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Performance of X-bar chart associated with Range under Three Delta Control Limits and Six Delta Initiatives

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Abstract:

A control chart is a graphical device for representation of the data for knowing the extent of variations from the expected standard. The technique of control chart was suggested by W.A. Shewhart of Bell Telephone Company based on three sigma limits. M. Harry, the engineer of Motorola has introduced the concept of six sigma in 1980. In six sigma initiatives, it is expected to produce 3.4 or less number of defects per million of opportunities. Moderate distribution proposed by Naik V.D and Desai J.M, is a sound alternative of normal distribution, which has mean and mean deviation as pivotal parameters and which has properties similar to normal distribution. Naik V.D and Tailor K.S. have suggested the concept of 3-delta control limits and developed various control charts based on this distribution. In this paper an attempt is made to construct a control chart based on three delta and six delta initiatives for X-bar chart associated with range. Suitable Table for mean deviation is also constructed and presented for the engineers for making quick decisions.

Key words: *Moderate distribution, X-bar, Mean deviation, Six Delta*

1. Introduction

The conventional quality control charts developed by W. A. Shewhart (1931) were based on normality assumptions and control limits are calculated using 3sigma distance from the expected level of quality. The concept of six-sigma was introduced by Motorola by the engineer Mikel Harry in 1980. The companies, which are practicing Six Sigma, are expected to produce 3.4 or less number of defects per million opportunities.

R.Radhakrishnan and P.Balamurugan (2010, 2011, and 2016) have developed six sigma based control charts for mean, Exponentially Weighted Moving Average(EWMA), X bar using standard deviation, standard deviation, range and moving averages.

Naik V.D and Desai J.M (2015) have proposed an alternative of normal distribution called moderate distribution, which has location parameter as mean(μ) and scale parameter as mean deviation(δ). Naik V.D. and Tailor K.S. (2015) have suggested $3\,\delta$ (3 mean deviation) control limits based on moderate distribution. On the basis of 3δ control limits, they have developed $\bar{X}$ -chart, R-chart, s-chart and d-chart Tailor K.S. (2016) has also developed moving average and moving range chart and exponentially moving average chart under moderateness assumption.

Tailor K.S (2017) has proposed the six sigma concept, similar to six delta concepts. The six sigma control limits are based normality assumption and the control limits are determined by using standard deviation (σ -sigma) of the statistic, whereas the six delta control limits are based on moderateness assumption and the control limits are determined by using mean deviation (δ-delta) of the statistic. In six sigma initiatives, it is expected to produce 3.4 or less number of defects per million of opportunities whereas in six delta initiatives, it is expected to produce 1.7 or less number of defects per million of opportunities. Tailor K.S. (2017, 2018) has proposed sample standard deviation(s) chart, sample mean deviation (d) chart, X-bar chart associated with mean deviation, exponentially weighted moving average (EWMA) chart and CUSUM chart based on six delta initiatives. In this paper X-bar chart associated with range under moderateness assumption is studied by using 6-delta methodology. A ready available table for mean deviation is also prepared for the quality control experts for taking fast actions.

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2. Some useful terms

A. upper specification limit (U.S.L)

It is the acceptable maximum value of an item suggested by the quality control expert.

B. lower specification limit (L.S.L)

It is the acceptable minimum value of an item suggested by the quality control expert.

C. tolerance level (T.L)

It is the difference between U.S.L and L.S.L, T.L = U.S.L – L.S.L

D. process capability (Cp)

This is the ratio of tolerance level to six times mean deviation of the process.

Cp = (T.L / $6\sqrt{\frac{\pi}{2}}\delta$) = (T.L/10.6369$\delta$) = (U.S.L – L.S.L)/10.6369δ

E. Mean deviation ($\delta_{6\delta}$):

It is the most intuitively and rationally defined measure of dispersion.

F. Quality Control Constant (H_{md})

The constant H_{md} is introduced to compute six delta based control limits for the said chart.

3. Three delta control limits for $\bar{X}$- chart associated with range:

Suppose that the main variable of the process x follows moderate distribution. The mean of x is E(x) = μ and mean deviation of x = $\delta_x = \delta'$. According to the central limit theorem for moderate distribution if the distribution of x is not moderate, it can be said that $\bar{X}$ follows moderate distribution for a large sample. So, mean of $\bar{X}$ = E ($\bar{X}$) = μ and mean deviation of $\bar{X}$ is $\delta_x = \frac{\delta_x}{\sqrt{n}}$.

On the basis of 3δ criteria suggested by Naik and Tailor, the control limits for x̄- chart associated with range can be represented as follows.

$$CL_{3\delta} = \mu_0 \quad (1)$$

$$LCL_{3\delta} = \mu_0 - 3\frac{\delta}{\sqrt{n}} = \mu_0 - 3\frac{1}{\sqrt{n}}\sqrt{\frac{2}{\pi}}\frac{\bar{R}}{d_2} = \mu_0 - A_2'\bar{R}, \quad (2)$$

$$UCL_{3\delta} = \mu_0 + 3\frac{\delta}{\sqrt{n}} = \mu_0 + 3\frac{1}{\sqrt{n}}\sqrt{\frac{2}{\pi}}\frac{\bar{R}}{d_2} = \mu_0 + A_2'\bar{R} \quad (3)$$

Where μ_0 is the target value of the mean, $\bar{R}$ is average range and $A_2' = \frac{3}{\sqrt{n}}\sqrt{\frac{2}{\pi}}\frac{1}{d_2}$ is a statistical quality control constant, which can be obtained from the readily available table.

4. Six delta based control limits for $\bar{X}$- chart associated with range:

Fix the tolerance level (T.L) and process capability (C_p) to determine the process mean deviation δ(termed as $\delta_{6\delta}$), which is calculated from Cp = (T.L/10.6369δ). For a specified T.L and C_p of the process, the values of $\delta_{6\delta}$ is calculated, and presented in table 2. The value of H_{md} is obtained by using $P(Z \le H_{md})$ = 1- α_1 , where α_1 = 1.7×10^{-6} and Z is a standard moderate variate. Thus, the control limits for six delta based control chart for moving average chart are determined as,

$$CL_{6\delta} = \mu_0 \quad (4)$$

$$LCL_{6\delta} = \mu_0 - \frac{H_{md}\delta_{6\delta}}{\sqrt{n}} \quad (5)$$

$$UCL_{6\delta} = \mu_0 + \frac{H_{md}\delta_{6\delta}}{\sqrt{n}} \quad (6)$$

5. An empirical study for $\bar{X}$- chart associated with range and comparison of three delta limits against six delta initiatives

To illustrate the use of x̄- chart associated with range with three delta and six delta limits, a data set is taken from Grant and Leavenworth (1988). The data, together with the corresponding values of mean and range are shown in Table 1.Three delta and six delta control limits are computed from this data set, and control charts are plotted under these two limits.

Table 1: Data set

Lot	X_1	X_2	X_3	X_4	X_5	$\bar{X}$	R
1	77	80	78	72	78	77	8
2	76	79	73	74	73	75	6
3	76	77	72	76	74	75	5
4	74	78	75	77	77	76.2	4
5	80	73	75	76	74	75.6	7
6	78	81	79	76	76	78	5
7	75	77	75	76	77	76	2
8	79	75	78	77	76	77	4
9	76	75	74	75	75	75	2
10	71	73	71	70	73	71.6	3
11	72	73	75	74	75	73.8	3
12	75	73	76	73	73	74	3
13	75	76	78	79	77	77	4
14	77	77	78	77	76	77	2
15	77	76	77	77	77	76.8	1
16	77	77	77	79	79	77.8	2

(a) Three delta control limits for x̄- chart associated with range:

Here the target mean (μ_0) is estimated by the average value of sample mean and process dispersion is estimated by the average value of sample range. Hence μ_0 = 75.8, $\bar{R}$ =3.81and for

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$= 5, A_2' = 0.4603$. The three delta control limits are computed using equations (1), (2), (3) and are found as

LCL = 74.05, CL = 75.8 and UCL = 77.55

(b) Control limits based on six delta initiatives for $\bar{x}$- chart associated with range :

For a given data set U.S.L = 78, L.S.L = 71.6, T.L = 78 – 71.6 = 6.4 and C_p = 1.5. The value of $\delta_{6\delta}$ = 0.4011, which is found from the Table 2, H_{md} = 5.815 which is calculated from $P(Z \leq H_{md})$ = 1- α_1, where $\alpha_1 = 1.7 \times 10^{-6}$. The value of μ_0 is taken to be 75.8. Hence, the control limits based on six delta initiatives for this chart for a specified T.L and H_{md} are computed using equations (4), (5), (6) and are found as,

$LCL_{6\delta}$ = 65.9210, $CL_{6\delta}$ = 75.8, $UCL_{6\delta}$ = 85.6789

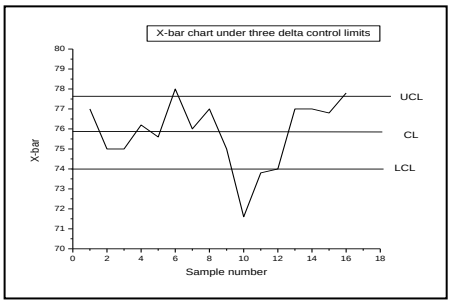
Table 2: Values of δ for a specified C_p and T.L

TL / C_p	6.1	6.2	6.3	6.4	6.5
1.0	0.5735	0.5829	0.5923	0.6017	0.6111
1.1	0.5213	0.5299	0.5384	0.5470	0.5555
1.2	0.4779	0.4857	0.4936	0.5014	0.5092
1.3	0.4411	0.4484	0.4456	0.4628	0.4701
1.4	0.4096	0.4163	0.4231	0.4298	0.4365
1.5	0.3823	0.3886	0.3949	0.4011	0.4074
1.6	0.3584	0.3643	0.3702	0.3761	0.3820
1.7	0.3373	0.3429	0.3484	0.3539	0.3595
1.8	0.3186	0.3238	0.3290	0.3343	0.3395
1.9	0.3018	0.3068	0.3117	0.3167	0.3216
2.0	0.2867	0.2914	0.2961	0.3008	0.3055
2.1	0.2731	0.2776	0.2820	0.2865	0.2910
2.2	0.2607	0.2649	0.2692	0.2734	0.2778
2.3	0.2493	0.2534	0.2575	0.2616	0.2657
2.4	0.2389	0.2429	0.2468	0.2507	0.2546
2.5	0.2294	0.2332	0.2369	0.2407	0.2444

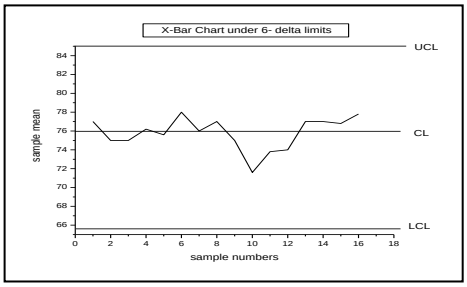
(c) $\bar{x}$- chart for data set given in Table 1 based on three delta and six delta limits

Figure 1

(a) $\bar{x}$- chart associated with range under 3-delta limits



(b) $\bar{x}$- chart associated with range under 6-delta limits



6. Summary and conclusion

In this paper, $\bar{x}$- chart associated with range is discussed under three delta and six delta control limits with an illustration. From figure 1, it can be seen that the production process is out of statistical control when we are applying 3-delta control limits but the process is under the statistical control when we are using six-delta based control limits. So it can be concluded that the chart under six delta control limits are more effective towards detecting the shift in the value of mean than the charts under three delta control limits. This is a next generation control chart technique and it can replace existing six sigma technique. So it is recommended that the control charts under six delta control limits should be used for the best results.

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