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An inventory model with modified Weibully distributed deterioration rate, quadratic demand, time dependent IHC and without shortages

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Abstract

In this paper, an inventory model is developed with quadratic demand. Three-parameters modified Weibull distribution is used to represent the distribution of time for deterioration. In the model considered here, shortages are not allowed to occur. The model is solved analytically to obtain the optimal solution of the problem. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.

Keywords: EOQ model, Modified Weibull Distributed Deterioration Rate, Quadratic Demand.

1. Introduction:

The pioneering work of Harris [11] inventory models are being treated by mathematical techniques. He developed the simplest inventory model, the Economic Order Quantity (EOQ) model which was later popularized by Wilson [23]. Relaxation of some assumptions in the formulation of the EOQ model led to the development of other inventory models that effectively tackle several other inventory problems occurring in day-to-day life.

Deterioration means damage, spoilage, dryness, vaporization, etc. It is defined as decay or damage such that the item cannot be used for its original purposes. The effect of deterioration is very important in many inventory systems. The products like fresh food (fish, meat, fruits and vegetables), photographic films, batteries, human blood etc. having a maximum usable lifetime are known as 'Perishable products'. It also refers 'direct spoilage' and the products like alcohol, gasoline, radioactive substance, paints, chemical ingredients, lubricants, glues etc. having no shelf-life at all are known as 'Decaying products'. No deterioration refers to inventories that their shelf life can be indefinite and hence they would fall under 'No deterioration category'. Inventory problems for deteriorating items have been studied extensively by many researchers from time to time. Inventory of deteriorating items first studied by Whitin [22], he considered the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader [7] extended the classical EOQ formula to include exponential decay, wherein a constant fraction of on hand inventory is assumed to be lost due to deterioration. They observed that items have been deteriorated in a negative exponential relation with time and the equation for inventory model for deteriorating products is $\frac{dI(t)}{dt} + \theta I(t) = -f(t)$ where θ is the constant decay rate, I(t) the inventory level at the time t and f(t) is the demand rate at time t.

Covert and Philip [4] and Shah and Jaiswal [18] carried out an extension to the above model by considering deterioration of Weibull and general distributions respectively. Dave and Patel [5] first developed an inventory model for deteriorating items with time proportional demand, instantaneous replenishment and no shortages allowed.

The consideration of exponentially decreasing demand for an inventory model was first proposed by Hollier and Mak [10], who obtained optimal replacement policies under both constant and variable replenishment intervals. Hariga and Benkherouf [9] generalized Hollier and Mak's model by taking into account both exponentially growing and declining markets. In this field some of the recent works has been done by Chung and Ting [3]. Wee [21] studied an inventory model with deteriorating items. Chang and Dye [2] developed an inventory model with time-varying demand and partial backlogging.

R. Begum, S.K.Shahu & R.R. Shahoo [16] developed model with time dependent on Quadratic demand rate. Chaintanya kumar Tripathy and Umakanth Mishra [1] developed another model with Quadratic demand when the deterioration rate depends on two parameters Weibull distribution without shortages. Garima Garg, Bindu Vaish and Shalini Gupta [6] developed a model based on demand and production rate without any shortages. Venkateswarlu and Mohan [19] developed an EOQ model with 2 parameters Weibull deterioration time dependent quadratic demand and salvage value. Venkateswarlu and Mohan [20] developed an EOQ model for time varying deterioration and price dependent quadratic demand with salvage value. Mohan and Venkateswarlu [13] studied an inventory management models with variable holding cost and salvage value. Mohan and Venkateswarlu [14] proposed an inventory model for time dependent quadratic demand with salvage considering deterioration rate is time dependent. R. Amutha and E. Chandrasekaran [15] developed an EOQ model for deteriorating items and quadratic demand and time dependent holding cost.

According to Sarhan and Zaindin [17], it is interesting to observe that the modified Weibull distribution has a nice physical interpretation. It represents the lifetime of a series system. This system consists of two independent components. The lifetime of one component follows exponential distribution and the lifetime of the other follows Weibull distribution. In this paper we have analysed an inventory system order level model for deteriorating items under quadratic demand and time dependent IHC. Three-parameters modified Weibull distribution is used to represent the distribution of time for deterioration and shortages are not allowed to occur.

Recently, Kirtan Parmar and U. B. Gothi [12] developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution. In this model, shortages are not allowed and holding cost is time-dependent. Here, we have extended above deterministic inventory model by taking three parameters modified Weibull distribution to represent the distribution of time to deterioration. In the model considered here, holding cost is time-dependent and shortages are not allowed to occur.

2. Notations:

We use the following notations for the developed mathematical model:

1. Q(t) : The instantaneous state of the inventory
   level at any time t. $(0 \le t \le T)$
2. R(t) : Demand rate varying over time.
3. $\theta(t)$: Deterioration rate per unit per unit time.
4. A : Ordering cost per order.
5. Q : Order Quantity
6. C_4: Deterioration cost per unit per unit time.
7. C_1 : Inventory holding cost per unit per unit time.
8. T : The fixed length of each cycle.
9. TC: The average total cost for the time period [0, T]

3. Assumptions:

The model is developed under the following assumptions:

1. Inventory system deals with a single item.
2. The annual demand rate is a function of time which is $R(t) = (u + vt + wt^2)$ $(u, v, w > 0)$
3. Holding cost is a linear function of time expressed by $C_1 = \psi + \xi t$ $(h, r > 0)$
4. $\theta(t) = \alpha + \beta\gamma t^{\gamma-1}$ Three parameters modified Weibull deterioration rate (unit/unit time) $(0 < \alpha < 1, 0 < \beta < 1, \gamma > 0)$.
5. Lead-time is zero.
6. Time horizon is finite.
7. Shortages are not allowed to occur.
8. No repair or replacement of the deteriorated items takes place during a given cycle.
9. Total inventory cost is a real, continuous function which is convex to the origin.
10. α and β are very small and so in the derivation of the model their higher powers are neglected.

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4. Mathematical Model And Analysis:

The objective of the model is to determine the optimal order quantity in order to keep the total relevant cost as low as possible. Let Q be the total amount of replenishment in the beginning of each cycle. Here we have taken the total duration T as fixed constant. The initial stock was ‘Q’ at time t = 0. Due to reasons of market demands and deterioration of the items, the inventory level gradually diminishes during the period [0, T] and ultimately falls to zero at t = T.

The graphical presentation is shown in **Figure 1**.

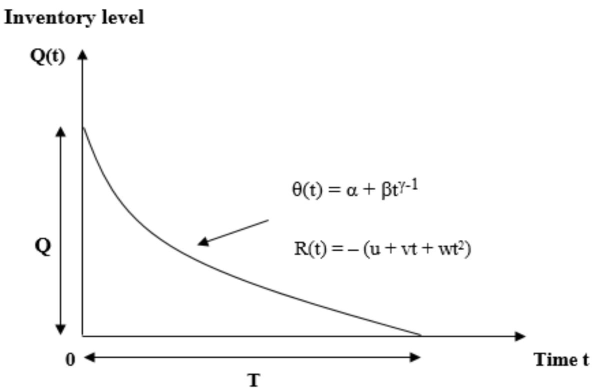

Figure 1: Graphical representation of the inventory system

The differential equations which describe the instantaneous state of Q(t) over the period [0, T] is given by

$$\frac{dQ(t)}{dt}+(\alpha+\beta r t^{\gamma-1})Q(t)=-(u+vt+wt^2),$$

$$(0\le t\le T) \qquad (1)$$

Under the boundary condition Q(0) = Q and Q(T) = 0, solutions of equations (1) is given by

$$Q(t)=\begin{Bmatrix} k\left(1-\alpha t-\beta t^{\gamma}\right)\text{-}\left(ut+\dfrac{vt^2}{2}+\dfrac{wt^3}{3}\right) \\ +\alpha\left(\dfrac{ut^2}{2}+\dfrac{vt^3}{6}+\dfrac{wt^4}{12}\right) \\ +\beta\gamma\left(\dfrac{ut^{\gamma+1}}{\gamma+1}+\dfrac{vt^{\gamma+2}}{2(\gamma+2)}+\dfrac{wt^{\gamma+3}}{3(\gamma+3)}\right)\end{Bmatrix}$$

$$\left(0<t<T\right) \qquad (2)$$

(where k is the constant of integration & neglecting higher powers of α, β)

Putting Q(0) = Q in equation (2), we get

$$Q=k=\begin{Bmatrix}\left(uT+\dfrac{vT^2}{2}+\dfrac{wT^3}{3}\right) \\ +\alpha\left(\dfrac{uT^2}{2}+\dfrac{vT^3}{3}+\dfrac{wT^4}{4}\right) \\ +\beta\left(\dfrac{uT^{\gamma+1}}{\gamma+1}+\dfrac{vT^{\gamma+2}}{\gamma+2}+\dfrac{wT^{\gamma+3}}{\gamma+3}\right)\end{Bmatrix} \quad (3)$$

Order costs or Setup Costs or Replenishment costs include the fixed cost associated with obtaining goods through placing of an order or purchasing or manufacturing or setting up a machinery before starting production.

The cost of placing order or replenishment costs is as follows:

$$OC = A \qquad (4)$$

Inventory is available in the system during the time interval [0, T]. Hence, the deterioration cost for inventory in stock is computed for time period [0, T].

The number of deteriorating items in the interval [0, T] is

$$DC=C_4\left\{Q-\int_0^T R(t)dt\right\}$$

$$DC=C_4\left\{Q-\int_0^T R\left(u+vt+\frac{wt^2}{2}\right)dt\right\}$$

$$\Rightarrow DC=C_4\begin{Bmatrix}\alpha\left(\dfrac{uT^2}{2}+\dfrac{vT^3}{3}+\dfrac{wT^4}{4}\right) \\ +\beta\left(\dfrac{uT^{\gamma+1}}{\gamma+1}+\dfrac{vT^{\gamma+2}}{\gamma+2}+\dfrac{wT^{\gamma+3}}{\gamma+3}\right)\end{Bmatrix}$$

(5)

Inventory is available in the system during the time interval [0, T]. Hence, the cost for holding

inventory in stock is computed for time period [0, T] only. Holding cost is as follows:

$$IHC = \int_0^T C_1 Q(t)dt$$

$$IHC=\int_0^T (\psi+\xi t)\begin{bmatrix} k\left(1-\alpha t-\beta t^{\gamma}\right) \\ -\left(ut+\frac{vt^2}{2}+\frac{wt^3}{3}\right) \\ +\alpha\left(\frac{ut^2}{2}+\frac{vt^3}{6}+\frac{wt^4}{12}\right) \\ +\beta\gamma\left(\begin{array}{l}\frac{ut^{\gamma+1}}{\gamma+1}+\frac{vt^{\gamma+2}}{2(\gamma+2)} \\ +\frac{wt^{\gamma+3}}{3(\gamma+3)}\end{array}\right)\end{bmatrix}dt$$

$$\Rightarrow IHC = \left\{\begin{array}{l} k\begin{bmatrix}\psi T+(\xi-\alpha\psi)\frac{T^2}{2} \\ -\alpha\xi\frac{T^3}{3}-\beta\psi\frac{T^{\gamma+1}}{\gamma+1}-\beta\xi\frac{T^{\gamma+2}}{\gamma+2}\end{bmatrix} \\ -u\psi\frac{T^2}{2}-(v\psi+2u\xi)\frac{T^3}{6} \\ -(2w\psi+3v\xi)\frac{T^4}{24}-w\xi\frac{T^5}{15} \\ +\alpha\begin{bmatrix}u\psi\frac{T^3}{6}+(v\psi+3u\xi)\frac{T^4}{24} \\ +(w\psi+2v\xi)\frac{T^5}{60}+w\xi\frac{T^6}{72}\end{bmatrix} \\ +\beta\gamma\begin{bmatrix}u\psi\frac{T^{\gamma+2}}{(\gamma+1)(\gamma+2)} \\ +\frac{[v\psi(\gamma+1)+2u\xi(\gamma+2)]T^{\gamma+3}}{2(\gamma+1)(\gamma+2)(\gamma+3)} \\ +\frac{[2w\psi(\gamma+2)+3v\xi(\gamma+3)]T^{\gamma+4}}{6(\gamma+2)(\gamma+3)(\gamma+4)} \\ +w\xi\frac{T^{\gamma+5}}{3(\gamma+3)(\gamma+5)}\end{bmatrix}\end{array}\right\} \quad (6)$$

Hence the total cost per unit time is given by

$$TC=\frac{1}{T}(OC+DC+IHC)$$

$$\Rightarrow TC=$$

$$\frac{1}{T}\left\{ \begin{array}{l} A + C_4\left[\begin{array}{l}\alpha\left(\frac{uT^2}{2}+\frac{vT^3}{3}+\frac{wT^4}{4}\right)\\ +\beta\left(\frac{uT^{\gamma+1}}{\gamma+1}+\frac{vT^{\gamma+2}}{\gamma+2}+\frac{wT^{\gamma+3}}{\gamma+3}\right)\end{array}\right] \\ + \left\{\begin{array}{l} k\left[\begin{array}{l}\psi T+(\xi-\alpha\psi)\frac{T^2}{2}\\ -\alpha\xi\frac{T^3}{3}-\beta\psi\frac{T^{\gamma+1}}{\gamma+1}-\beta\xi\frac{T^{\gamma+2}}{\gamma+2}\end{array}\right]\\ -u\psi\frac{T^2}{2}-(v\psi+2u\xi)\frac{T^3}{6}\\ -(2w\psi+3v\xi)\frac{T^4}{24}-w\xi\frac{T^5}{15}\\ +\alpha\left[\begin{array}{l}u\psi\frac{T^3}{6}+(v\psi+3u\xi)\frac{T^4}{24}\\ +(w\psi+2v\xi)\frac{T^5}{60}+w\xi\frac{T^6}{72}\end{array}\right]\\ +\beta\gamma\left[\begin{array}{l} u\psi\frac{T^{\gamma+2}}{(\gamma+1)(\gamma+2)}\\ +\frac{[v\psi(\gamma+1)+2u\xi(\gamma+2)]T^{\gamma+3}}{2(\gamma+1)(\gamma+2)(\gamma+3)}\\ +\frac{[2w\psi(\gamma+2)+3v\xi(\gamma+3)]T^{\gamma+4}}{6(\gamma+2)(\gamma+3)(\gamma+4)}\\ +w\xi\frac{T^{\gamma+5}}{3(\gamma+3)(\gamma+5)}\end{array}\right]\end{array}\right\}\end{array}\right\} \qquad (7)$$

Our objective is to determine optimum value of T so that TC(T) is minimum. The value of T for which total cost TC(T) is minimum, is the solution of equation $\frac{\partial TC(T)}{\partial T}=0$ satisfying the condition $\frac{\partial^2 TC(T)}{\partial T^2}>0.$

5. Numerical Example:

To illustrate the proposed model, an inventory system with the following hypothetical values is considered. By taking A = 100, α = 0.09, β = 0.08, γ = 3, C_4 = 8, ψ = 1, ξ = 0.5, u = 25, v = 17 and w = 10 (with appropriate units).

Solution is obtained by using appropriate software, we obtain the optimal value of T = 1.352852397 units, Q = 65.43170180 units and optimal total cost TC = 164.8067417 units.

6. Sensitivity Analysis:

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes or errors in its input parameter values. In this section, we study the sensitivity of the cycle length (T), optimal inventory level (Q) and total cost per time unit (TC) with respect to the changes in the values of the parameters A, α, β, γ, C_4, ψ, ξ, u, v and w.

The sensitivity analysis is performed by considering 25% and 50% increase or decrease in each one of the above parameters keeping all other parameters the same. The results are presented in table – 1.

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Table: Partial Sensitivity Analysis

Parameter	% Change	T	Q	TC	% Change in Q	% Change in TC
A	– 50	1.175380274	51.26795073	108.8113877	-21.65	-33.98
	– 25	1.275986048	58.97230903	138.2181565	-9.87	-16.13
	+ 25	1.41581926	71.13144307	189.5401766	8.71	15.01
	+ 50	1.469557678	76.31100297	212.9220744	16.63	29.20
α	– 50	1.423860034	69.54852477	159.5037485	6.29	-3.22
	– 25	1.387536267	67.43307042	162.1633604	3.06	-1.60
	+ 25	1.319805076	63.54377984	167.4392189	-2.89	1.60
	+ 50	1.288370928	61.76693769	170.0639175	-5.60	3.19
β	– 50	1.564672465	82.50380448	173.0482465	26.09	5.00
	– 25	1.437062636	71.89961328	167.1357557	9.88	1.41
	+ 25	1.290823261	60.92355623	163.9517544	-6.89	-0.52
	+ 50	1.242145767	57.53242451	158.8273237	-12.07	-0.59
γ	– 50	1.625513242	90.55357187	189.3854301	38.39	14.91
	– 25	1.452558006	74.11884551	173.097106	13.28	5.03
	+ 25	1.288420073	60.10281988	159.8226225	-08.14	-3.02
	+ 50	1.243544608	56.51732721	156.5044505	-13.62	-5.04
Parameter	**% Change**	**T**	**Q**	**TC**	**% Change in Q**	**% Change in TC**
C_4	– 50	1.806548459	96.8266464	171.1361236	47.98	3.84
	– 25	1.499640021	79.34514852	167.8936581	21.26	1.87
	+ 25	1.256427719	57.41059759	165.8195612	-12.26	0.61
	+ 50	1.18532846	51.99444854	158.2787627	-20.54	2.11
ψ	– 50	1.283882189	59.61194902	141.9343275	-8.89	-13.88
	– 25	1.317281916	62.37719655	152.7028565	-4.67	-7.34
	+ 25	1.390852342	68.8253159	178.471151	5.19	8.29
	+ 50	1.431611983	72.62251163	193.9779002	10.99	17.7
ξ	– 50	1.334537815	63.84458552	158.8922295	-2.43	-3.59
	– 25	1.343539308	64.62077879	161.782892	-1.24	-1.83
	+ 25	1.362500650	66.28033863	167.974661	1.3	1.92
	+ 50	1.372510668	67.17008147	171.2989353	2.66	3.94
u	– 50	1.388122227	49.21325347	145.5819305	-24.79	-11.67
	– 25	1.369918588	57.40872391	155.3257223	-12.26	-5.75
	+ 25	1.336822638	73.29787718	174.0488963	12.02	5.61
	+ 50	1.32173826	81.02098997	183.073174	23.83	11.08

v	– 50	1.41286237	60.88646632	155.3160973	-6.95	-5.76
	– 25	1.381087631	63.2092786	160.2055023	-3.4	-2.79
	+ 25	1.327506305	67.56685575	169.1621759	3.26	2.64
	+ 50	1.304557089	69.6251265	173.3049318	6.41	5.16
w	– 50	1.412439326	65.14220915	161.0196065	-0.44	-2.3
	– 25	1.380366156	65.24840067	162.9504275	-0.28	-1.13
	+ 25	1.32878084	65.6660328	166.590365	0.36	1.08
	+ 50	1.30740112	65.93515045	168.3054394	0.77	2.12

7. Graphical Presentation of Sensitivity Analysis:

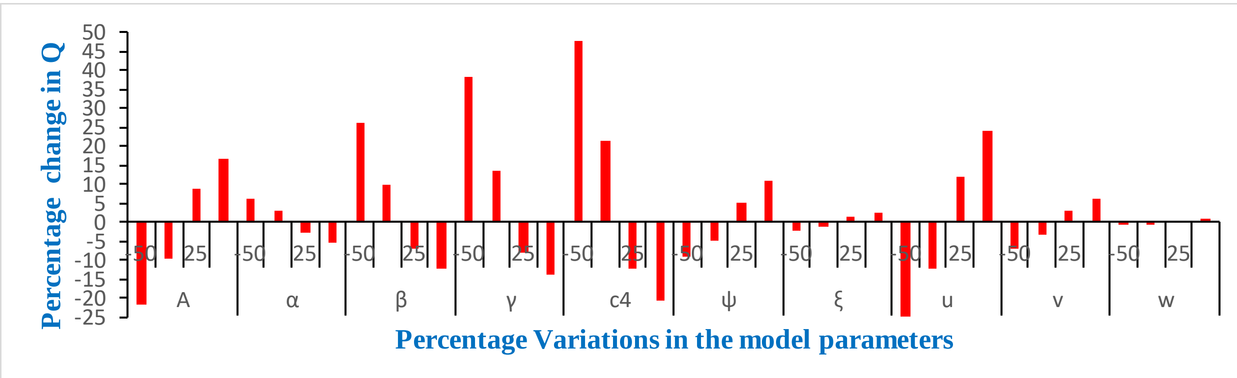

Figure 1

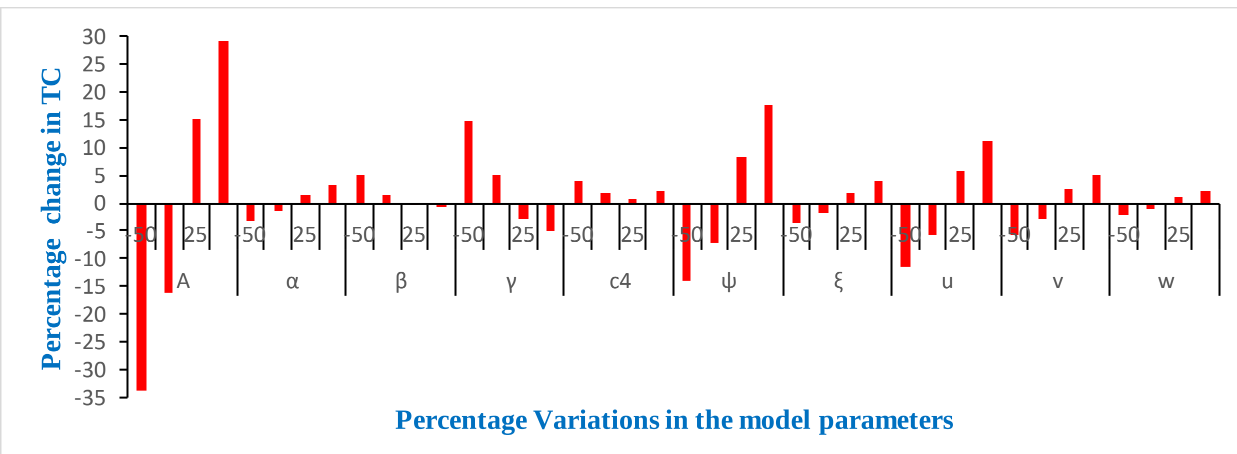

Figure 3

8. Conclusions:

(i) It is observed from table that the cycle time (T) increases due to increment in the value of the parameters A, ψ, ξ and cycle time (T) decreases due to increment in the value α, β, γ, C_4, u, v, w.

(ii) It is observed from table that the inventory order level (Q) increases due to increment in the value of the parameters A, ψ, ξ, u, v, w and inventory order level (Q) decreases due to increment in the value α, β, γ, C_4.

(iii) We may conclude from table that the average total cost per time per unit (TC) increases due to increment in the value of the parameters A, α, ψ, ξ, u, v, w and TC decreases due to increment in the value of parameter β, γ, C_4.
(iv) We may also conclude from table that average total cost per time per unit (TC) highly sensitive with respect to our model parameters A, γ, u, ψ, C_4 and moderately sensitive with respect to parameters α, β, ξ, v, w.

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