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Correspondence between the quintessence and dilaton scalar fields with the polytropic gas dark energy model

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Abstract

In this paper, we study the correspondence between the quintessence and dilaton scalar fields with the polytropic gas dark energy model. This correspondence allows us to reconstruct the dynamics and potential of the**se** scalar fields in the context of the polytropic gas dark energy model, which describes the accelerated expansion of the universe.

Keywords: *Dark energy, Polytropic gas, quintessence, dilaton*

1. Introduction

Cosmologist's belief that our universe expands under an accelerated expansion [1]-[7].In the standard Friedman Lemaitre Robertson Walker (FLRW) cosmology, a new energy with negative pressure, called dark energy (DE) is responsible for this expansion [8]. The nature of the DE is still unknown and various problems have been proposed by the researchers in this field**.** About 70% of the present energy of the universe is contained in the DE. The cosmological constant with the time independent equation of state is the earliest and simplest candidate for the dark energy. Besides the cosmological constant, there are many dynamical dark energy models with the time dependent equation of state that have been proposed to explain the cosmic acceleration. Polytropic gas is one of the dynamical dark energy models [9]. In this work, we focus on the polytropic gas model as a DE model. The polytropic gas DE model is a phenomenological model of dark energy where the pressure is a function of energy density [10]. The quintessence [12, 13] and dilaton [14]-[16] scalar fields are considered as a source of dark energy. In this paper, we establish a correspondence between the quintessence and dilaton scalar fields with the polytropic gas dark energy model. This correspondence allows us to reconstruct the dynamics and potential of the**se** fields in the context of the polytropic gas dark energy model, which describes the accelerated expansion of the universe.

2. Polytropic quintessence model

Equation of state (EOS) of the polytropic gas is given by [11]

$$P_\Lambda = K\rho_\Lambda^{1+\frac{1}{n}} \tag{1}$$

Where P_Λ, ρ_Λ, K and n are the pressure, energy density, polytropic constant and polytropic index respectively.

The conservation equation for the dark energy in the FRW universe is given by

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0 \tag{2}$$

Where H is the Hubble parameter and overhead dot denotes the differentiation with respect to the cosmological time

Using the EOS (1) into the conservation equation (2) and integrating we get

$$\rho_\Lambda = \left[Ba^{3/n} - K\right]^{-n} \tag{3}$$

Where B is a positive integration constant and $a(t)$ is a time scale factor of the universe .

The corresponding pressure takes the following form

$$P_\Lambda = K\left[Ba^{3/n} - K\right]^{-n-1} \qquad (4)$$

Using equations (3) & (4), the EOS parameter for the polytropic gas dark energy model is obtained as

$$\omega_\Lambda = \frac{P_\Lambda}{\rho_\Lambda} = -1 + \frac{Ba^{3/n}}{Ba^{3/n} - K} \qquad (5)$$

When $K > Ba^{3/n}$, from (5), we see that $\omega_\Lambda < -1$ which corresponds to a universe dominated by phantom field. The phantom field lead to accelerated expansion of the universe

The action for the quintessence scalar field φ is given by [12], [13]

$$\mathrm{S} = \int\left[\frac{1}{2}\mathrm{g}^{\mathrm{ij}}\,\partial_{\mathrm{i}}\varphi\,\partial_{\mathrm{j}}\varphi - \mathrm{V}(\varphi)\right]\sqrt{-\mathrm{g}}\,\mathrm{d}^4\mathrm{x} \quad (6)$$

Where φ *is* the quintessence field with the potential $V(\varphi)$ and g is the determinant of g_{ij}

The pressure and energy density for the quintessence scalar field φ are given by

$$P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \qquad (7)$$

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \qquad (8)$$

From the equations (3), (4), (7) and (8), we get the scalar potential and the kinetic energy terms for the polytropic gas as

$$V(\varphi) = \frac{\frac{B}{2}a^{3/n - K}}{\left(Ba^{3/n - K}\right)^{n+1}} \qquad (9)$$

$$\dot{\varphi}^2 = \frac{Ba^{3/n}}{\left(Ba^{3/n - K}\right)^{n+1}} \qquad (10)$$

The equation of state for the scalar field φ is given by

$$\omega_\varphi = \frac{P_\varphi}{\rho_\varphi} = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)} \qquad (11)$$

So that $-1 \le \omega_\varphi \le 1$

If the kinetic term $\dot{\varphi}^2$ dominates then $\omega_\varphi \approx 1$ and if the potential term $V(\varphi)$ dominates then $\omega_\varphi \approx -1$

The equation of motion for the field $\varphi = \varphi(t)$ on a FLRW background is given by the Klein Gordon equation

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0 \qquad (12)$$

Where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, overhead dot denotes differentiation with respect to cosmic time t and $V'(\varphi) = \frac{dV}{d\varphi}$

The Friedmann equation and acceleration equation are given by

$$H^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right] \qquad (13)$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3}\left[\dot{\varphi}^2 - V(\varphi)\right] \qquad (14)$$

If $\dot{\varphi}^2 < V(\varphi)$ then $\ddot{a} > 0$ this corresponds to the accelerated expansion of the universe

If $\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi)$ (Slow roll approximation) then $H^2 \approx \frac{1}{3}V(\varphi)$ and $\omega_\varphi \approx -1$ (like cosmological constant) which represents a potential dominated scalar field. In the slow roll approximation as the term $\dot{\varphi}^2$ is considered negligible and the potential $V(\varphi)$ can be considered to fulfill

$$3H\dot{\varphi} \approx -V'(\varphi) \qquad (15)$$

Therefore this slow rolling potential dominated scalar field can accelerated the expansion of the universe and act as a dark energy candidate.

3. Polytropic dilaton model

The pressure and energy density of the dilaton scalar field are given by [14]-[16]

$$P_\varphi = -\chi + ce^{\lambda\varphi}\chi^2 \qquad (16)$$

$$\rho_\varphi = -\chi + 3ce^{\lambda\varphi}\chi^2 \qquad (17)$$

Where c and λ are positive constants and $\chi = \frac{\dot{\varphi}^2}{2}$

So the equation of state parameter for the dilaton scalar field is obtain as

$$\omega_d = \frac{P_\varphi}{\rho_\varphi} = \frac{-1 + ce^{\lambda\varphi}\chi}{-1 + 3ce^{\lambda\varphi}\chi} \qquad (18)$$

To establish the correspondence between the dilaton equation of state parameter(ω_d) and polytropic equation of state parameter(ω_Λ) , equating the equations (5) and (18) we get,

$$ce^{\lambda\varphi}\chi = \frac{\omega_\Lambda - 1}{3\omega_\Lambda - 1} = \frac{2K - Ba^{3/n}}{4K - Ba^{3/n}} \quad (19)$$

Putting $\chi = \frac{\dot{\varphi}^2}{2}$ in the above equation (19) we get,

$$ce^{\lambda\varphi}\dot{\varphi}^2 = \frac{4K - 2Ba^{3/n}}{4K - Ba^{3/n}} \quad (20)$$

The above equation can be written as

$$e^{\frac{\lambda\varphi}{2}}\dot{\varphi} = \left[\frac{1}{c}\frac{4K - 2Ba^{3/n}}{4K - Ba^{3/n}}\right]^{\frac{1}{2}} \quad (21)$$

And its integration yields

$$\varphi(a) = \frac{2}{\lambda} ln\left[e^{\frac{\lambda\varphi(a_0)}{2}} + \frac{\lambda}{2\sqrt{c}}\int_{a_0}^{a}\frac{1}{aH}\left[\frac{4K - 2Ba^{3/n}}{4K - Ba^{3/n}}\right]^{\frac{1}{2}} da\right] \quad (22)$$

Thus we reconstructed the potential and dynamics of the dilaton scalar field in the context of the polytropic gas.

3. Conclusion

In this paper, we have studied the polytropic gas dark energy model with the quintessence and dilaton scalar fields. The polytropic quintessence model indicates a potential dominated scalar field universe in the slow roll approximation that corresponds to the accelerated expansion of the universe. We established a correspondence between the polytropic gas and the scalar fields. We also reconstructed the potential and dynamics of the scalar fields in the context of polytropic gas.

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