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Numerical Solution of Fuzzy Pure Multiple Neutral Delay Differential Equations

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Abstract

This article verifies the accuracy of Runge Kutta method of order four for Fuzzy multiple retarded delay differential equations to solve Fuzzy multiple neutral delay differential equations and to implement the technique to solve Fuzzy pure multiple neutral delay differential equations. The idea is to analyze the adoption of the method found for Retarded delay system to solve neutral delay systems where the two systems contains multiple delay. An numerical example is presented to verify the theory

Keywords: *Pure delay, Multiple delay, Neutral delay, Fuzzy delay differential equations, Numerical solution, Runge-Kuta method.*

1. Introduction

Evolution of daily life without Engineering and Technology is not possible. Numerical solutions deal evolution of problems involving modeling of physical, chemical and Biological systems. The approximation theory considered to be very important in any of such modeling. The Natural phenomena are always approximate where it is not possible to find the exactness. The case study of Patients health involving various tests like urine test, blood test, glucose level, cholesterol level etc., determined by medical laboratories vary from time to time, place to place and even the mental stress or stamina controls the result. So they are not constant. Approximation with some error exists in such tests. The periodical reading of planetary motion, weather prediction are also subject to error in exactness. So the numerical solutions come into effect to study any such natural systems. Here in this article we adopt one such numerical solution of fuzzy multiple neutral delay differential equations. Alfredo Bellen, Marino Zennaro [2] analyzed numerical methods for delay differential equations in detail. S. Abbasbandy and T. Allahviranloo [1] discussed numerical solution of fuzzy differential equation by Runge Kutta method . Suha Najeeb Al Rawi, Raghad Kadhim Salih and Amaal Ali Mohammed solved the linear delay differential equations of N^{th} order by Runge-Kutta method [8]. Guang Da Hu, Guang Di Hu and S. A Meguid discussed on Stability of Runge-Kutta methods for delay differential systems with multiple delays [5]. Baruh Cahlon, Darrell Schmidt analyzed the stability of systems of delay differential equations [3]. T. Jayakumar et.al., solved fuzzy delay differential equations numerically by Runge-Kutta method [6]. D. Prasantha Bharathi et.al., presented the numerical solutions of Fuzzy Retarded delay differential equations [7]. In section 2, we present the concept of fuzzy multiple neutral delay differential equations, In section 3, we explain fuzzy pure multiple neutral delay differential equations. In section 4, we propose Runge –Kutta method of order four for solving fuzzy multiple retarded delay differential equations. In Section 5 a numerical example is studied to test the theory. We conclude the study of this paper finally in Section 6.

2. Fuzzy Multiple Neutral Delay Differential Equations (FMNDDE)

Let us consider the Initial value problem in ordinary differential equations,

$$\begin{cases} y'(t) = f(t, y(t)) \\ y(0) = y_0 \end{cases} \quad (1)$$

Where $f:[0,\infty)\times R \to R$ is continuous. We would like to interpret Eq (1) using Seikkala's derivative and $y_0 \in E^1$. Let $[y_0]^\alpha = [\underline{y}^\alpha(t), \overline{y}^\alpha(t)]$. By Zadeh's extension principle, we get $f:[0,\infty)\times E^1 \to E^1$, where ,

$$[f(t,y)]^\alpha = [\min\{f(t,u) : u \in [\underline{y}^\alpha(t), \overline{y}^\alpha(t)]\}, \max\{f(t,u) : u \in [\underline{y}^\alpha(t), \overline{y}^\alpha(t)]\}] \quad (2)$$

.

So from Eq (2), the Eq (1) became fuzzy differential equation. The concept of uncertainty and randomness arises whenever we deal with natural parameters so fuzzy is adopted in modeling any physical problems.

The concept of fuzzy introduced by Chang and Zadeh[4] comes in to effect to deal the problems involving uncertainty. Differential equations just used to study the rate of change but it is not possible to know when and how does the change occurred plays a vital role in predicting any future occurrence of same incidence. In delay differential equation we are evaluating the past with the present data. In nature, Let us consider a simple problem of medication as an example. Suppose a person is found to infected by virus like Dengue, Malaria, etc.,. It was unnoticed till the infection causes him trouble. The virus particles does not affect the host all of the sudden. They will take some time to multiple themselves and to attack healthy cells. Such period is called a delay period. After the patient identified with influenza virus the Doctor would prescribe him some medicine. But the medicine taken at once will not cure the disease all of the sudden. It will take some time for the medicine to boost up the healthy cells, immunity power etc., to cure the disease. Then there comes also the delay period. In such case multiple delay differential equations play a important role.The mathematical structure of multiple neutral delay differential equations is given by

$$\begin{cases} y'(t) = f(t, y(t), y'(t-\tau_1), y'(t-\tau_2), \ldots, \\ \qquad y'(t-\tau_n)) & t_0 \le t \le t_n \\ y(t) = \phi(t) & -\tau \le t \le t_0 \\ y(t_0) = \phi(t_0) = \phi_0 \end{cases} \tag{3}$$

Similar to that of Eq(2), Eq(3) follows as $f : [0,\infty) \times R \times R \times \ldots \times R \to R$ which is continuous and

$$[f(t, y(t), y'(t-\tau_1), \ldots, y'(t-\tau_n))]^{\alpha} =$$
$$[\min\{ f(t, u(t), u'(t-\tau_1), u'(t-\tau_2), \ldots, u'(t-\tau_n))) :$$
$$u(t) \in [\underline{y}^{\alpha}(t), \overline{y}^{\alpha}(t)],$$
$$u'(t-\tau_1) \in [\underline{y}^{\alpha}(t-\tau_1), \overline{y}^{\alpha}(t-\tau_1)], \ldots,$$
$$u'(t-\tau_n) \in [\underline{y}^{\alpha}(t-\tau_n), \overline{y}^{\alpha}((t-\tau_n))]\},$$
$$\max\{ f(t, u(t), u(t-\tau_1), u(t-\tau_2), \ldots, u(t-\tau_n))) :$$
$$u(t) \in [\underline{y}^{\alpha}(t), \overline{y}^{\alpha}(t)],$$
$$u'(t-\tau_1) \in [\underline{y}^{\alpha}(t-\tau_1), \overline{y}^{\alpha}(t-\tau_1)], \ldots,$$
$$u'(t-\tau_n) \in [\underline{y}^{\alpha}(t-\tau_n), \overline{y}^{\alpha}((t-\tau_n))]\}]$$
.

(4)

Since it is not possible to directly solve neutral delay differential equation, We apply the technique of transforming it to retarded delay differential equations proposed by Alfredo Bellen,et.al

$$\begin{cases} y'(t) = f(t, \phi(t), \phi(t-\tau_1), \phi(t-\tau_2), \ldots, \\ \qquad \phi(t-\tau_n)) & t_0 \le t \le t_n \\ \phi(t) = \xi(t) & -\tau \le t \le t_0 \\ \phi(t_0) = \xi(t_0) = \xi_0 \end{cases} \tag{5}$$

Then it follows that

$$[f(t, \phi(t), \phi(t-\tau_1), \ldots, \phi(t-\tau_n))]^{\alpha} =$$
$$[\min\{ f(t, v(t), v(t-\tau_1), v(t-\tau_2), \ldots, v(t-\tau_n))) :$$
$$v(t) \in [\underline{\phi}^{\alpha}(t), \overline{\phi}^{\alpha}(t)],$$
$$v(t-\tau_1) \in [\underline{\phi}^{\alpha}(t-\tau_1), \overline{\phi}^{\alpha}(t-\tau_1)], \ldots,$$
$$v(t-\tau_n) \in [\underline{\phi}^{\alpha}(t-\tau_n), \overline{\phi}^{\alpha}((t-\tau_n))]\},$$
$$\max\{ f(t, u(t), u(t-\tau_1), u(t-\tau_2), \ldots, u(t-\tau_n))) :$$
$$v(t) \in [\underline{\phi}^{\alpha}(t), \overline{\phi}^{\alpha}(t)],$$
$$v(t-\tau_1) \in [\underline{\phi}^{\alpha}(t-\tau_1), \overline{\phi}^{\alpha}(t-\tau_1)], \ldots,$$
$$v(t-\tau_n) \in [\underline{\phi}^{\alpha}(t-\tau_n), \overline{\phi}^{\alpha}((t-\tau_n))]\}] \tag{6}$$
.

3. Fuzzy Pure Multiple Neutral Delay Differential Equations (FPMNDDE)

The Pure Multiple neutral delay arises as special case of delay differential equation. The equation is so called because there is no $\phi(t)$ and the retarded delay terms [3]. Such a system is transformed as

$$\begin{cases} \phi'(t) = f(t, \phi(t-\tau_1), \phi(t-\tau_2), \ldots, \\ \qquad \phi(t-\tau_n)), & t_0 \le t \le t_n \\ \phi(t) = \xi(t) & -\tau \le t \le t_0 \\ \phi(t_0) = \xi(t_0) = \xi_0 \end{cases} \tag{7}$$

From Eq(7), it follows that $f : [0,\infty) \times R \times R \times \ldots \times R \to R$ which is continuous and $[f(t, \phi(t-\tau_1), \ldots, \phi(t-\tau_n))]^{\alpha} =$

$$[\min\{f(t, v(t-\tau_1),\ldots,v(t-\tau_n))):$$
$$v(t-\tau_1)\in[\underline{\phi}^{\alpha}(t-\tau_1),\overline{\phi}^{\alpha}(t-\tau_1)],\ldots,$$
$$v(t-\tau_n)\in[\underline{\phi}^{\alpha}(t-\tau_n),\overline{\phi}^{\alpha}((t-\tau_n))],\},$$
$$\max\{f(t, v(t-\tau_1),\ldots,v(t-\tau_n))):$$
$$v(t-\tau_1)\in[\underline{\phi}^{\alpha}(t-\tau_1),\overline{\phi}^{\alpha}(t-\tau_1)],\ldots,$$
$$v(t-\tau_n)\in[\underline{\phi}^{\alpha}(t-\tau_n),\overline{\phi}^{\alpha}((t-\tau_n))],\}]$$

.

The FPMNDDE obeys all the properties, definitions and tests, etc., of multiple neutral delay differential equations. Because of its initial value was same as that of neutral delay differential equations Eq(1). The purpose of studying pure neutral delay differential equations is it directly gives the solution by result of delays in the derivatives of the past state without the solution of present state.

4.Fourth Order Runge-Kutta Method

In this section, for a fuzzy multiple retarded delay differential equation Eq(5), we develop the fourth order Runge-Kutta method for multiple delay $f(t,\phi(t),\phi(t-\tau_1),\phi(t-\tau_2),\ldots,\phi(t-\tau_n))$ by an application of Runge-Kutta method for fuzzy differential equation when f in Eq(1) can be obtained via the Zadeh extension principle from. $f\in C[R^{+}\times R\times\ldots\times R,R]$. We are using Runge-Kutta method for multiple delay to solve the system of the differential equation because of its stability was already discussed by Guang DaHu, Gurang Di Hu and S.A. Meguid [4]. We assume that the existence and uniqueness of solutions of Eq (1) hold for each $[t_k,t_{k+1}]$. The following method is the extension of [5]. The Runge Kutta method is the fourth order approximation of $\underline{\Phi'_k}(t,\alpha)$ and $\overline{\Phi'_k}(t,\alpha)$.

We define

$$\underline{y}(t_{n+1};\alpha)-\underline{y}(t_n;\alpha)=\sum_{i=1}^{4}w_i\underline{K}_i(t_n;y(t_n;\alpha)),$$

$$\overline{y}(t_{n+1};\alpha)-\overline{y}(t_n;\alpha)=\sum_{i=1}^{4}w_i\overline{K}i(t_n;y(t_n;\alpha))$$

Where w_1,w_2,w_3 and w_4 are constants and

$$\underline{K}_1(t;\phi(t;\alpha))=\min\{hg(t,\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_2))\mid$$
$$\phi(t)\in[\underline{\phi}(t_{k,n};\alpha),\overline{\phi}(t_{k,n};\alpha)],$$
$$\phi(t-\tau_1)\in[\underline{\phi}(t_{k,n}-\tau_1;\alpha),\overline{\phi}(t_{k,n}-\tau_1;\alpha)],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{\phi}(t_{k,n}-\tau_n;\alpha),\overline{\phi}(t_{k,n}-\tau_2;\alpha)]\}$$

$$\overline{K}1(t;\phi(t;\alpha))=\max\{hg(t,\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_n))\mid$$
$$\phi(t)\in[\underline{\phi}(t_{k,n};\alpha),\overline{\phi}(t_{k,n};\alpha)],$$
$$\phi(t-\tau_1)\in[\underline{\phi}(t_{k,n}-\tau_1;\alpha),\overline{\phi}(t_{k,n}-\tau_1;\alpha)],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{\phi}(t_{k,n}-\tau;\alpha),\overline{\phi}(t_{k,n}-\tau;\alpha)]\}$$

$$\underline{K}_2(t;\phi(t;\alpha))=\min\{hg(t+\frac{h}{2},\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_n))\mid$$
$$\phi(t)\in[\underline{z}_1(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),\overline{z}1(t_{k,n},\overline{\phi}(t_{k,n};\alpha))],$$
$$\phi(t-\tau_1)\in[\underline{z}_1(t_{k,n}-\tau_1,\underline{\phi}(t_{k,n}-\tau_1;\alpha)),\overline{z}1(t_{k,n}-\tau_1,\overline{\phi}(t_{k,n}-\tau_1;\alpha))],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{z}_1(t_{k,n}-\tau_n,\underline{\phi}(t_{k,n}-\tau_n;\alpha)),\overline{z}1(t_{k,n}-\tau_n,\overline{\phi}(t_{k,n}-\tau_n;\alpha))]\}$$

$$\overline{K}2(t;\phi(t;\alpha))=\max\{hg(t+\frac{h}{2},\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_n))\mid$$
$$\phi(t)\in[\underline{z}_1(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),\overline{z}1(t_{k,n},\overline{\phi}(t_{k,n};\alpha))],$$
$$\phi(t-\tau_1)\in[\underline{z}_1(t_{k,n}-\tau_1,\underline{\phi}(t_{k,n}-\tau_1;\alpha)),\overline{z}1(t_{k,n}-\tau_1,\overline{\phi}(t_{k,n}-\tau_1;\alpha))],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{z}_1(t_{k,n}-\tau_n,\underline{\phi}(t_{k,n}-\tau_n;\alpha)),\overline{z}1(t_{k,n}-\tau_n,\overline{\phi}(t_{k,n}-\tau_n;\alpha))]\}$$

$$\underline{K}3(t;\phi(t;\alpha)) = \min\{hg(t+\frac{h}{2},\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_n))\mid \phi(t)\in[\underline{z}2(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),\bar{z}2(t_{k,n},\bar{\phi}(t_{k,n};\alpha))],$$
$$\phi(t-\tau_1)\in[\underline{z}2(t_{k,n}-\tau_1,\underline{\phi}(t_{k,n}-\tau_1;\alpha)),\bar{z}2(t_{k,n}-\tau_1,\bar{\phi}(t_{k,n}-\tau_1;\alpha))],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{z}2(t_{k,n}-\tau_n,\underline{\phi}(t_{k,n}-\tau_n;\alpha)),\bar{z}2(t_{k,n}-\tau_n,\bar{\phi}(t_{k,n}-\tau_n;\alpha))]\}$$

$$\bar{K}3(t;\phi(t;\alpha)) = \max\{hg(t+\frac{h}{2},\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_n))\mid \phi(t)\in[\underline{z}2(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),\bar{z}2(t_{k,n},\bar{\phi}(t_{k,n};\alpha))],$$
$$\phi(t-\tau_1)\in[\underline{z}2(t_{k,n}-\tau,\underline{\phi}(t_{k,n}-\tau_1;\alpha)),\bar{z}2(t_{k,n}-\tau_1,\bar{\phi}(t_{k,n}-\tau_1;\alpha))],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{z}2(t_{k,n}-\tau,\underline{\phi}(t_{k,n}-\tau_n;\alpha)),\bar{z}2(t_{k,n}-\tau_n,\bar{\phi}(t_{k,n}-\tau_n;\alpha))]\}$$

$$\underline{K}4(t;\phi(t;\alpha)) = \min\{hg(t+h,\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_n))\mid \phi(t)\in[\underline{z}3(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),\bar{z}3(t_{k,n},\bar{\phi}(t_{k,n};\alpha))],$$
$$\phi(t-\tau_1)\in[\underline{z}3(t_{k,n}-\tau_1,\underline{\phi}(t_{k,n}-\tau_1;\alpha)),\bar{z}3(t_{k,n}-\tau_1,\bar{\phi}(t_{k,n}-\tau_1;\alpha))],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{z}3(t_{k,n}-\tau_n,\underline{\phi}(t_{k,n}-\tau_n;\alpha)),\bar{z}3(t_{k,n}-\tau_n,\bar{\phi}(t_{k,n}-\tau_n;\alpha))]\}$$

$$\bar{K}4(t;\phi(t;\alpha)) = \max\{hg(t+h,\phi(t),\phi(t-\tau_1),\ldots,\phi(t-\tau_n))\mid \phi(t)\in[\underline{z}3(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),\bar{z}3(t_{k,n},\bar{\phi}(t_{k,n};\alpha))],$$
$$\phi(t-\tau_1)\in[\underline{z}3(t_{k,n}-\tau,\underline{\phi}(t_{k,n}-\tau_1;\alpha)),\bar{z}3(t_{k,n}-\tau_1,\bar{\phi}(t_{k,n}-\tau_1;\alpha))],\ldots,$$
$$\phi(t-\tau_n)\in[\underline{z}3(t_{k,n}-\tau,\underline{\phi}(t_{k,n}-\tau_n;\alpha)),\bar{z}3(t_{k,n}-\tau_n,\bar{\phi}(t_{k,n}-\tau_n;\alpha))]\}$$

Next we define,

$$\underline{z}1(t_{k,n},\underline{\phi}(t_{k,n};\alpha)) = \underline{\phi}(t_{k,n};\alpha)+\frac{1}{2}\underline{K}1(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),$$
$$\bar{z}1(t_{k,n},\bar{\phi}(t_{k,n};\alpha)) = \bar{\phi}(t_{k,n};\alpha)+\frac{1}{2}\bar{K}1(t_{k,n},\bar{\phi}(t_{k,n};\alpha)),$$
$$\underline{z}2(t_{k,n},\underline{\phi}(t_{k,n};\alpha)) = \underline{\phi}(t_{k,n};\alpha)+\frac{1}{2}\underline{K}2(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),$$
$$\bar{z}2(t_{k,n},\bar{\phi}(t_{k,n};\alpha)) = \bar{\phi}(t_{k,n};\alpha)+\frac{1}{2}\bar{K}2(t_{k,n},\bar{\phi}(t_{k,n};\alpha)),$$
$$\underline{z}3(t_{k,n},\underline{\phi}(t_{k,n};\alpha)) = \underline{\phi}(t_{k,n};\alpha)+\underline{K}3(t_{k,n},\underline{\phi}(t_{k,n};\alpha)),$$
$$\bar{z}3(t_{k,n},\bar{\phi}(t_{k,n};\alpha)) = \bar{\phi}(t_{k,n};\alpha)+\bar{K}3(t_{k,n},\bar{\phi}(t_{k,n};\alpha)),$$

Then we define

$$S[(t,\underline{\phi}(t;\alpha),\bar{\phi}(t;\alpha)] = \underline{K}_1(t;\phi(t;\alpha))+2\underline{K}_2(t;\phi(t;\alpha))+2\underline{K}_3(t;\phi(t;\alpha))+\underline{K}_4(t;\phi(t;\alpha)),$$
$$T[(t,\underline{\phi}(t;\alpha),\bar{\phi}(t;\alpha)] = \bar{K}1(t;\phi(t;\alpha))+2\bar{K}2(t;\phi(t;\alpha))+2\bar{K}3(t;\phi(t;\alpha))+\bar{K}4(t;\phi(t;\alpha)).$$

The approximate solution (3) is given by

$$\begin{cases}\underline{\phi}(t_{n+1};\alpha) = \underline{\phi}(t_n;\alpha)+\underline{\phi}(t_n-\tau;\alpha)+\frac{1}{6}S[(t_n,\underline{\phi}(t_n;\alpha),\underline{\phi}_n(t;\alpha))],\\ \bar{\phi}(t_{n+1};\alpha) = \bar{\phi}(t_n;\alpha)+\bar{\phi}(t_n-\tau;\alpha)+\frac{1}{6}T[(t_n,\bar{\phi}(t_n;\alpha),\bar{\phi}_n(t;\alpha))]\end{cases} \quad (8)$$

The approximate solution Eq(5) is given by

$$\begin{cases}\underline{\phi}(t_{n+1};\alpha) = \underline{\phi}(t_n-\tau;\alpha)+\frac{1}{6}S[(t_n,\underline{\phi}(t_n;\alpha),\underline{\phi}_n(t;\alpha))],\\ \bar{\phi}(t_{n+1};\alpha) = \underline{\phi}(t_n-\tau;\alpha)+\frac{1}{6}T[(t_n,\bar{\phi}(t_n;\alpha),\bar{\phi}_n(t;\alpha))]\end{cases} \quad (9)$$

The (8) and (9) are then transformed to

$$\underline{y}(t_{n+1};\alpha) = \underline{\phi}(t_{n+1};\alpha) + \underline{y}(t_n - \tau;\alpha),$$
$$\overline{y}(t_{n+1};\alpha) = \overline{\phi}(t_{n+1};\alpha) + \overline{y}(t_n - \tau;\alpha). \qquad (10)$$

5. Numerical Examples

In this section we apply the RK-4 of FMRDDE to FMNDDE. Consider the following fuzzy pure multiple neutral delay differential equation

$$\begin{cases} y'(t;\alpha) = \\ (0.75+0.25\alpha) \\ y'(t-1)+y'(t-2)+y'(t-3), \\ (1.125-0.125\alpha) \\ y'(t-1)+y'(t-2)+y'(t-3), \quad 0 \le t \le 5 \\ y(t;\alpha) = (0.75+0.25\alpha)t^2, \\ (1.125-0.125\alpha)t^2, \qquad -3 \le t \le 0 \end{cases} \qquad (11)$$

The analytic solution is given by

$$Y(t;\alpha) = (0.75+0.25\alpha)Y(t),$$
$$(1.125-0.125\alpha)Y(t), \quad 0 \le t \le 5,\ 0 \le \alpha \le 1 \qquad (12)$$

$$\text{Where, } Y(t) = \begin{cases} t^2, & (t<0) \\ 3(-4t+t^2), & (0 \le t < 1) \\ (14-28t+5t^2), & (1 \le t < 2) \\ (78-68t+9t^2), & (2 \le t < 3) \\ (294-164t+17t^2), & (3 \le t \le 4) \\ (854-360t+31t^2), & (4 \le t < 5) \end{cases}$$

Transforming Eq(11) is to Fuzzy retarded delay differential equation and then applying RK-4 method, the approximate solutions found in Table:1 are obtained.

Theorem: 5.1

The analytic and approximate solutions of ordinary differential equations coincide exactly without any error when the initial function or condition is $y(x) = x^2$ when x=0..

Proof

Let us consider the simple ordinary differential equations $y'(x) = f(x), y(0) = 0$ So $dy(x) = f(x)dx$. Integrating it we get $y(x) = f(x) + c$, Here $c = 0$ because of initial condition. In such cases $y(x) = f(x)$, So analytic and approximate solutions obtained by numerical method will posses equations of algebraic type and coincide exactly.

Corollary: 5.2

For the given system, Eq(11), both exact and approximate solutions coincide exactly.

Proof:

From Theorem:1,the system Eq (11) has initial condition as $y(t) = t^2$. So both analytic and approximate solutions posses algebraic type and the values will coincide exactly.

The analytic solution (12) and the approximate solution obtained by using above RK-4 method is given in Table:1 and plotted in Figure:1 and in Figure: 2.

Table:1

Approximate & Analytic solutions of FPMNDDE.				
t	Approximate Solution		Analytic Solution	
	$\underline{y}(t;\alpha)$	$\overline{y}(t;\alpha)$	$\underline{Y}(t;\alpha)$	$\overline{Y}(t;\alpha)$
0	-128.25	-192.375	-128.25	-192.375
0.1	-132.525	-190.2375	-132.525	-190.2375
0.2	-136.8	-188.1	-136.8	-188.1
0.3	-141.075	-185.9625	-141.075	-185.9625
0.4	-145.35	-183.825	-145.35	-183.825
0.5	-149.625	-181.6875	-149.625	-181.6875
0.6	-153.9	-179.55	-153.9	-179.55
0.7	-158.175	-177.4125	-158.175	-177.4125
0.8	-162.45	-175.275	-162.45	-175.275
0.9	-166.725	-173.1375	-166.725	-173.1375
1.0	-171	-171	-171	-171

Approximate and Analytic Solution

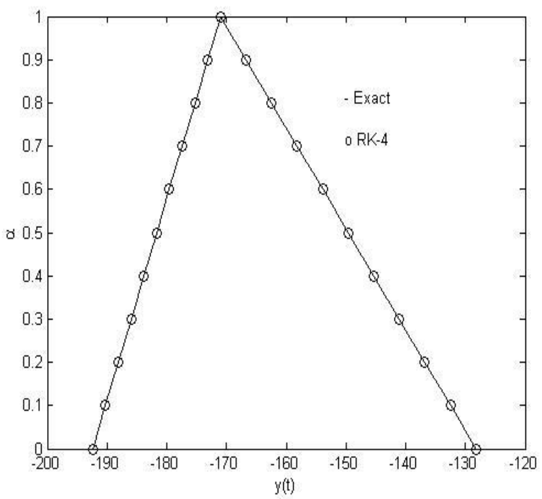

Figure:1 For $\alpha \in [0,1], t = 5$

Approximate and Analytic Solution

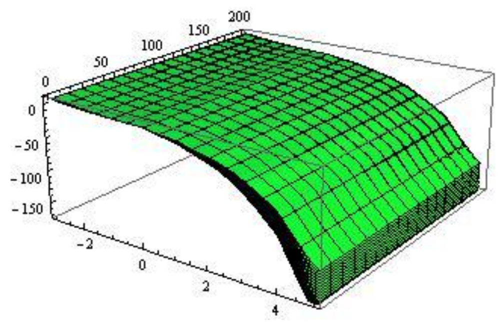

Figure:2 For $\alpha \in [0,1], t \in [0,5]$

6. Conclusion

In this article, we have developed numerical solution of Fuzzy pure multiple retarded delay differential equations and applied the method to solve fuzzy pure multiple neutral delay differential equations. Thus by this transformation technique of Neutral delay to retarded delay is more appreciable to solve a Fuzzy neutral delay systems. The approximate solutions coincide with the analytic solutions as shown in the figure 1 and in 2. Thus the numerical solution holds good to solve any kind of multiple neutral delay differential equations. So it can be implemented to any such pure multiple neutral delay differential equations modeling the problems in real life. From the Table:1 the accuracy of the method is up to four decimal places.

Future Work

The modeling of efficiency of various medicines treating various diseases as an application of FMNDDE is our future work.

Conflict of interest

The author confirms that there is no conflict of interest to declare for this publication.

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