IJASRM

International Journal of Advanced Scientific Research and Management, Volume 4 Issue 4, April 2019

www.ijasrm.com

ISSN 2455-6378

A Different Approach on Homo Cordial Labeling of Spider Graph

S.Sriram[1] and R.Govindarajan[2]

[1] Department of Mathematics, Patrician College of Arts and Science, Adyar, Chennai, Tamil Nadu ,India

[2]P.G&U.GDepartment of Mathematics, D.G.Vaishnav College, Arumbakkam, Chennai, Tamil Nadu, India

Abstract

Let G= (V, E) be a graph with p vertices and q edges. A Homo Cordial Labelling of a graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 1 if f(u)=f(v) or 0 if $f(u) \neq f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. The graph that admits a Homo- Cordial labelling is called Homo Cordial graph. In this paper we prove that the spider graph $SP(1^m, 2^t, 3)$ is homo Cordial labelling graph and further study on the generalization of labelling spider graph $SP(1^m, 2^t, 3)$

***Keywords:*Homo Cordial graphs, Homo Cordial labelling, Spider graph**

2000 Mathematics Subject Classification: 05C78

1. *Introduction*

A graph G is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. Gallian[1] has given a dynamic survey of graph labelling. The origin of graph labeling can be attributed to Rosa. The concept of Homo Cordial labelling of graph was introduced by .Nellai Murugan. A and Mathubala. A [2,3,4]. Motivated towards the labelling of homo cordial labelling of graphs and the different types of spider graph discussed by P.Jeyanthi and T. Saratha Devi [5].In this paper we prove that spider graph $SP(1^m, 2^t, 3)$ is Homo Cordial labelling graph. Further to generalize the concept of homo cordial labelling of spider graph $SP(1^m, 2^t, 3)$ we have ascertained the ways in which the number of labels assigned with 0 and number of labels assigned with 1 so as to identify the phenomena of spider graph $SP(1^m, 2^t, 3)$ to be called a homo cordial labelling graph. Also we have studied on the characteristics of labelling a spider graph $SP(1^m, 2^t, 3)$

2.Preliminaries

Definition 2.1: A tree is called a spider if it has a centre vertex C of degree R>1 and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has $X_1's$ of length a_1, $X_2's$ paths of length a_2 etc. We shall denote the spider by $SP(a_1^{x_1}, a_2^{x_2}, a_3^{x_3} \dots a_m^{x_m})$ where

a_1 p a_2 p a_3 p $\dots a_m$ and $x_1 + x_2 + \dots + x_m = R$

IJASRM

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www.ijasrm.com

ISSN 2455-6378

3. Main Results

Theorem .3.1: The Spider graph $SP\left(1^m, 2^t, 3\right)$ is a homo cordial labelling graph

Proof: Let $G = SP\left(1^m, 2^t, 3\right)$ be a Spider graph.

We know that a tree is called a spider if it has a Centre vertex C of degree R>1 and all the other vertex is either a leaf or with degree 2. Thus a spider is an amalgamation of k paths with various lengths. If it has X_1's of length a_1, X_2's paths of length a_2 etc. We shall denote the spider by

$SP\left(a_1^{x_1}, a_2^{x_2}, a_3^{x_3} \ldots a_m^{x_m}\right)$ where

$a_1 \prec a_2 \prec a_3 \prec \ldots a_m$ and $x_1 + x_2 + \ldots + x_m = R$

Define the vertex set as

$$V(SP(1^m, 2^t, 3)) = \{u,\ v_i,\ u_j, w_k, v\}$$

where $1 \le i \le m, 1 \le j \le t+1$ and

$$t+2 \le k \le 2t+2$$

The corresponding edge set

$$E(SP(1^m, 2^t, 3)) = \{e_i = uv_i :\ 1 \le i \le m\} \cup \{\ e'_i = u\ u_j\}$$

$$\cup \{e''_j = u_j w_{t+1+j}\} \cup \{e'''_1 = w_{2t+2} v\}$$

where $1 \le j \le t+1\}$

Now to label the vertices let us consider the bijective function $f : V \to \{0,1\}$ such that such that each edge uvis assigned the label 1 if f (u) =f (v) or 0 if $f(u) \ne f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. We define the labelling of vertices as follows

$$f(u) = 0$$

$$f(v_i) = 1 \text{ for } i \equiv 1 (\bmod m)$$

$$f(v_i) = 0 \text{ for } i \equiv 0 (\bmod m) \text{ where } 1 \le i \le m$$

$$f(u_j) = 0 \text{ for } 1 \le j \le t+1$$

$$f(w_k) = 1 \text{ for } t+2 \le k \le 2t+2$$

$$f(v) = 1$$

Then the induced edge labelling for the graph $G = SP\left(1^m, 2^t, 3\right)$ are

$$f^*(uv_i) = 0 \text{ for } i \equiv 1 (\bmod m)$$

$$f^*(uv_i) = 1 \text{ for } i \equiv 0 (\bmod m)$$

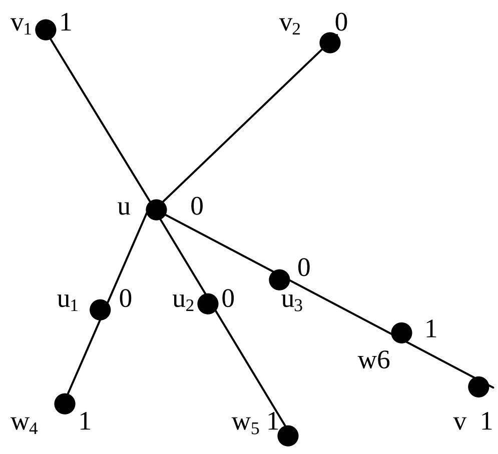
$$f^*(uu_j) = 1 \text{ for } 1 \le j \le t+1$$

$$f\left(u_j w_k\right) = 0 \text{ for } 1 \le j \le t+1 \text{ and } t+2 \le k \le 2t+2$$

$$f\left(w_{2t+2} v\right) = 1$$

Noticing the induced edge labelling we find that the number of vertices labelled with 0 is n and the number of vertices labelled with 1 is n and that the number of edges labelled with 0 is n+1 and the number of edges labelled with 1 is n. Hence $\left|v_f(0) - v_f(1)\right| \le 1$ and $\left|e_f(0) - e_f(1)\right| \le 1$. Therefore the Spider graph $G = SP\left(1^m, 2^t, 3\right)$ is a homo cordial labelling graph.

Figure.1 Homo Cordial Labeling of $SP\left(1^2, 2^2, 3\right)$



ISSN 2455-6378

Definition 3.3: Basic Spider graph $SP(1^m, 2^t, 3)$ for m=1 and t=1. The Spider graph $SP(1^1, 2^1, 3)$ is defined to have the vertex set $V(SP(1^1, 2^1, 3)) = \{u, v_1, u_1, u_2, w_3, w_4, v\}$ and $E(SP(1^1, 2^1, 3)) = \{uv_1, uu_1, uu_2, u_1w_3, u_2w_4, w_4v\}$ with number of vertices of $SP(1^1, 2^1, 3)$ =7 and number of edges $SP(1^1, 2^1, 3)$ = 6.

Note: By the labelling procedure adopted in Theorem.3.1 we have for the basic spider graph . $SP(1^1, 2^1, 3)$ the number of vertices labelled with 1 is 4 and number of vertices labelled with 0 is 3 and the number of edges labelled with 1 is 3 and the number of edges labelled with 0 is 3.

Now let us construct the different spider graphs from the basic spider graph $SP(1^1, 2^1, 3)$

Let us consider the spider graph by fixing the value of t as 1 and increasing the value of m by 1 to form different spider graphs namely $SP(1^2, 2^1, 3), SP(1^3, 2^1, 3)$ and so on.

We note that the spider graphs $SP(1^2, 2^1, 3), SP(1^3, 2^1, 3)$ and so on has the following labels for the vertices and edges from the following table.1

Table.1: Spider Graph $SP(1^m, 2^t, 3)$ labelling with number of vertices and number of edges labelled with 1 and 0 by fixing t=1 and increasing the value of m by 1

SpiderGraph	**Number of vertices labelled with**		**Number of Edges labelled with**	
	1	**0**	**1**	**0**
$SP(1^1, 2^1, 3)$	4	3	3	3
$SP(1^2, 2^1, 3)$	4	4	4	3
$SP(1^3, 2^1, 3)$	5	4	4	4
$SP(1^4, 2^1, 3)$	5	5	5	4
$SP(1^5, 2^1, 3)$	6	5	5	5

The procedure can be continued to form different spider graphs**.**

Observation.1: From the above table we find that from the basic spider graph $SP(1^1, 2^1, 3)$ by increasing the value of m by 1 and fixing the value of t as 1 each part of m increases the number of vertices labelled with 0 by 1 and edges labelled with 1 by 1 and number of vertices labelled with 1 by 1 and edges labelled with 0 by 1 alternatively.

Observation.2: From the above table we also find that the number of vertices labelled with 0 and 1 are the same in case of m is even and number of edges labelled with 0 and 1 are same if m is odd**.**

Definition 3.4: Let us denote the number of vertices labelled with 1 of spider graph $SP(1^m, 2^t, 3)$ by $V_1(SP(1^m, 2^t, 3))$ and the number of vertices labelled with 0 of spider graph $SP(1^m, 2^t, 3)$ by $V_0(SP(1^m, 2^t, 3))$.

Definition 3.5: Let us denote the number of edges labelled with 1 of spider graph $SP(1^m, 2^t, 3)$ by $E_1(SP(1^m, 2^t, 3))$ and the number of vertices labelled with 0 of spider graph $SP(1^m, 2^t, 3)$ by $E_0(SP(1^m, 2^t, 3))$.

Observation.3: From the above table we find that the number of vertices labelled with 1 and number of edges labelled with 0 has the following characteristic When m is even

$$V_1(SP(1^m, 2^1, 3)) = V_1(SP(1^{m-1}, 2^1, 3))$$

$$E_0(SP(1^m, 2^1, 3)) = E_0(SP(1^{m-1}, 2^1, 3))$$

Observation .4: From the above table we find that the number of vertices labelled with 0 and number of edges labelled with 1 has the following characteristic When m is odd

IJASRM
International Journal of Advanced Scientific Research and Management, Volume 4 Issue 4, April 2019
www.ijasrm.com
ISSN 2455-6378

$V_0\left(SP\left(1^m,2^1,3\right)\right)=V_1\left(SP\left(1^{m-1},2^1,3\right)\right)$

$E_1\left(SP\left(1^m,2^1,3\right)\right)=E_1\left(SP\left(1^{m-1},2^1,3\right)\right)$

Further let us characterise the properties of Spider graph $SP\left(1^m,2^t,3\right)$ by separating based on m is odd or even.

Theorem 3.6: For the spider graph $SP\left(1^m,2^t,3\right)$ by fixing t=1 and increasing the value of m by 1 we can obtain the following result for $m\geq 1$

$V_1\left(SP(1^{2m+1},2^1,3)\right)=V_1(SP(1^{2m-1},2^1,3)+1$

$V_0\left(SP(1^{2m+1},2^1,3)\right)=V_0(SP(1^{2m-1},2^1,3)+1$

$E_1\left(SP(1^{2m+1},2^1,3)\right)=E_1(SP(1^{2m-1},2^1,3)+1$

$E_0\left(SP(1^{2m+1},2^1,3)\right)=E_0(SP(1^{2m-1},2^1,3)+1$

Proof: Let us prove the results by applying Mathematical induction on m. Let us prove for m=1
We know that when m=1 the spider graph $SP\left(1^m,2^t,3\right)$ fixing t=1 becomes the basic spider graph $SP\left(1^1,2^1,3\right)$ and for which the number of vertices labelled with 1 is 4 and number of vertices labelled with 0 is 3, number of edges labelled with 1 is 3 and number of edges labelled with 0 is 3. Hence we can compute $V_1(SP(1^1,2^1,3)$ =4; $V_0(SP(1^1,2^1,3)$ =3; $E_1(SP(1^1,2^1,3)$ =3 and $E_0(SP(1^1,2^1,3)$ =3. We can also compute $V_1(SP(1^3,2^1,3)=5$; $V_0(SP(1^3,2^1,3)=4$; $E_1(SP(1^3,2^1,3)=4$ and $E_0(SP(1^3,2^1,3)=4$.
Hence the result for m=1 can be verified.
Now let us assume that the result is true for m=k
i.e. it is true that the following results holds good

$V_1\left(SP(1^{2k+1},2^1,3)\right)=V_1(SP(1^{2k-1},2^1,3)+1$

$V_0\left(SP(1^{2k+1},2^1,3)\right)=V_0(SP(1^{2k-1},2^1,3)+1$

$E_1\left(SP(1^{2k+1},2^1,3)\right)=E_1(SP(1^{2k-1},2^1,3)+1$

$E_0\left(SP(1^{2k+1},2^1,3)\right)=E_0(SP(1^{2k-1},2^1,3)+1$

Now let us prove that it is true for m=k+1
i.e. to prove

$V_1\left(SP(1^{2k+3},2^1,3)\right)=V_1(SP(1^{2k+1},2^1,3)+1$

$V_0\left(SP(1^{2k+3},2^1,3)\right)=V_0(SP(1^{2k+1},2^1,3)+1$

$E_1\left(SP(1^{2k+3},2^1,3)\right)=E_1(SP(1^{2k+1},2^1,3)+1$

$E_0\left(SP(1^{2k+3},2^1,3)\right)=E_0(SP(1^{2k+1},2^1,3)+1$

We consider to prove only for odd values of m and hence we understand that m= k and m=k+1 are odd terms with a common difference 2
Since we have assumed that it is true for m=k we have

$V_1\left(SP(1^{2k+1},2^1,3)\right)=V_1(SP(1^{2k-1},2^1,3)+1$

We have from the understanding that next term to 2k+1 is 2k+3 and hence replacing 2k+1 by 2k+3 on the L.H.S. We also know that next odd term to 2k-1 is 2k+1. Hence replacing 2k-1 by 2k+1 on the R.H.S we have the result namely

$V_1\left(SP(1^{2k+3},2^1,3)\right)=V_1(SP(1^{2k+1},2^1,3)+1$.

Similarly we can prove the remaining results. Hence the proof by Mathematical induction.

Theorem 3.7: For the spider graph $SP\left(1^m,2^t,3\right)$ by fixing t=1 and increasing the value of m by 1 we can obtain the following result for $m\geq 1$

$V_1\left(SP(1^{2m+2},2^1,3)\right)=V_1(SP(1^{2m},2^1,3)+1$

$V_0\left(SP(1^{2m+2},2^1,3)\right)=V_0(SP(1^{2m},2^1,3)+1$

$E_1\left(SP(1^{2m+2},2^1,3)\right)=E_1(SP(1^{2m},2^1,3)+1$

$E_0\left(SP(1^{2m+2},2^1,3)\right)=E_0(SP(1^{2m},2^1,3)+1$

Proof: Let us prove the results by applying Mathematical induction on m. Let us prove for m=1We know that when m=1 the spider graph $SP\left(1^m,2^t,3\right)$ fixing t=1 becomes the basic spider graph $SP\left(1^1,2^1,3\right)$ and for which the number of vertices labelled with 1 is 4 and number of vertices labelled with 0 is 3, number of edges labelled with 1 is 3 and number of edges labelled with 0 is 3. Hence we can compute $V_1(SP(1^2,2^1,3)$ =4; $V_0(SP(1^2,2^1,3)$ =4; $E_1(SP(1^2,2^1,3)$ =4 and $E_0(SP(1^2,2^1,3)$ =3. We can also compute $V_1(SP(1^4,2^1,3)=5$; $V_0(SP(1^4,2^1,3)=5$; $E_1(SP(1^4,2^1,3)=5$ and $E_0(SP(1^4,2^1,3)=4$.
Hence the result for m=1 can be verified.
Now let us assume that the result is true for m=k
i.e. it is true that the following results holds good

$V_1\left(SP(1^{2k+2},2^1,3)\right)=V_1(SP(1^{2k},2^1,3)+1$

$V_0\left(SP(1^{2k+2},2^1,3)\right)=V_0(SP(1^{2k},2^1,3)+1$

$E_1\left(SP(1^{2k+2},2^1,3)\right)=E_1(SP(1^{2k},2^1,3)+1$

$E_0\left(SP(1^{2k+2},2^1,3)\right)=E_0(SP(1^{2k},2^1,3)+1$

Now let us prove that it is true for m=k+1
i.e. to prove

$V_1\left(SP(1^{2k+4},2^1,3)\right)=V_1(SP(1^{2k+2},2^1,3)+1$

$V_0\left(SP(1^{2k+4},2^1,3)\right)=V_0(SP(1^{2k+2},2^1,3)+1$

$E_1\left(SP(1^{2k+4},2^1,3)\right)=E_1(SP(1^{2k+2},2^1,3)+1$

$E_0\left(SP(1^{2k+4},2^1,3)\right)=E_0(SP(1^{2k+2},2^1,3)+1$

We consider to prove only for even values of m and hence we understand that m= k and m=k+1 are even terms with a common difference 2

Since we have assumed that it is true for m=k we have

$V_1\left(SP(1^{2k+2},2^1,3)\right)=V_1(SP(1^{2k},2^1,3)+1$

We have from the understanding that next term to 2k+2 is 2k+4 and hence replacing 2k+2 by 2k+4 on the L.H.S. We also know that next even term to 2k is 2k+2. Hence replacing 2k by 2k+2 on the R.H.S we have the result namely

$V_1\left(SP(1^{2k+4},2^1,3)\right)=V_1(SP(1^{2k+2},2^1,3)+1$.

Similarly we can prove the remaining results. Hence the proof by Mathematical induction.

Theorem 3.8: For the spider graph $SP\left(1^m,2^t,3\right)$ for $m\geq 1$ we have

$V_1\left(SP(1^{2m+1},2^1,3)\right)=V_1(SP(1^1,2^1,3)+m$

$V_0\left(SP(1^{2m+1},2^1,3)\right)=V_0(SP(1^1,2^1,3)+m$

$E_1\left(SP(1^{2m+1},2^1,3)\right)=E_1(SP(1^1,2^1,3)+m$

$E_0\left(SP(1^{2m+1},2^1,3)\right)=E_0(SP(1^1,2^1,3)+m$

Proof: We know that for the basic spider graph $SP\left(1^1,2^1,3\right)$ $V_1(SP(1^1,2^1,3)$ =4 ; $V_0(SP(1^1,2^1,3)$ =3; $E_1(SP(1^1,2^1,3)$ =3 and $E_0(SP(1^1,2^1,3)$ =3. For the next odd number m=3 we have $V_1(SP(1^3,2^1,3)=5$; $V_0(SP(1^3,2^1,3)=4$; $E_1(SP(1^3,2^1,3)=4$ and $E_0(SP(1^3,2^1,3)=4$ which increases by 1 . Now for m odd numbers it can be computed from the basic spider graph $SP\left(1^1,2^1,3\right)$ by increasing the number of vertices labelled with 1 at each step we find that the total number of vertices labelled with 1 increases by m, increasing the number of vertices labelled with 0 at each step we find that the total number of vertices labelled with 0 increases by m , increasing the number of edges labelled with 1 at each step we find that the total number of edges labelled with 1 increases by m and increasing the number of edges labelled with 0 at each step we find that the total number of edges labelled with 0 increases by m. Which proves the result. Hence the proof.

Theorem 3.9: For the spider graph $SP\left(1^m,2^t,3\right)$ for $m\geq 1$ we have

$V_1\left(SP(1^{2m+2},2^1,3)\right)=V_1(SP(1^2,2^1,3)+m$

$V_0\left(SP(1^{2m+2},2^1,3)\right)=V_0(SP(1^2,2^1,3)+m$

$E_1\left(SP(1^{2m+2},2^1,3)\right)=E_1(SP(1^2,2^1,3)+m$

$E_0\left(SP(1^{2m+2},2^1,3)\right)=E_0(SP(1^2,2^1,3)+m$

Proof: We know that from the basic spider graph $SP(1^2,2^1,3)$ $V_1(SP(1^2,2^1,3)=4$; $V_0(SP(1^2,2^1,3)=4$; $E_1(SP(1^2,2^1,3)=4$ and $E_0(SP(1^2,2^1,3)=3$. For the next even number m =4 we have $V_1(SP(1^4,2^1,3)=5$; $V_0(SP(1^4,2^1,3)=5$; $E_1(SP(1^4,2^1,3)=5$ and $E_0(SP(1^4,2^1,3)=4$ which increases by 1 . Now for m even numbers it can be computed from the spider graph $SP(1^2,2^1,3)$ by increasing the number of vertices labelled with 1 at each step we find the total number of vertices labelled with 1 increases by m, increasing the number of vertices labelled with 0 at each step we find the total number of vertices labelled with 0 increases by m , increasing the number of edges labelled with 1 at each step we find the total number of edges labelled with 1 increases by m and increasing the number of edges labelled with 0 at each step we find the total number of edges labelled with 0 increases by m. Which proves the result. Hence the proof.

Definition 3.10: Let us define for a spider graph $SP\left(1^m,2^t,3\right)$ by fixing t=1

$V\left(SP(1^m,2^1,3)\right)=V_1(SP(1^m,2^1,3)+V_0(SP(1^m,2^1,3)$

$E\left(SP(1^m,2^1,3)\right)=E_1(SP(1^m,2^1,3)+E_0(SP(1^m,2^1,3)$

Theorem 3.11: For the Spider graph $SP\left(1^m,2^t,3\right)$ by fixing t=1 and for $m\geq 2$ we have

$V\left(SP(1^m,2^1,3)\right)=V\left(SP(1^1,2^1,3)+(m-1)\right)$

$E\left(SP(1^m,2^1,3)\right)=E\left(SP(1^1,2^1,3)+(m-1)\right)$

Proof: Let $G=SP\left(1^m,2^1,3\right)$ be the spider graph. The total number of vertices labelled with 0 and 1 for the basic spider graph is 7 and is denoted by $V\left(SP(1^1,2^1,3)\right)$ and the total number of edges labelled with 0 and 1 for the basic spider graph is 6

IJASRM
International Journal of Advanced Scientific Research and Management, Volume 4 Issue 4, April 2019
www.ijasrm.com
ISSN 2455-6378

and is denoted by $E\ (SP(1^1,2^1,3)$. Hence for the spider graph $SP\left(1^2,2^1,3\right)$ we have

$V\ \left(SP(1^2,2^1,3)\right)=V\ (SP(1^1,2^1,3)+1$

for the spider graph $SP\left(1^3,2^1,3\right)$

$V\ \left(SP(1^3,2^1,3)\right)=V\ (SP(1^1,2^1,3)+2$ and so on. Hence in general we have

$V\ \left(SP(1^m,2^1,3)\right)=V\ (SP(1^1,2^1,3)+(m-1)$

Similarly

$E\ \left(SP(1^2,2^1,3)\right)=E\ (SP(1^1,2^1,3)+1$ for the spider graph $SP\left(1^2,2^1,3\right)$

$E\ \left(SP(1^3,2^1,3)\right)=E\ (SP(1^1,2^1,3)+2$ for the spider graph $SP\left(1^3,2^1,3\right)$ Hence in general we have

$E\ \left(SP(1^m,2^1,3)\right)=E\ (SP(1^1,2^1,3)+(m-1)$.

Hence the proof of the theorem.

Now let us now consider spider graph by fixing the value of m as 1 and increasing the value of t by 1 to form different spider graphs namely $SP(1^1,2^2,3), SP(1^1,2^3,3)$ and so on from the basic spider graph $SP(1^1,2^1,3)$.

On constructing the spider graph from the basic spider graph $SP(1^1,2^1,3)$ by fixing m as 1 and increasing the value of t by 1 we have the following table.2

Table.2 : Spider Graph $SP\left(1^m,2^t,3\right)$ labelling with number of vertices and number of edges labelled with 1 and 0 by fixing m=1 and increasing the value of t by 1

Spider Graph	No of vertices labelled with		Number of edges labelled with	
	1	0	1	0
$SP\left(1^1,2^1,3\right)$	4	3	3	3
$SP(1^1,2^2,3)$	5	4	4	4
$SP(1^1,2^3,3)$	6	5	5	5
$SP(1^1,2^4,3)$	7	6	6	6
$SP(1^1,2^5,3)$	8	7	7	7

We understand from the table that spider graph $SP\left(1^m,2^t,3\right)$ by fixing m as 1 and increasing the value of t by 1 from the basic spider graph $SP(1^1,2^1,3)$ has labels assigned to both vertices and edges with 1 and 0 increases by 1 .

Observation.5: From the above table we find that the number of edges labelled with 1 and 0 are same for all the powers of t of a spider graph $SP\left(1^m,2^t,3\right)$ by fixing m=1

Observation.6: From the above table we find that the number of vertices labelled with 1 is one more than the number of vertices labelled with 0 for all the powers of t of a spider graph $SP\left(1^m,2^t,3\right)$ by fixing m=1.

Theorem 3.12: For a spider graph $SP\left(1^m,2^t,3\right)$ by fixing m=1 and increasing the value of t by 1, for $t\geq 1$ we find the following result

$V_1\left(SP(1^1,2^{t+1},3)\right)=V_1(SP(1^1,2^1,3)+t$

$V_0\left(SP(1^1,2^{t+1},3)\right)=V_0(SP(1^1,2^1,3)+t$

$E_1\left(SP(1^1,2^{t+1},3)\right)=E_1(SP(1^1,2^1,3)+t$

$E_0\left(SP(1^1,2^{t+1},3)\right)=E_0(SP(1^1,2^1,3)+t$

Proof: Let G be a spider graph $SP\left(1^m,2^t,3\right)$.

For the basic spider graph $SP\left(1^1,2^1,3\right)$ the number of vertices labelled with 1 denoted by $V_1\left(SP(1^1,2^1,3)\right)$ is 4 and the number of vertices labelled with 0 denoted by $V_0\left(SP(1^1,2^1,3)\right)$ is 3 and the number of edges labelled with 1 denoted by $E_1\left(SP(1^1,2^1,3)\right)$ is 3 and the number of edges labelled with 0 denoted by $E_0\left(SP(1^1,2^1,3)\right)$ is 3.

On constructing spider graph by fixing m=1 and increasing the value of t by 1 we have the number of vertices labelled with 1 and 0 each increases by 1 from the basic spider graph and the number of edges labelled with 1 and 0 each increases by 1 from the basic spider graph. Hence in general we have the result

$V_1\left(SP(1^1,2^{t+1},3)\right)=V_1(SP(1^1,2^1,3)+t$

$V_0\left(SP(1^1,2^{t+1},3)\right)=V_0(SP(1^1,2^1,3)+t$

$E_1\left(SP(1^1,2^{t+1},3)\right)=E_1(SP(1^1,2^1,3)+t$

$E_0\left(SP(1^1,2^{t+1},3)\right)=E_0(SP(1^1,2^1,3)+t$

Hence the proof of the theorem.

IJASRM
International Journal of Advanced Scientific Research and Management, Volume 4 Issue 4, April 2019
www.ijasrm.com
ISSN 2455-6378

Theorem 3.13: For the Spider graph $SP\left(1^m, 2^t, 3\right)$ by fixing m=1 and for $t \geq 1$ we have

$$V\left(SP(1^1, 2^{t+1}, 3)\right) = V\left(SP(1^1, 2^1, 3)\right) + 2t$$

$$E\left(SP(1^1, 2^{t+1}, 3)\right) = E\left(SP(1^1, 2^1, 3)\right) + 2t$$

Proof: Let $G = SP\left(1^1, 2^t, 3\right)$ be the spider graph. The total number of vertices labelled with 0 and 1 for the basic spider graph is 7 and is denoted by $V\left(SP(1^1, 2^1, 3)\right)$ and the total number of edges labelled with 0 and 1 for the basic spider graph is 6 and is denoted by $E\left(SP(1^1, 2^1, 3)\right)$. Hence for the spider graph $SP\left(1^1, 2^2, 3\right)$ we have

$$V\left(SP(1^1, 2^2, 3)\right) = V\left(SP(1^1, 2^1, 3)\right) + 2$$

for the spider graph $SP\left(1^1, 2^3, 3\right)$

$V\left(SP(1^1, 2^3, 3)\right) = V\left(SP(1^1, 2^1, 3)\right) + 4$ and so on. Hence in general we have

$$V\left(SP(1^1, 2^{t+1}, 3)\right) = V\left(SP(1^1, 2^1, 3)\right) + 2t$$

Similarly

$E\left(SP(1^1, 2^2, 3)\right) = E\left(SP(1^1, 2^1, 3)\right) + 2$ for the spider graph $SP\left(1^1, 2^2, 3\right)$

$E\left(SP(1^1, 2^3, 3)\right) = E\left(SP(1^1, 2^1, 3)\right) + 4$ for the spider graph $SP\left(1^1, 2^3, 3\right)$

Hence in general we have

$$E\left(SP(1^1, 2^{t+1}, 3)\right) = E\left(SP(1^1, 2^1, 3)\right) + 2t.$$

Hence the proof of the theorem.

4. Results

In this paper we have considered Spider Graph $SP\left(1^m, 2^t, 3\right)$ and proved that it is homo cordial labelling graph and have identified a generalization method for Spider graph

5. Concluding Remarks

We are investigating on the other types of Spider graphs which can be also labelled so as to prove that they are homo cordial labelling graph.

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