350	2.520	2.062	1.390	323.993
3.40	3	13	4	2.364	2.340	2.502	2.066	1.387	327.826
3.50	3	13	4	2.351	2.376	2.486	2.071	1.379	330.440

10. Conclusion

Here, a real-life inventory model for deteriorating seasonal product is developed whose demand depends upon the unit cost of the product in fuzzy environment. Unit cost of product is time dependent. Lifetime of each item is finite and rate of deterioration depend on the age of the item. Unique contribution of the paper is fourfold:

- The model is developed for such items like food grains, pulses, potato, onion etc., whose stable demand exists in the market throughout the year but it fluctuates for a part of the year when they are produced in the field. Here modeling is done for such products during their season of grown. These items are normally stored in cold storage and when bought in the market items are fully deteriorated after a finite time R, which is considered here as lifetime of the product. For the best of author's knowledge none have considered this type of inventory model.

- Here for the first time unit cost of an item is modeled following real life situation, which gradually decreases with time during grown of the item in the field, then it retains the lowest value for a period and again gradually increases with time to normal price of the year. Though it is found for above mentioned items in every year, inventory practitioners overlooked this real life phenomenon.
- It is assumed that time horizon of the season is fuzzy in nature. For the first time season of an item is considered as a combination of three imprecise intervals. In fact three parts in which unit cost function can be divided are considered as fuzzy numbers, which agree with reality.
- As optimization of fuzzy objective is not well defined, optimistic/pessimistic return of the objective function (using possibility/necessity measure of the fuzzy event) is optimized. A fuzzy simulation based region reducing genetic algorithm is proposed to evaluate this fuzzy objective value.

At length, though the model is formulated in fuzzy environment, demand or lifetime/deterioration of the product is not considered as imprecise in nature, though it is appropriate for these types of products. In fact, consideration of fuzzy demand or deterioration the inventory model leads to fuzzy differential equation for formulation of the model. Using proposed solution approach one cannot consider imprecise demand which is the major limitation of the approach. So, further research work can be done incorporating fuzzy demand and or deterioration in the imprecise planning horizon. Though the model is presented in crisp environment and fuzzy, it can be formulated in stochastic, fuzzy-stochastic environment.

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References

[1]Banerjee, S. and Sharma, A.(2010A), Inventory model for seasonal demand with option to change the market, Computers & Industrial Engineering, Vol-59, pp. 807-818.

[2]Banerjee, S. and Sharma, A.(2010B), Optimal procurement and pricing policies for inventory models with price and time dependent seasonal demand, Mathematical and Computer Modelling,Vol-51, pp.700-714.

[3]Bellman, R. E. and Zadeh, L. A. (1970), Decision making in a fuzzy environment, Management Science,Vol-17, No.4, B141-B164.

[4]Bera, U. K., Maiti, M. K. and Maiti, M. (2012), Inventory model with fuzzy lead-time and dynamic demand over finite time horizon using a multi-objective genetic algorithm, Computers and Mathematics with Applications, doi:10.1016/j.camwa.2012.02.060.

[5]Bera, U. K., Maiti, A. K. (2012), A soft computing approach to multi-item fuzzy EOQ model incorporating discount, Int. J. of Information and Decision Science ,Vol-4, No.4, pp. 313-328 .

[6]Bera, A.K. & Jana, D.K. (2017), Multi-item imperfect production inventory model in Bi-fuzzy environment, Opsearch, Vol-54, No.2, pp.260-282.

[7]Bessaou, M. and Siarry, P. (2001), A genetic algorithm with real-value coding to optimize multimodal continuous function, Structural and Multidisciplinary Optimization, Vol-23, No.1, pp.63–74.

[8]Chen,K.K. and Chang, C.-T. (2007), A seasonal demand inventory model with variable lead time and resource constraints, Applied Mathematical Modelling, Vol-31, pp.2433-2445.

[9]Chowdhury, R. Roy , Ghosh, S. K. , Chaudhuri, K. S.(2015), An Inventory Model for Deteriorating Items with Stock and Price Sensitive Demand,International Journal of Applied and Computational Mathematics, Vol-1, No.2, pp.187–201.

[10]Das, D., Roy, A. and Kar, S. (2012), A multi-item inventory model for two-stage production system with imperfect processes using differential evolution and credibility measure, International Journal of Operations Research, Vol-9, No.2, pp.87–99.

[11]Davis, T.E. and Principe, J.C. (1991), A simulated annealing like convergence for the simple genetic algorithm', Proceedings of 4th International Conference on Genetic Algorithms ICGA, pp.174–182.

[12]De, S.K. and Mahata, G.C (2017), Decision of a fuzzy inventory with fuzzy backorder model under

cloudy fuzzy demand rate, International Journal of Applied and Computational Mathematics, Vol-3,No,3, pp.2593-2609.
[13]De, S.K. & Sana, S.S. (2015), Backlogging EOQ model for promotional effort and selling price sensitive demand-an intuitionistic fuzzy approach, Annals of Operations Research , Vol.-233, No.1,pp. 57-76.
[14]Dutta,D & Kumar, P. (2015), A partial backlogging inventory model for deteriorating items with time-varying demand and holding cost: An interval number approach , Croatian Operational Research Review, Vol- 6, pp. 321–334.
[15]Dubois, D. & Prade,H.(1983), Ranking Fuzzy numbers in the Setting of Possibility Theory, Information Sciences, Vol-30, pp.183-224.
[16]Garai,G., Chakraborty, D. & Roy, T.K. (2016), A multi-item periodic review probabilistic fuzzy inventory model with possibility and necessity constraints, International Journal of Business Forecasting and Marketing Intelligence , Vol- 2, No.3, pp.175–189.
[17]Goldberg, D.E. (1989), Genetic Algorithms: Search, Optimization and Machine Learning, Addison Wesley, Massachusetts.
[18]Guchhait, P., Maiti, M.K. & M.Maiti (2014), Inventory model of a deteriorating item with price and credit linked fuzzy demand : A fuzzy differential equation approach, Opsearch, Vol-51, No.3, pp. 321-353.
[19]Jaber, M.Y., Bonney, M., Rosen, M.A., & Moualek, I., (2009), Entropic order quantity (EOQ) model for deteriorating items, Applied Mathematical Modelling, Vol-33, No.1, pp. 564-578.
[20]Janssen, L., Claus, T. & Sauer, J. (2016), Literature review of deteriorating inventory models by key topics from 2012 to 2015, Int. Jr. of Production Economics, Vol-182, pp.86-112.
[21]Karmakar, B. & Dutta, K. (2014), Inventory models with ramp-type demand for deteriorating items with partial backlogging and time varying holding cost, Yugoslav Journal of Operations Research,Vol-24,No. 2, pp.249 – 266.
[22]Kumar, S. and Rajput, U. S. (2015), Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging, International Journal of Applied Mathematics, Vol-6, No.3, pp. 496–509.
[23]Krommyda, I.P., Skouri, K.., Konstantaras, I. & Ganas, I.(2017), Two-Warehouse Inventory Systems for Seasonal Deteriorating Products with Permissible Delay in Payments, Operational Research in Business and Economics, Springer Proceedings in Business and Economics, DOI 10.1007/978-3-319-33003-7_13, pp.246-269.
[24]Kao, C. and Hsu, W. K. (2002), Lot Size-Reorder Point Inventory Model with Fuzzy Demands, Computers & Mathematics with Applications, Vol-43, pp.1291-1302.
[25]Lam, S. M. and Wong, D. S. (1996), A fuzzy mathematical model for the joint economic lot size problem with multiple price breaks, European Journal of Operational Research, Vol-95, pp.611-622.
[26]Lee, Y. Y., Kramer, A. B. and Hwang, C. L. (1991), A comparative study of three lot-sizing methods for the case of fuzzy demand, International Journal of Production Management, Vol-9, pp.72-80.
[27]Liu, B. and Iwamura, K. (1998A), Chance constraint Programming with fuzzy parameters, Fuzzy Sets and Systems, Vol- 94, pp.227-237.
[28]Liu, B. and Iwamura, K. (1998B), A note on chance constrained programming with fuzzy coefficients, Fuzzy Sets and Systems, Vol-100, pp. 229-233.
[29]Mahata, G. C. and Goswami, A. (2010), The Optimal Cycle Time for EPQ Inventory Model of Deteriorating Items under Trade Credit Financing in the Fuzzy Sense, International Journal of Operations Research, Vol-7,No.1, pp.26-40.
[30]Manna, A.K., Das, B., Dey, J.K. & Mondal, S.K. (2016), Multi-item EPQ model with learning effect on imperfect production over fuzzy random planning horizon, Journal of Management Analytics, DOI:10.1080/23270012.2016.1217755.
[31]Maiti. A.K., Maiti, M.K. and Maiti, M. (2014), An EOQ model of an item with imprecise seasonal time via genetic algorithm, International Journal of Operational Research, Vol-19, No.3, pp.358- 384.
[32]Maiti, M.K. and Maiti, M. (2006), Fuzzy inventory model with two warehouses under possibility constraints, Fuzzy sets and fuzzy systems, Vol-157,pp.52-73.
[33]Maiti, M.K. and Maiti, M. (2007), Two storage inventory model in a mixed environment, Fuzzy Optimization and Decision Making, Vol-6, pp.391-426.
[34]Maiti, M. K.(2008), Fuzzy inventory model with two warehouses under possibility measure on fuzzy goal, European Journal of Operational Research, Vol-188, pp.746-774.
[35]Maiti, M. K.(2011), A fuzzy genetic algorithm with varying population size to solve an inventory model with credit-linked promotional demand in an imprecise planning horizon, European Journal of Operational Research, Vol-213, pp. 96-106.

[36]Maiti, A. K., Maiti, M.K. and Maiti, M. (2009), Inventory model with stochastic lead time and price dependent demand incorporating advance payment, Applied Mathematical Modelling, Vol-33, pp.2433-2443.

[37]Mandal, S. and Maiti, M. (2002), Multi-item fuzzy EOQ modelsusing Genetic Algorithm, Computers & Industrial Engineering, Vol-44, pp.105-117.

[38]Michalewicz, Z. (1992), Genetic Algorithms + Data Structures= Evolution Programs, Springer, Berlin.

[39]Mohanty, D.J., Kumar, R. & Goswami, A. (2018), Trade-credit modeling for deteriorating item inventory system with preservation technology under random planning horizon, Sådhanå, 43:45, https://doi.org/10.1007/s12046-018-0807-0

[40]Pal, S., Maiti, M. K. and Maiti, M. (2009), An EPQ model with price discounted promotional demand in an impreciseplanning horizon via Genetic Algorithm, Computers & Industrial Engineering, Vol-57, pp.181-187.

[41]Panda, S., Senapati, S. and Basu, M. (2008), Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand, Computers& Industrial Engineering, Vol-54, pp.301-314.

[42]Roy T. K. and Maiti, M. (2000), A Multi-item displayed EOQ model in fuzzy environment, The Journal of Fuzzy Mathematics, Vol- 8, pp.881-888.

[43]Roy, A. Pal, S. & Maiti, M.K. (2009), A production inventory model with stock dependent demand incorporating learning and inflationary effect in a random planning horizon: A fuzzy genetic algorithm with varying population size approach, Compuetrs & Industrial Engineering, Vol-57, No.4, pp.1324-1335.

[44]Rastogi, M., Kushwah, P., Singh, S. R. (2018), An inventory model for non-instantaneous deteriorating products having price sensitive demand and partial backlogging of occurring shortages, International Journal of Operations and Quantitative Management, Vol-24, No.1, pp.59-73.

[45]Sarkar, B., (2011) An EOQ model with delay in payments and time varying deterioration rate, Mathematical and Computer Modelling,Vol-55, No.4,367-377.

[46]Sana, S.S., (2011), Price-sensitive demand for perishable items - an EOQ model, Applied Mathematics and Computation, Vol-217, pp.6248-6259.

[47]Skouri,K, and Konstantaras, I. (2013), Two-warehouse inventory models for deteriorating products with ramp type demand rate. Journal of Industrial & Management Optimization, Vol-9, No.4, pp. 855-883.

[48]Simon, D., Rarick, R., Ergezer, M. and Du, D. (2011) , Analytical and numerical comparisons of biogeography-based optimization and genetic algorithms, Information Sciences, Vol.-181, No.7, pp.1224–1248.

[49]Tayal, S., Singh, S & Sharma, R. (2015). An inventory model for deteriorating items with seasonal products and an option of an alternative market. Uncertain Supply Chain Management, Vol-3, No.1, pp.69-86.

[50]Yadav, D, Pundir, S. & Kumari, R., (2011), A fuzzy multi-item production model with reliability and flexibility under limited storage capacity with deterioration via geometric programming, Int. J. of Mathematics in Operational Research, Vol.-3, No.1, pp.78-98.

[51]Zadeh, L.A. (1978), Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets and Systems, Vol-1, pp.3-28.

[52]Zegordi, S.H., Kamal Abadi, I.N. and Beheshti Nia, M.A. (2010), A novel genetic algorithm forsolving production and transportation scheduling in a two-stage supply chain, Computers & Industrial Engineering, Vol,-58, No.3, pp.373–381.

[53]Zhou, Y-W., Lau, H-S. and Yang, S-L. (2004), A finite horizon lot-sizing problem with time-varying deterministic demand and waiting-time-dependent partial backlogging, International Journal of Production Economics, Vol.-91, pp.109-119.