IJASRM
International Journal of Advanced Scientific Research and Management, Special Issue 5, April 2019
www.ijasrm.com
ISSN 2455-6378

Chromatic number and chromatic index of product of two isomorphic regular graphs

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Abstract

Graph theory is nothing but the study of graphs: in mathematics, mathematical structures used to model pair-wise relations between objects from a certain collection. Graph coloring is one of the most important, well-known and studied sub fields of graph theory and extensively researched subject in the field of graph theory, having many applications. In this paper we present two objectives, our first objective we study about vertex coloring of regular graph and find the chromatic number of regular graph and our second objective we study edge coloring of regular graph and find chromatic index of regular graph. Lastly, we turn our attention to the product and sum of two isomorphic regular graphs, which has been found to be very interesting to study and color.

Key words:- *graph coloring, vertex coloring, edge coloring, product of graph, sum of graph, isomorphic graph.*

1. Introduction

The concept of coloring of regular graphs arise in many real situations that occur in computer science, physical science, mathematics, communication science and many other areas. The graphs can be used to represent almost any problem involving discrete arrangements of objects. Some basics terminology and then discuss some important concepts in graph theory with many applications of graphs. Graph coloring is one of the most popular topics in graph theory. Suppose that graph G with n vertices and asked to color the vertices such that no two adjacent vertices have the same color. Then we think about the minimum number of colors that we would require, color the vertices. This constitutes a coloring problem. The coloring performed on edges or as well as on vertices of a graph. In the case of regular graph, we may even interest in coloring the regions. In coloring graphs there is no disconnected graphs, color vertices in one component of a disconnected graph has no effect on coloring of the other components. So, it's usual to investigate coloring of the connected graphs only.

2. Methodology

Proper coloring of a graph is simple enough, but proper coloring with the minimum number of colors is, in general, a difficult task. In fact, there has not yet been found a simple way of characterising a *k*-chromatic graph. Chromatic number and chromatic index of some specific types of graph is discussed in the rest of sections.

Graph Coloring: The assign of colors to the vertices of G, one color to each vertex, so that adjacent vertices are assigned different colors is called the proper coloring of G or simply vertex coloring. Then n - coloring of G is coloring of G using $n-$colors. If G has n -coloring, then G is said to be n -colorable. There are two types of coloring.

1. Vertex coloring
2. Edge coloring

Vertex-Coloring: The assign of colors to the vertices of G, one color to each vertex, so that adjacent vertices are assign different colors is called the proper coloring of G or simply vertex coloring.

Chromatic number: The chromatic number of a graph G, denoted (G), is the least number of distinct colors with which G can be properly colored. It is denoted by $\chi(G)$.

Edge coloring

An edge coloring of graph G is an assignment of colors to the edges of G so that no two edges with a common vertex receive the same color.

Chromatic index

The minimum numbers of colors required to proper edge coloring of G is called the chromatic index of G is denoted by $\chi^{'}(G)$.

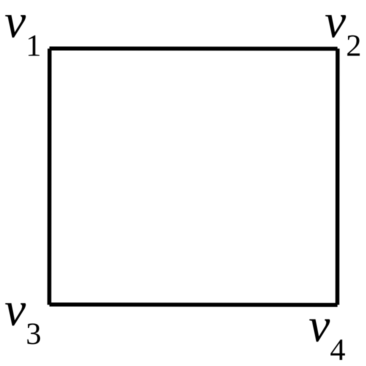
***K*-Coloring**

A coloring of a graph using a set of k colors is called a k -coloring. A graph which has a k -coloring is said to be k -colorable.

Product of Graphs: To define the product $G_1 \times G_2$ of two graphs consider any two points $u=(u_1,u_2)$ and $v=(v_1,v_2)$ in $V=V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever [$u_1 = v_1$ and u_2 adj v_2] or [$u_2 = v_2$ and u_1 adj v_1].

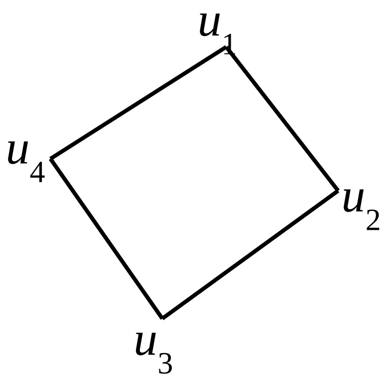
Chromatic number of product of two isomorphic regular graphs $C_4 \times C_4$

Let two isomorphic regular graphs G_1 and G_2



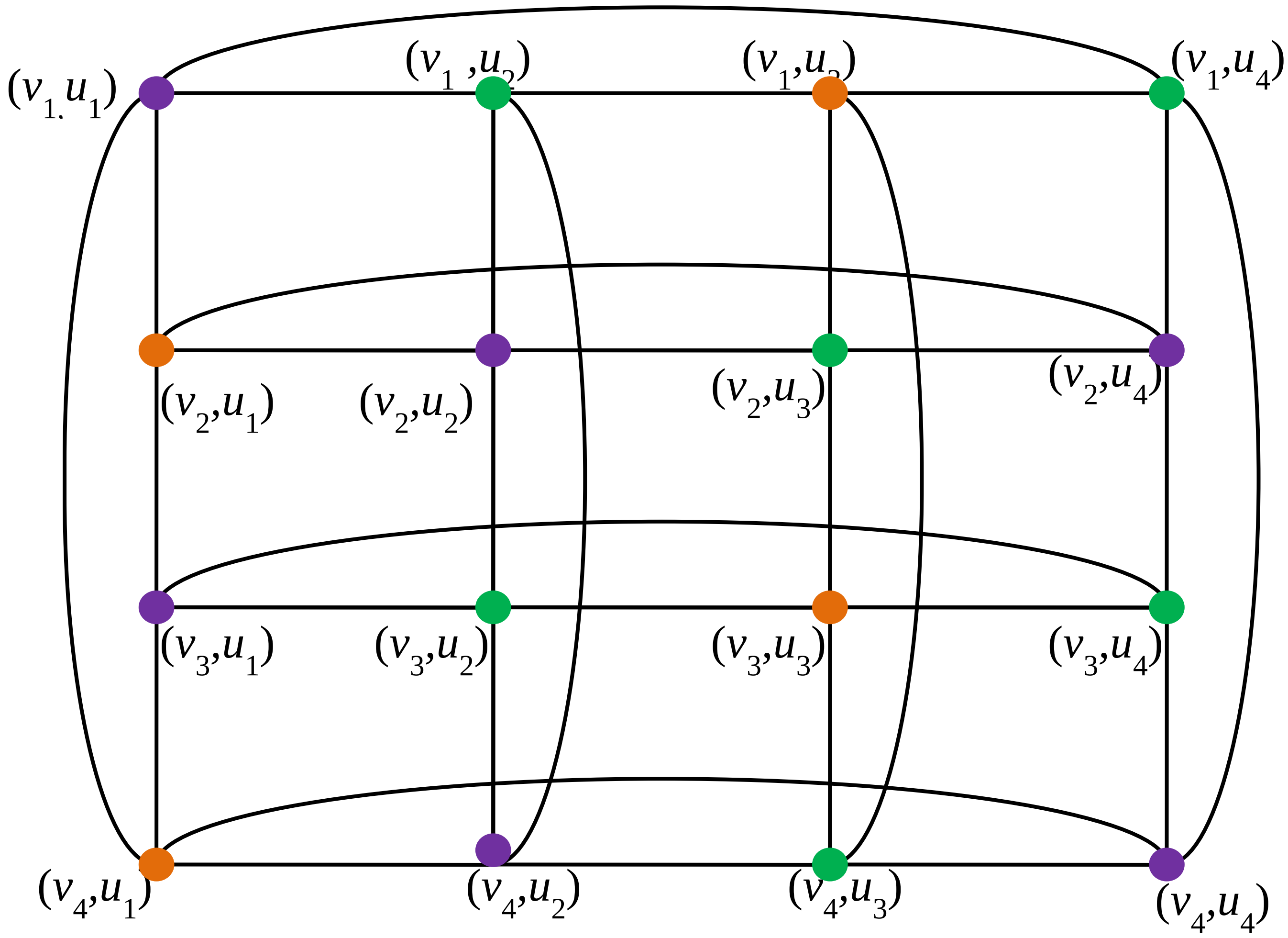
G_1 (regular graph**)**

Fig 1(a)



G_2 (isomorphic graph**)**

Fig 1(b)



$G_{1\times}G_2$ (product of two graphs)

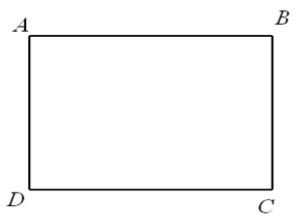
Fig 1(c)

The chromatic number $\chi(G_1 \times G_2) = 3$

Chromatic number of sum of two isomorphic regular graphs

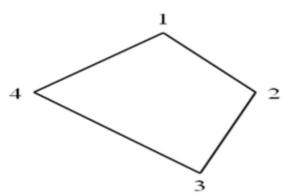
The sum of two isomorphic regular graph (C_4+C_4)

Let the two isomorphic regular graph G_1 and G_2



G_1 (regular graph)

Fig 2(a)



G_2 (isomorphic of graph)

Fig 2(b)

The proper vertex coloring of sum of (G_1+G_2)

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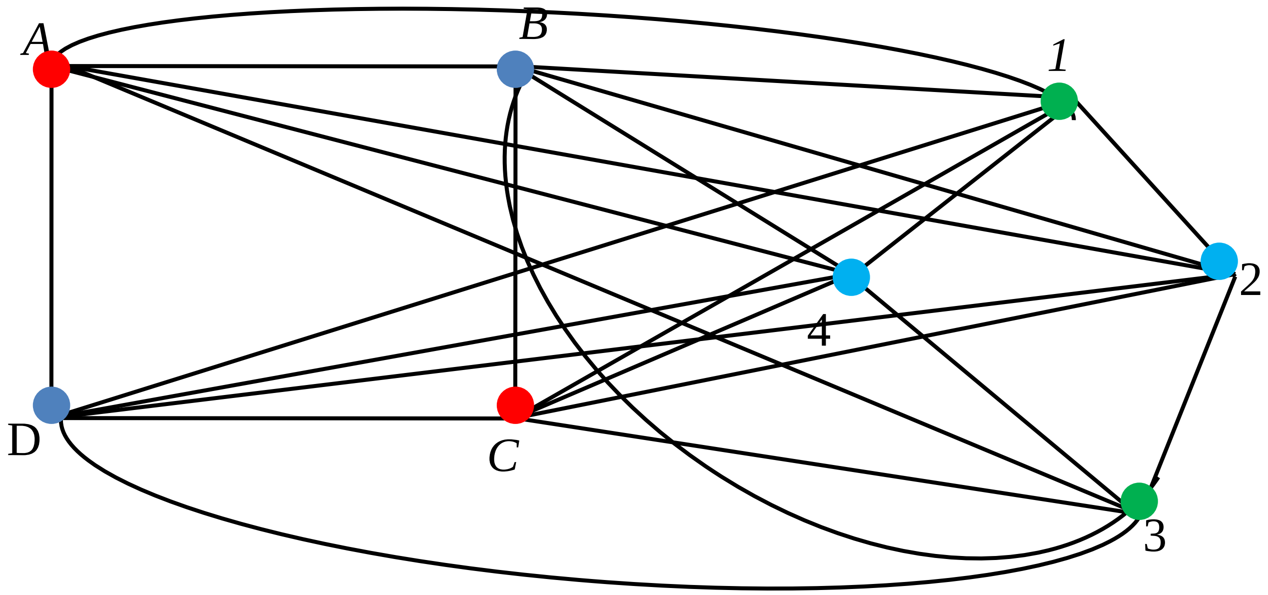


G_1+G_2 (sum of two graphs)

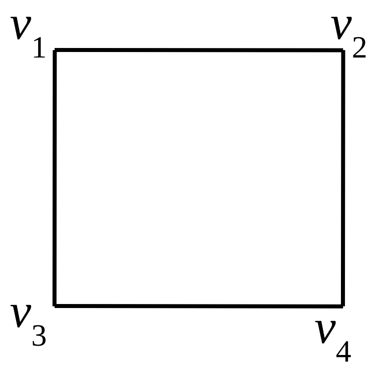
Fig 2(c)

The chromatic number $\chi(G_1 + G_2) = 4$

The chromatic index of product of two isomorphic regular graph

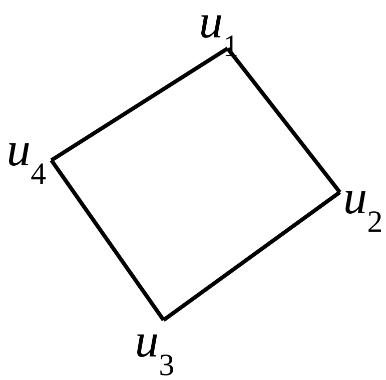
$C_4 \times C_4$

Let the two isomorphic regular graph



G_1 (regular graph)

Fig 3(a)



G_2 (isomorphic of graph)

Fig 3(b)

The proper vertex coloring of product of $G_1 \times G_2$

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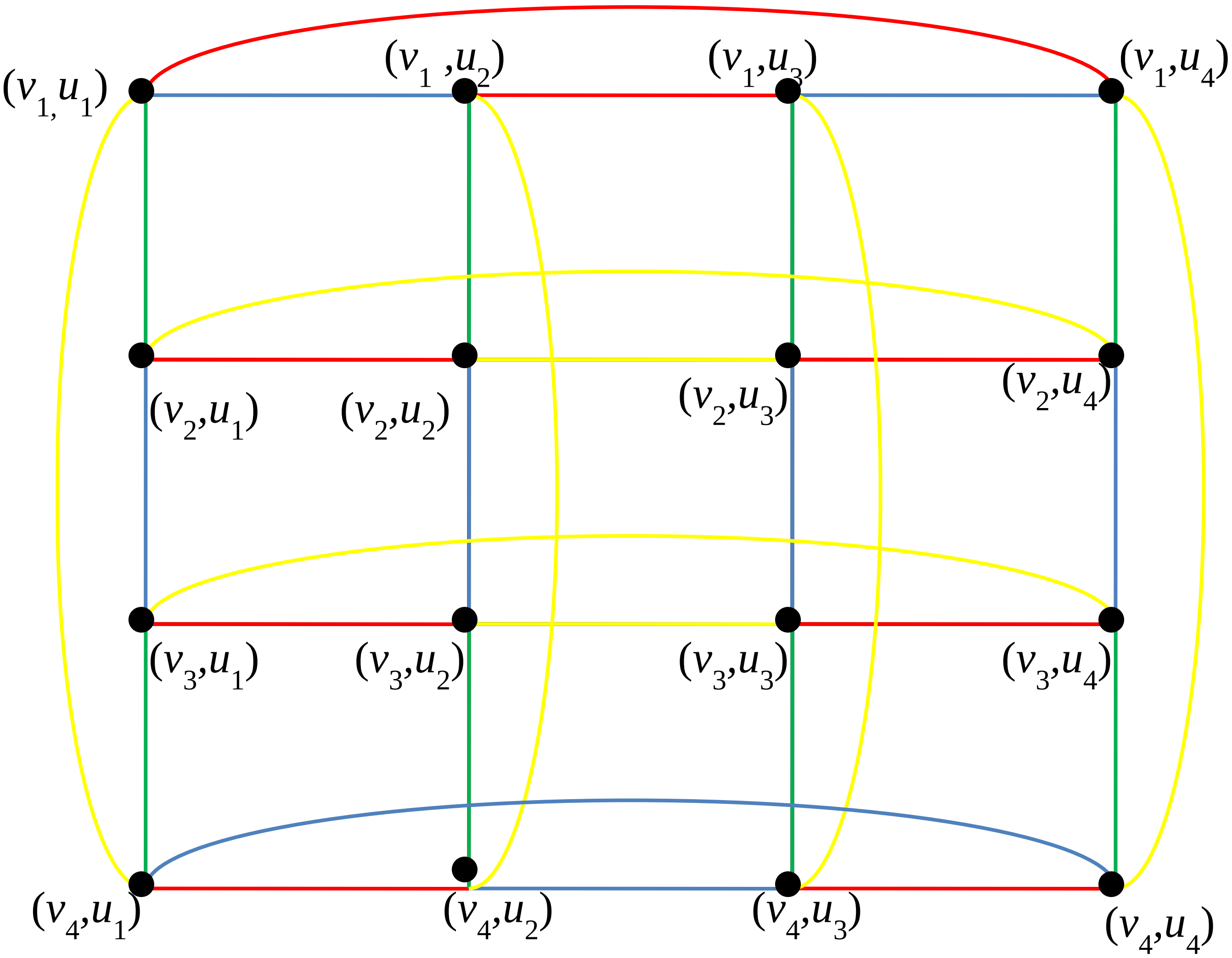

G_1×G_2 (product of two graphs)

Fig 3(c)

The chromatic index $\chi'(G_1 \times G_2) = 4$

The chromatic index of sum of two isomorphic regular graph C_3+C_3

Let the two isomorphic regular graph G_1 and G_2

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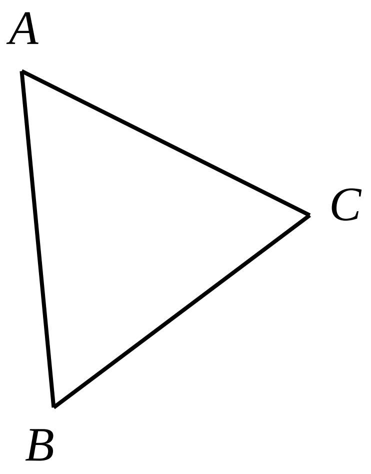
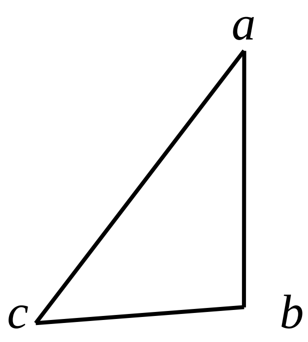
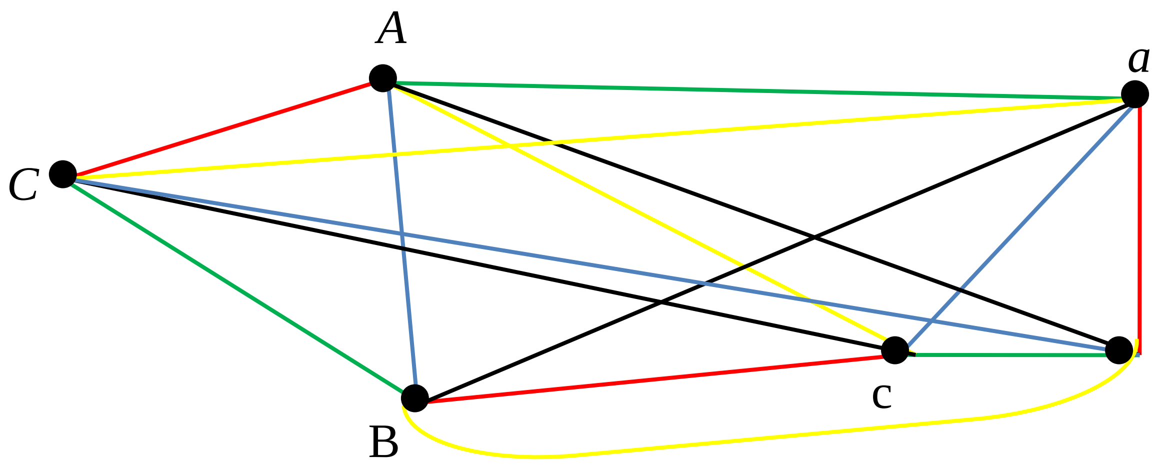


G_1 (regular graph)
Fig 4(a)



G_2 (isomorphic of graph)
Fig 4(b)



G_1+G_2 (sum of two graphs)
Fig 4(c)

The chromatic index $\chi'(G_1)+G_2 = 5$

CONCLUSION If G be a k - regular graph with n vertices, where n even. If $k \geq \frac{n}{2}$ then $\chi'(G) = k$. If the maximum vertex degree of a regular graph G is Δ then $\chi(G) \leq \Delta + 1$. The product of two regular graphs is regular graph if both graphs are isomorphic. The sum of two regular graphs is regular graph if both graphs are isomorphic.

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