IJASRM

International Journal of Advanced Scientific Research and Management, Volume 4 Issue 6, June 2019

www.ijasrm.com

ISSN 2455-6378

On Semi*δ- Homeomorphism in Topological Spaces

Reena C[1], Vijayalakshmi P[2] and S. Pious Missier[3]

[1] Assistant Professor, PG and Research Department of Mathematics, St.Mary'sCollege,Tuticorin. Affliated to Manonmaniam Sundaranar University, Tirunelveli, India

[2]M.Phil Scholar, PG and Research Department of Mathematics, St.Mary'sCollege,Tuticorin. Affliated to Manonmaniam Sundaranar University, Tirunelveli, India

[3] Associate Professor (Retd), PG and Research Department of Mathematics, V.O.C College,Tuticorin. Affliated to Manonmaniam Sundaranar University, Tirunelveli, India.

Abstract: The purpose of this paper to introduce a new class of function namely semi*δ-homeomorphism and strongly semi*δ-homeomorphism and study their properties. Also we relate and compare these functions with some other functions in topological spaces.

Keywords: Semi*δ –homeomorphism, Strongly Semi* δ –homeomorphism

AMS subject classification : 54A05,54C08

1.INTRODUCTION

Many researchers have generalized the notion of homeomorphisms in topological spaces. Maki et al [3] introduced generalised homeomorphism which are generalizations of homeomorphism in topological spaces. The concept of semi*δ-open sets [4], have been initiated by Pious Missier .S and Reena .C. The purpose of this paper is to introduced a new class of functions called semi*δ-homeomorphism and strongly semi*δ –homeomorphism in topological spaces

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, ω) or X, Y, Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and interior of A respectively. The power set of X is denoted by P(X).

Definition 2.1: A subset *A* of a topological space (X, τ) is called a **semi*δ-open set** [2] if there exists a δ-open set U in X such that $U \subseteq A \subseteq Cl^*(U)$.

Definition 2.2: The **semi*δ-interior** [2] of *A* is defined as the union of all semi*δ-open sets of *X* contained in *A*. It is denoted by ***s*δInt*(A).**

Definition 2.3 : If A is a subset of a topological space X, the **semi*δ-closure [5]**of A is defined as the intersection of all semi*δ-closed sets in X containing A. It is denoted by s*δCl(A).

Definition 2.4: A function f: (X, τ) → (Y, σ) is said to be **semi*δ-continuous** [4] if f^{-1}(V)is semi*δ-open in (X, τ) for every open set V in (Y, σ).

Definition 2.5: A function f: (X, τ) ⟶ (Y, σ) is said to be **semi*δ-open** [4] if *f* (*U*) is semi*δ-open in *Y* for every open set *U* in *X*.

Definition 2.6: A function f : (X, τ)⟶ (Y, σ)is said to be **semi*δ-closed** [5] if *f*(*F*) is semi*δ-closed in *Y* for every closed set *F* in *X*.

Definition 2.7: A map f: (X, τ) →(Y, σ) is called a **Homeomorphism** [6] if f is both open and continuous.

Definition 2.8: A function is f:(X, τ) →(Y, σ) is said to be **semi-homeomorphism** [1] if f is both continuous and semi-open.

Definition 2.9: A function f :(X, τ) ⟶(Y, σ) is said to be **semi*δ-irresolute** [4] if f^{-1}(V) is semi*δ-open in (X, τ) for every semi*δ-open set V in (Y, σ).

Theorem 2.10: [2] Every δ-open set is semi*δ-open.

Theorem 2.11: [2] In any topological space,
(i) Every semi*δ-open set is δ-semi-open.
(ii) Every semi*δ-open set is semi - open.
(iii) Every semi*δ-open set is semi* - open.
(iv) Every semi*δ-open set is semi*-preopen.
(v) Every semi*δ-open set is semi-preopen.
(vi) Every semi*δ-open set is semi*α-open
(vii) Every semi*δ-open set is semiα-open.

3. SEMI*δ- HOMEOMORPHISM

Definition 3.1: A bijection $f:(X,\tau)\to(Y,\sigma)$ is called **semi*δ homeomorphisms** if f is both semi* δ- continuous and semi*δ- open.

Example 3.2: Let X=Y={a, b, c, d}, τ={φ,{a},{b},{a, b},X}andσ={φ,{a},{c},{a, c}, Y} S*δO(X , τ) = {φ,{a},{b},{a, b}, {b, c}{c, a}, X} and S*δO(Y ,σ)={φ,{a},{c}, {a, b}{b, c},{a, c},Y} Let $f:(X,\tau)\to(Y,\sigma)$ be defined by f(a)=a, f(b)=c, f(c)=b. Clearly f is semi*δ- homeomorphism.

Theorem 3.3: Let $f:(X,\tau)\to(Y,\sigma)$ be a bijective, semi*δ-continuous map then the following statements are equivalent.

(i) f is a semi*δ-open map
(ii) f is a semi*δ-homeomorphism
(iii) f is a semi*δ- closed map

Proof: $(i)\Rightarrow(ii)$ It is clearly by definition

$(ii)\Longrightarrow(iii)$ Let F be a closed set in (X, τ). Then F^c is open set in (X, τ), By (ii), f(F^c) is semi*δ open set in (Y, σ). Since f(F^c) =$[f(F)]^c$, $[f(F)]^c$ is semi*δ-open set in (Y, σ). That is f(F) is semi*δ-closed in (Y, σ). Therefore f is a semi*δ-closed map.

$(iii)\Longrightarrow(i)$ Let f be a semi*δ-closed map. Let G be open set in (X, τ), then G^C is closed set in (X, τ). By hypothesis, f(G^c) is semi*δ-closed in (Y, σ), since f(G^c) = $[f(G)]^c$, $[f(G)]^c$ is semi*δ-closed in (Y, σ). That is f(G) is semi*δ-open in (Y, σ). Therefore f is semi*δ-open map.

Theorem 3.4: Every δ-homeomorphisms is semi*δ-homeomorphism.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a δ-homeomorphism, then f is bijective, δ-continuous and δ-open. Let V be a open set in Y. Since f is δ-continuous, f^{-1}(V) is δ-open in X. Since, every δ-open set is semi*δ-open, by theorem 2.10. f^{-1}(V) is semi*δ-open in X which implies f is semi*δ-continuous. Let W be an open set in X. Since, f is δ-open, f (W) is δ-open in Y. Since every δ-open set is semi*δ-open, by theorem 2.10. f (W) is semi*δ-open in Y which implies f is semi*δ-open. Thus, f is semi*δ- homeomorphism.

Remark 3.5: The converse of the above theorem need not be true

Example 3.6: Let X=Y= {a, b, c, d}, τ={φ,{c},{d},{a, c},{c, d},{a, c, d},X} and σ = {φ, {b},{c},{b, c}Y}, S*δO(X, τ) = {φ, {d}, {a, c},{b, d},{a, b, c},{a, c, d},X}, δ O(X ,τ) = {φ, {d},{a, c}, X} and S*δO(Y, σ) = {φ,{b}, {c}, {b, c}, Y}, δO(Y, σ)={φ,{b},{c},{b, c},Y}. Let f:(X, τ) → (Y, σ) be a map defined by f(a)=f(c)=c, f(b)=f(d)=b. Clearly, f is semi*δ-homeomorphisms. Here, {b} is open in Y, but f^{1}({b})={b, d} is not δ-open in X. Therefore, f is not δ-homeomorphism.

Theorem 3.7: Every semi*δ- homeomorphism is δ-semi homeomorphism.

Proof: Let $f:(X,\tau)\to$ (Y, σ) be an semi*δ–homeomorphism, then f is bijective, semi*δ-continuous and semi*δ-open. Let G be an open set in Y. Since f is semi*δ-continuous, $f^{-1}(G)$ is semi*δ-open in X. Since every semi*δ open set is δ –semi open, by theorem 2.11, $f^{-1}(G)$ is δ- semi open in X. Therefore f is δ-semi continuous. Let V be an open set in X. since f is semi*δ-open , f(V) is semi*δ-open in Y. Since every semi*δ-open set is δ-semi open, by theorem 2.11, f(V) is δ-semi open in Y. Therefore f is δ-semi-open. Thus, f is δ-semi homeomorphism.

Remark 3.8: The converse of the above theorem need not be true.

Example 3.9: Let X = Y = {a, b, c, d}, τ={φ,{a},{b},{a, b}, {a, b, c}, X} and σ ={φ,{b},{c},{b, c},Y}, S*δ O(X, τ) = {φ, {a}, {b}, {a, b}{b, d},{a, d},{a, b, d},X}, δ-SO(X, τ) = {φ, {a},{b}, {a, b},{b, c}, {a, c}, {b, d},{a, d}, {a, b, c}, {a, b, d},{a, c, d}, {b, c, d}, X} and S*δO(Y, σ) = {φ, {b},{c}, {b, c}, Y}, δ-SO(Y, σ) ={φ, {b},{c},{a, b}, {b, c}, {a, c},{c, d},{b, d}, {a, b, c},{a, c, d}, {a, b, d},Y}. Let f:(X,τ)→(Y,σ) be a map defined by f(a)=c, f(b)=b, f(c)=a, f(d)=d. Clearly, f is δ-semi homeomorphism. Here, {a, b, c} is open in X, but f({a, b, c}) = {a, b, c} is not semi*δ- open in Y. Therefore, f is not semi*δ-homeomorphism.

Theorem 3.10: Every semi*δ- homeomorphism is semi-homeomorphism.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an semi*δ-homeomorphism, then f is bijective, semi*δ-continuous and semi*δ-open. Let G be an open set in Y. Since f is semi*δ-continuous, $f^{-1}(G)$ is semi*δ-open in X. Since every semi*δ-open set is semi–open, by theorem 2.11, $f^{-1}(G)$ is semi –open in X. Therefore f is semi-continuous. Let V be an open set in X. Since f is semi*δ-open , f(V) is semi*δ-open in Y. Since every semi*δ-open set is semi open, by theorem 2.11, f(V) is semi-open in

Y. Therefore f is semi-open. Thus, f is semi-homeomorphism.

Remark 3.11: The converse of the above theorem need not be true.

Example 3.12: Let X=Y= {a, b, c, d}, τ={φ, {a}, {b}, {a, b}, {a, b, c},X} and σ ={φ, {a}, {b}, {a, b},{b, c}{a, b, c},Y}, S*δO(X, τ) ={φ, {a},{b},{a, b}, {a, d},{b, d},{a, b, d},X}, SO(X, τ)= {φ,{a},{b},{a, b},{b, c},{a, d},{a, c}, {b, d},{a, b, c},{b, c, d},{a, b, d},X} and S*δO(Y, σ)={φ, {a}, {a, d}, {b, c}, {a, b, c}, {b, c, d}, Y}, SO(Y, σ)={φ, {a},{b},{a, b}, {b, c},{a, d},{b, d},{a, b, c}, {a, b, d},Y}. Let f: (X, τ)→(Y, σ) be a map defined by f(a)=b, f(b)=a, f(c)=d, f(d)=c. Clearly, f is semi-homeomorphism. Here, {a} is open in X, but, f({a})= {b} is not semi*δ- open in Y. Therefore f is not a semi*δ- homeomorphism.

Theorem 3.13: Every semi*δ-homeomorphism is semi*- homeomorphism.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an semi*δ-homeomorphism, then f is bijective, semi*δ-continuous and semi*δ-open. Let G be an open set in Y. Since f is semi*δ- continuous, $f^{-1}(G)$ is semi*δ-open in X. Since every semi*δ-open set is semi*-open, by theorem 2.11, $f^{-1}(G)$ is semi*-open in X. Therefore f is semi*-continuous. Let V be an open set in X. Since f is semi*δ-open, f(V) is semi*δ-open in Y. Since, every semi*δ-open set is semi*-open, by theorem 2.11, f(V) is semi*-open in Y. Therefore f is semi-open. Thus, f is semi*-homeomorphism.

Remark 3.14: The converse of the above theorem need not be true.

Example 3.15: Let X =Y= {a, b, c. d}, τ={φ,{a},{c},{a, d},{a, c},{a, c, d},X} and σ={φ,{a}, {b},{a, b},{b, c},{a, b, c}, Y}, S*δO(X, τ) = {φ,{c},{b, c},{a, d}, {a, c, d}, {a, b, d}}, S*O(X, τ)={φ, {a},{c},{a, b},{b, c}, {a, c} , {a, d},{a, b, c},{a, b, d}, {a, c, d},X} and S*δO(Y, σ) ={φ, {a}, {a, d}, {b, c} ,{a, b, c}, {b, c, d},Y}, S*O(Y, σ)={φ, {a},{b},{a, b}, {b, c},{a, d},{b, d},{a, b, c}, {a, b, d},{b, c, d},Y}. Let f: (X, τ)→(Y, σ) be a map defined by f(a)= a, f(b)= c, f(c)=b, f(d)=d. Clearly, f is semi*-homeomorphism. Here, {c} is open in X, but, f({c})= {b}is not semi*δ- open in Y. Therefore f is not a semi*δ- homeomorphism.

Theorem3.16: Every semi*δ- homeomorphism is semi*-pre homeomorphism.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an semi*δ-homeomorphism, then f is bijective, semi*δ-continuous and semi*δ-open. Let G be an open set in Y. Since f is semi*δ- continuous, $f^{-1}(G)$ is semi*δ- open in X. Since every semi*δ- open set is semi*pre-open, by theorem 2.11, $f^{-1}(G)$ is semi*pre-open in X. Therefore f is semi*pre-continuous. Let V be an open set in X. Since f is semi*δ- open, f(V) is semi*δ-open in Y. Since , every semi*δ-open set is semi*pre-open, by theorem 2.11, f(V) is semi*pre-open in Y. Therefore f is semi*pre-open. Thus, f is semi*pre-homeomorphism.

Remark 3.17: The converse of the above theorem is need not be true.

Example 3.18: Let X = Y = {a, b, c, d}, τ={φ,{a},{c}, {a, d},{a, c}, {a, c, d},X} and σ={φ,{b}, {c}, {b, d},{b, c},{b, c, d}, Y}, S*PO(X, τ) ={φ,{a},{c},{a, b},{b, c},{a, c},{a, d} {a, b, c}, {a, c, d}, {a, b, d}, X}, S*δO(X, τ) = {φ, {c},{b, c},{a, d},{a, b, d}, {a, c, d},X} and S*PO(Y, σ) = {φ, {b}, {c}, {a, b},{a, c},{b, c}, {b, d},{a, b, d},{a, b, c}, {b, c, d},Y}, S*δO(Y, σ)={φ, {c},{a, c}, {b, d},{a, b, d}, {b, c, d} ,Y}. Let f: (X, τ) → (Y, σ) be a map defined by f(a)=b, f(b)=a, f(c)=c, f(d)=d. Clearly, f is semi*pre- homeomorphism. Here, {a} is open in X, but, f({a})= {b}is not semi*δ- open in Y. Therefore f is not a semi*δ- homeomorphism.

Theorem 3.19: Every semi*δ- homeomorphism is semi pre-homeomorphism.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an semi *δ-homeomorphism, then f is bijective, semi*δ-continuous and semi*δ-open. Let G be an open set in Y. Since f is semi*δ- continuous, $f^{-1}(G)$ is semi*δ-open in X. Since every semi*δ-open set is semi pre-open, by theorem 2.11, $f^{-1}(G)$ is semi pre -open in X. Therefore f is semi pre-continuous. Let V be an open set in X. Since f is semi*δ-open, f(V) is semi*δ-open in Y. Since, every semi*δ-open set is semi pre-open, by theorem 2.11, f(V) is semi pre-open in Y. Therefore f is semi pre-open. Thus, f is semi pre-homeomorphism.

Remark 3.20: The converse of the above theorem need not be true.

Example 3.21: Let X=Y= {a, b, c, d}, τ={φ,{b},{c}, {b, d},{b, c}, {b, c, d},X} and σ={φ,{a}, {b}, {a, b},{b, c},{a, b, c}, Y}, S*δO(X, τ)={φ,{c},{a, c},{b, d}, {b, c, d}, {a, b, d}},SPO(X, τ) ={φ, {b},{c},{a, b}, {b, c},{a, c}, {b, d},{a, b, c},{a, b, d}, {b, c, d}, X} and S*δO(Y, σ)={φ, {a}, {a, d}, {b, c}, {a, b, c}, {b, c, d},Y}, SPO(Y, σ)={φ, {a},{b}, {a, b}{b, c},{a, d},{b, d},{a, b, c}, {a, b, d}, {b, c, d},Y}. Let f: (X, τ)→(Y, σ) be a map defined by f(a)=c, f(b)=b, f(c)=a, f(d)=d. Clearly, f is semi pre- homeomorphism. Here, {b} is open in X, but, f({b})= b is not semi*δ- open in Y. Therefore f is not a semi*δ- homeomorphism.

Theorem 3.22: Every semi*δ –homeomorphism is semi*α-homeomorphism.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an semi*δ-homeomorphism, then f is bijective, semi*δ-continuous and semi*δ-open. Let G be an open set in Y. Since f is semi*δ- continuous, $f^{-1}(G)$ is semi*δ-open in X. Since every semi*δ-open set is semi*α-open, by theorem 2.11, $f^{-1}(G)$ is semi*α-open in X. Therefore f is semi*α-continuous. Let V be an open set in X. Since f is semi*δ- open, f(V) is semi*δ-open in Y. Since every semi*δ- open set is semi*α-open, by theorem 2.11, f(V) is semi*α-open in Y. Therefore f is semi* α-open. Thus f is semi*α-homeomorphism.

Remark 3.23: The converse of the above theorem need not be true.

Example 3.24: Let X=Y= {a, b, c, d}, τ={φ,{a},{b}, {a, d},{a, b}, {a, b, d},X} and σ={φ,{a}, {c},{a, d},{a, c},{a, c, d}, Y}, S*δO(X, τ)={φ,{b},{b, c},{a, d}, {a, c, d},{a, b, d}},S*αO(X, τ)={φ, {a},{b},{a, b},{b, c},{a, c}, {a, d},{a, b, c},{a, b, d},X} and S*δO(Y, σ)={φ, {c}, {a, d}, {b, c}, {a, c, d},{a, b, d},Y}, S*αO(Y, σ)={φ, {a},{c},{a, b}{b, c},{a, d},{a, c},{a, b, c}, {a, b, d}, {a, c, d},Y}. Let f: (X, τ)→(Y, σ) be a map defined by f(a)=a, f(b)=c, f(c)=b, f(d)=d. Clearly, f is semi*α -homeomorphism. Here, {a} is open in X, but, f({a})= {a}is not semi*δ- open in Y. Therefore f is not a semi*δ- homeomorphism.

Theorem 3.25: Every semi*δ- homeomorphism is semi α-homeomorphism.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an semi*δ-homeomorphism, then f is bijective, semi*δ –continuous and semi*δ –open. Let G be an open set in Y. Since f is semi*δ-continuous, $f^{-1}(G)$ is semi*δ –open in X. Since every semi*δ-open set is semi α-open, by theorem 2.11, $f^{-1}(G)$ is semi α-open in X. Therefore f is semi α-continuous. Let V be open set in X. Since f is semi*δ-open, f(V) is semi*δ- open in Y. Since every semi*δ-open set is semi α-open , by theorem 2.11, f(V) is semi α-open in Y. Therefore f is semi α-open. Thus, f is semi α-homeomorphism.

Remark 3.26: The converse of the above theorem need not be true.

Example 3.27: Let X =Y= {a, b, c. d}, τ={φ,{a},{c},{a, d},{a, c}, {a, c, d},X} and σ={φ,{b}, {c},{b, d},{b, c},{b, c, d}, Y}, S*δ O(X, τ)={φ,{c},{b, c},{a, d}, {a, c, d},{a, b, d}},SαO(X, τ)={φ, {a},{c},{a, b}, {b, c},{a, c}, {a, d},{a, b, c},{a, b, d}, {a, c, d},X} and S*δO(Y, σ)={φ, {c}, {a, c}, {b, d},{a, b, d},{b, c, d},Y}, SαO(Y, σ)={φ, {b},{c},{a, b},{b, c},{a, c},{b, d},{a, b, c}, {a, b, d},{b, c, d},Y}. Let f: (X, τ)→(Y, σ) be a map defined by f(a)=b, f(b)=a, f(c)=c, f(d)=d. Clearly, f is semi α-open homeomorphism. Here, {a} is open in X, but, f({a})= {b}is not semi*δ- open in Y. Therefore f is not a semi*δ- homeomorphism.

Remark 3.28: The concept of semi*δ-homeomorphism and homeomorphism are independent.

Example 3.29: Let X=Y ={a, b, c, d}, τ={φ, {b}, {c}, {b, c},{a, b, c}, X} and σ = {φ, {a}, {b}, {a, b}, {a, b, c}, Y}, S*δO (X, τ) = {φ,{b},{c},{b, c}{c, d},{b, d},{b, c, d}, X} and S*δO(Y ,σ)= {φ , {a}, {b},{a, b},{a, d}, {b, d},{a, b, d}, Y} . Let $f:(X,\tau)\to(Y,\sigma)$ be a map defined by f(a)= d, f(b)= b, f(c)=a, f(d)=c. Clearly f is semi*δ- homeomorphism. Here {a, b, c} is open in X, but f({a, b, c}) = {a, b, d} not open in Y. Hence f is not an open –map. Therefore f is not homeomorphism

Example 3.30: Let X=Y ={a, b, c, d}, τ={φ, {a}, {b},{a, b},{b, c},{a, b, c}, X} and σ = {φ,{a}, {b},{a, b},{a, d},{a, b, d},Y}, S*δO(X, τ) = {φ,{a},{b, c}{a, d},{a, b, c},{b, c, d}, X} and S*δO(Y ,σ)= {φ ,{b}, {b, c},{a, d},{a, c, d},{a, b, d}, Y}. Let $f:(X,\tau)\to(Y,\sigma)$ be a map defined by f(a)= b, f(b)= a, f(c)=d, f(d)=c. Clearly f is homeomorphism. Here {a, b, c} is open in X, but f({ b})={a} not semi*δ- open in Y. Hence f is not semi*δ-open. Therefore f is not semi*δ-homeomorphism

Remark 3.31: The concept of semi*δ-homeomorphism and α-homeomorphism are independent.

Example 3.32: Let X=Y ={a, b, c, d}, τ={φ, {a}, {b}, {a, b},{b, c},{a, b, c}, X} and σ = {φ,{a},{c},{a, b},{a,d},{a, b, c},Y}, S*δO(X, τ) = {φ,{a},{a, d},{b, c},{a, b, c},{a, c, d}, X}, αO (X, τ) = {φ, {a}, {b}, {a, b},{b, c} , {a, b, c},{a, b, d},X} and S*δO(Y ,σ)= {φ,{a}, {b}{a, b}, {a, d}, {b, d}, {a, b, d}, Y}, αO(Y, σ) ={φ, {a}, {c}, {a, c}, {a, d}, {a, b, c},{a, c, d}}. Let $f:(X,\tau)\to(Y,\sigma)$ be a map defined by f(a)= c, f(b)= a, f(c)=d, f(d)=b. Clearly f is α homeomorphism. Here {b} is open in X, but f({b})={a} not semi*δ-open in Y. Therefore f is not semi*δ- homeomorphism

Example 3.33: Let X=Y ={a, b, c, d}, τ={φ, {c}, {d},{a, c},{ c, d},{a, c, d}, X} and σ = {φ,{b}, {c},{ b, c}Y}, S*δO(X, τ)= {φ,{d},{a, c}{b, d},{a, b, c},{a, c, d}, X} , αO(X, τ) =

{φ, {c},{d},{c, d},{a, c},{b, c ,d}, {a, c, d},X} and S*δO(Y ,σ) = {φ ,{b},{c}, {b, c}, Y}, αO(Y,σ) = { φ ,{b},{c} ,{b, c}, Y}. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)= f(c)=c, f(b)=f(d)=b. Clearly f is semi*δ-homeomorphism. Here { b} is open in Y, but f^{-1}({b})={b, d} not α-open in X. Hence f is not α-continuous. Therefore f is not α-homeomorphism

Remark 3.34: The concept of semi*δ-homeomorphism and α*-homeomorphism are independent.

Example 3.35: Let X=Y ={a, b, c, d}, τ={φ, {a}, {b}, {a, b},{a, b, c}, X} and σ ={φ,{a}, {b},{a, b},{b, c},{a, b, c},Y}, S*δO(X, τ)={φ,{a},{b},{a, b},{b, d},{a, d}, {a, b, d}, X} , α*O (X,τ) = {φ,{a},{b},{c}, {a, b}, {a, c},{b, c},{a, b, c},{a, b, d},X} and S*δO(Y ,σ)= {φ , {a}, {b, c}{a, d},{a, b, c}, {b, c, d}, Y} , α*O(Y,σ)={φ,{a},{b},{c}, {a, b},{a, c},{b, c}, {a, b, c},{a, b, d},Y}. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)= a, f(b)= b, f(c)=c, f(d)=d. Clearly f is α*-homeomorphism. Here {b} is open in X, but f({b})={b} not semi*δ-open in Y. Hence f is not open –map. Therefore f is not semi*δ-homeomorphism.

Example 3.36: Let X=Y ={a, b, c, d}, τ={φ, {c}, {d},{a, c},{ c, d},{a, c, d}, X} and σ = {φ,{b}, {c},{ b, c}Y}, S*δO(X, τ) = {φ,{d}, {a, c}{b, d},{a, b, c},{a, c, d}, X} , α*O(X,τ)={φ,{a}, {c}, {d}, {a, d},{c, d}, {a, c} , {b, c ,d},{a, c, d},X} and S*δO(Y ,σ) = {φ , {b},{c},{b,c}, Y}, α*O(Y,σ)=P(X). Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)= f(c)=c, f(b)=f(d)=b. Clearly f is semi*δ-homeomorphism. Here {b} is open in Y, but f^{-1}({ b})={b, d} not α*-open in X. Hence f is not α*-continuous. Therefore f is not α*-homeomorphism

Remark 3.37: The concept of semi*δ-homeomorphism and pre-homeomorphism are independent.

Example 3.38: Let X=Y ={a, b, c, d}, τ={φ, {a}, {c}, {a, d},{a, c},{a, c, d}, X} and σ = {φ,{b},{c},{b, d},{b, c},{b, c, d},Y}, S*δO(X, τ)={φ,{c},{b, c},{a, d},{a, c, d}, {a, b, d}, X}, PO (X, τ) = {φ,{a},{c} {a, d},{a, c}, {a, b, c},{a, b, d},{a, c, d},X} and S*δO(Y ,σ) = {φ ,{c}, {a, c}{b, d},{a, b, d}, {b, c, d}, Y}, PO(Y, σ)={φ,{b},{c},{b, c}, {b, d},{b, c, d}, {a, b, c},Y}. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a) = b, f(b) = a, f(c) = c, f(d) = d. Clearly f is pre-homeomorphism. Here {a} is open in X, but f({a})={b} is not semi*δ-open in Y. Therefore f is not semi*δ- homeomorphism

Example 3.39: Let X=Y ={a, b, c, d}, τ={φ, {c}, {d},{a, c},{c, d},{a, c, d}, X} and σ = {φ,{b}, {c},{ b, c}Y}, S*δO(X, τ)= {φ,{d},{a, c}{b, d},{a, b, c},{a, c, d}, X} , PO(X,τ)={φ, {c}, {d},{c, d},{a, c},{b, c ,d}, {a, c, d},X} and S*δO(Y ,σ)= {φ ,{b},{c}, {b, c}, Y}, PO(Y, σ) = {φ, {b},{c},{b, c}, {a, b, c},{b, c, d},Y}. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)= f(c)=c, f(b)=f(d)=b. Clearly f is semi*δ-homeomorphism. Here { b} is open in Y, but f^{-1} ({b})={b, d} not pre-open in X. Hence f is not pre-continuous. Therefore f is not pre-homeomorphism

Remark 3.40: The concept of semi*δ-homeomorphism and pre*-homeomorphism are independent.

Example 3.41: Let X=Y ={a, b, c, d}, τ={φ, {b}, {c}, {b, d},{b, c},{b, c, d}, X} and σ = {φ,{a}, {b},{a d},{a, b},{a, b, d},Y}, S*δO(X, τ)={φ,{c},{a, c},{b, d},{b, c, d}, {a, b, d}, X}, P*O (X, τ) = {φ,{b},{c},{d}, {b, c},{c, d},{b, d},{a, b, c},{b, c, d},X} and S*δO(Y , σ)= {φ , {b}, {b, c}{a, d}, {a, b, d}, {a, c, d}, Y} , P*O(Y, σ)= {φ, {a}, {b}, {d}, {a, b},{a, d},{b, d},{a, b, d},{a, b, c},Y}. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)=c, f(b)= b, f(c)=a, f(d)=d. Clearly f is pre*-homeomorphism. Here {c} is open in X, but f({c})={a} not semi*δ-open in Y. Hence f is not open –map. Therefore f is not semi*δ-homeomorphism

Example 3.42: Let X=Y ={a, b, c, d}, τ={φ, {c}, {d},{a, c},{ c, d},{a, c, d}, X} and σ = {φ , {b},{c},{b, c}Y}, S*δ O(X, τ)= {φ, {d}, {a, c}{b, d},{a, b, c},{a, c, d}, X} , P*O(X, τ)= {φ,{a}, {c}, {d},{c, d},{a, c}, {a, d}, {b, c ,d},{a, c, d},X} and S*δO(Y ,σ) = {φ , {b}, {c},{b, c}, Y}, P*O(Y, σ)=P(X). Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)=f(c)=c, f(b)=f(d)=b. Clearly f is semi*δ-homeomorphism. Here { b} is open in Y, but f^{-1}({ b})={b, d} not pre*-open in X. Hence f is not pre*-continuous. Therefore f is not pre*-homeomorphism.

Remark 3.43: The concept of semi*δ-homeomorphism and g-homeomorphism are independent.

Example 3.44: Let X=Y ={a, b, c, d}, τ={φ, {a},{b}, {a, b}, {a, d},{a, b, d}, X} and σ ={φ,{a}, {c},{a d},{a, c},{a, c, d},Y}, S*δO(X, τ)={φ,{b},{b, c},{a, d},{a, c, d}, {a, b, d}, X} , gO (X, τ) = {φ,{a},{b},{d}, {b, d},{a, b},{a, d},{a, b, d},X} and S*δO(Y ,σ) = {φ , {c}, {b, c}{a, d}, {a, c, d} , {a, b, d}, Y} , gO(Y, σ)={φ,{a},{c},{d}, {a, c}, {c, d},{a, d},{a, c, d},Y}. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a map defined by f(a)=a, f(b)= c, f(c)=b, f(d)=d. Clearly f is g-homeomorphism. Here {a} is open in

X, but f({a})={a} not semi*δ-open in Y. Therefore f is not semi*δ- homeomorphism

Example 3.45: Let X=Y ={a, b, c, d}, τ={φ, {c}, {d},{a, c},{ c, d},{a, c, d}, X} and σ = {φ,{b}, {c},{ b, c}Y}, S*δO(X, τ)= {φ, {d},{a, c}{b, d},{a, b, c},{a, c, d}, X} , gO(X, τ)={φ, {a}, {c}, {d},{c, d},{a, c}, {a, d}, {a, c ,d},X} and S*δ O(Y ,σ) = {φ , {b}, {c}, {b, c}, Y}, P*O(Y, σ)={φ, {a},{b}, {c}, {d},{a, b}{b, c},{c, d},{a, c}{b, d}, {a, b, d}{b, c, d},Y}). Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a map defined by f(a)= f(c)=c, f(b)=f(d)=b. Clearly f is semi*δ-homeomorphism. Here {b} is open in Y, but f^{-1}({ b})={b, d} not g-open in X. Hence f is not g-continuous. Therefore f is not g-homeomorphism

Remark 3.46: The composition of two semi*δ-homeomorphism need not to be a semi*δ-homeomorphism.

Example 3.47: Let X=Y=Z={a, b, c, d} , τ={φ,{c},{d},{a, c},{c, d},{a, c, d},X} and σ = {φ,{b},{c},Y}, ω={ φ, {a}, {b}, {a, b}, Z} S*δO(X, τ)={φ, {d},{a, c},{b, d}{a, b, c},{a, c, d}, X} and S*δO(Y, σ)={φ, {b},{c}, {b, c}, Y}, S*δO(Z, ω)={φ, {a},{b}, {a, b}, Z}. Let f:(X, τ)→(Y, σ) be a map defined by f(a)=f(c)=c, f(b)=f(d)=b. Clearly f is semi*δ-homeomorphism. Let g: (Y, σ)→(Z, ω) be a map defined by g(a)=c, g(b)=b, g(c)=a, g(d)=d. Clearly, g is semi*δ-homeomorphism Here f and g are semi *δ-homeomorphism. But$(f\circ g)$({c})=f(g(c))=f(a)=c. But {c} is not semi* δ-open in (Z, ω). Therefore $(f\circ g)$ is not semi*δ- homeomorphism.

4. STRONGLY SEMI*δ HOMEOMORPHISM

Definition 4.1: A bijection f:(X,τ)→(Y,σ) is said to be **strongly semi*δ –homeomorphism** if both f and f^{-1} are semi*δ –Irresolute. We denote the family of all strongly semi*δ-homeomorphism of a topological space X into itself by **s*δ –*h^s(X)***.

Example 4.2: Let X=Y= {a, b, c}, τ={φ, {a},{b},{a, b},X} and σ={φ, {a}, {c}, {a, c}, Y} , S*δ O(X ,τ)={φ, {a},{b},{a, b},{b, c}{a, c}, X} and S*δ O(Y, σ)= {φ, {a},{c}, {a, b},{a, c}, {b, c}, Y}. Let f:(X, τ) →(Y, σ) be a map defined by f(a)=a, f(b)=c, f(c)=b. Clearly, f is strongly semi*δ- homeomorphism.

Theorem 4.3: If f:(X,τ)→(Y,σ) and g:(Y,σ) → (Z, ω) are strongly semi*δ-homeomorphism then $(g\circ f):(X,\tau)\rightarrow(Z,\omega)$ is also strongly semi*δ-homeomorphism.

Proof:

(i) Let U be a semi *δ –open in Z. Now, $(g\circ f)^{-1}(U)=f^{-1}(g^{-1}(U))=f^{-1}(V)$ where $V=g^{-1}(U)$. By hypothesis, $V=g^{-1}(U)$ is semi*δ-open in Y and so again, by hypothesis $f^{-1}(V)$ is semi*δ-open in X. Thus $(g\circ f)$ is semi*δ- irresolute.

(ii) Let G be a semi*δ-open in X. By hypothesis, f(G) is semi*δ-open in Y. Again, by hypothesis $(g\circ f)(G)=g(f(G))$ is semi*δ-open in Z .Thus, $(g\circ f)^{-1}$is semi*δ –irresolute. Hence $(g\circ f)$ strongly semi*δ-homeomorphism.

Theorem 4.4: Every strongly semi*δ-homeomorphism is semi*δ-irresolute.
Proof: It is the consequence of the definition.

Remark 4.5: The converse of the above theorem need be true.

Example 4.6: Let X=Y= {a, b, c, d}, τ={φ, {a},{b},{c},{a, b},{a, c},{b, c},{a, b, c},X} and σ={ φ, {b}, {c}, {b, c}, Y} , S*δO(X, τ)= {φ, {a},{b},{c},{a, b},{a, d},{b ,c},{c, d},{a, c},{b, d},{a, b, c}{a, b, d},{a, c, d},{b, c, d}X} and S*δO(Y, σ)={φ, {b},{c}, {b, c}, Y}. Let f: (X, τ)→(Y, σ) be an identity map. Clearly, f is semi*δ- irresolute. But for the open set {b} in(X, τ), $(f^{-1})^{-1}(\{a\})=f(\{a\})=a$ is not open in (Y, σ).

Theorem 4.7: The set s*δ-h^s(X) is a group under the composition of maps.
Proof: Define a binary operation '∗' as follows. ∗ : s*δ –*h^s(X)* × s*δ –*h^s(X)* ⟶ s*δ-*h^s(X)* , by f ∗ g = g o f for all f , g ∊ s*δ-*h^s(X)* and o is the usual operation of composition of maps. By the above result g o f ∊ s*δ-*h^s(X)* . We know that the composition of maps are associative and the identity map I: X ⟶ X ∊ s*δ-*h^s(X)* serves as the identity element. If f ∊ s*δ –*h^s(X)* then f^{-1} ∊ s*δ-*h^s(X)* such that f o f^{-1} = f^{-1} o f = I and so inverse exists for each element of s*δ-*h^s(X)*. Therefore, s*δ-*h^s(X)* is a group under the composition of maps.

Theorem 4.8: Let f*:* (X, τ) ⟶(Y, σ) be a strongly semi*δ -homeomorphism. Then f induces an isomorphism from the group s*δ-*h^s(X)* onto the group s*δ-*h^s(Y)*.
Proof: Using the map f, we define a map ψ_f : s*δ-*h^s(X)* ⟶ s*δ-*h^s(Y)* by ψ_f (ϕ) = f o ϕ o f^{-1} for every ϕ ∊ s*δ-*h^s(X)*. By theorem 4.6, ψ_f is well defined in general, because f o ϕ o f^{-1} is a strongly semi*δ-homeomorphism for every strongly semi*δ -homeomorphism ϕ: X ⟶ Y. We have to show that ψ_f is a bijective homeomorphism. Bijection of ψ_f

is clear. Further for all ϕ_1 , $\phi_2 \in$ s*δ –$h^s(X)$, $\psi_f(\phi_1 \circ \phi_2) = f \circ (\phi_1 \circ \phi_2) \circ f^{-1} = (f \circ \phi_1 \circ f^{-1}) \circ (f \circ \phi_2 \circ f^{-1}) = \psi_f(\phi_1) \circ \psi_f(\phi_2)$. Therefore, ψ_f is a homeomorphism and hence it induces an isomorphism induced by f.

Theorem 4.9: If f: (X, τ)→(Y, σ) is strongly semi*δ-homeomorphism then s*δ cl(f^{-1}(B)) = f^{-1}(s*δcl(B)) for all B⊆Y.

Proof: Since f is strongly semi*δ-homeomorphism, f is semi*δ-irresolute. Since s*δcl(f(B)) is a semi*δ-closed set in Y, f^{-1}(s*δcl(f(B))) is semi*δ-closed in (X, τ). Now f^{-1}(B)⊆ f^{-1} (s*δ cl (B)) and so by theorem 4.5 [6], s*δcl(f^{-1}(B))⊆f^{-1}(s*δcl(B)). Again since f is strongly semi*δ-homeomorphism, f^{-1} is semi*δ-irresolute. Since s*δcl(f^{-1}(B)) is semi*δ-closed in X, $(f^{-1})^{-1}$ (s*δcl(f^{-1}(B)))=f(s*δcl (f^{-1}(B))) is semi*δ-closed in Y. Now B⊆$(f^{-1})^{-1}$ (f^{-1} (B))⊆$(f^{-1})^{-1}$(s*δcl(f^{-1} (B)))=f(s*δcl(f^{-1}(B))) and so s*δcl(B) ⊆ f(s*δcl(f^{-1}(B))) . Hence f^{-1}(s*δcl(B)) ⊆ f^{-1} (f(s*δcl(f^{-1} (B)))) ⊆ s*δcl(f^{-1}(B)) and hence the equality holds.

Corollary4.10: If f: (X, τ)→(Y, σ) is strongly semi*δ-homeomorphism then s*δcl(f(B))=f(s*δcl(B)) for all B⊆ X.

Proof: Since f: (X, τ)→(Y, σ) is strongly semi*δ-homeomorphism, f^{-1}: (Y, σ) →(X, τ) is also strongly semi*δ-homeomorphism. Therefore by the above theorem s*δcl((($f^{-1})^{-1}$)(B))=$(f^{-1})^{-1}$ (s*δcl(B)) for all B⊆X. That is s*δcl(f(B))=f(s*δcl(B))
.

Corollary 4.11: If f: (X, τ)→(Y, σ) is strongly semi*δ-homeomorphism, then f(s*δint(B))=s*δint(f(B)) for all B⊆X.

Proof: For any set B ⊆ X , $(s*\delta int(B))^c$ = s*δcl(B^c).Thus s*δint(B) =$(s*\delta cl(B^c))^c$. Then f(s*δint(B)) = f($(s*\delta cl(B^c))^c$) = $(f(s*\delta cl(B^c)))^c$ = $(s*\delta cl(f(B^c)))^c$ =s*δint(f(B)).

Corollary 4.12: If f: (X, τ)→(Y, σ) is strongly semi*δ-homeomorphism, then f^{-1}(s*δint(B)) = s*δint (f^{-1}(B)) for all B⊆Y.

Proof: If f: (X, τ)→(Y, σ) is strongly semi*δ-homeomorphism then f^{-1}: (Y, σ) → (X, τ) is also strongly semi*δ-homeomorphism, the proof follows from the above corollary.

REFERENCES

[1] Levine. N, Semi – open sets and semi – continuity in topological spaces, Amer.Math. (1963), 36-41.
[2] Pious Missier.S and Reena.C, On Semi*δ-Open Sets in Topological Spaces, IOSR Journal
[3] Pious Missier S and Robert A, More functions associated with semi-star-open sets, IJSIMR, Vol 2(5), May 2014, 460-468.
[4] Pious Missier.S and Reena.C, Continuous and Irresolute Functions Via Star Generalised Closed Sets, IJMER- 7(2),Feb 2017, 58-64.
[5] Pious Missier, S, Reena, C &Mahadevan, G, 2017, 'Contra Semi*δ-Continuous Functions in Topological Spaces', International Journal of Mathematics And its Applications, Vol. 5, no. 4-A, pp 19-30.
[6] Willard, S., General Topology, Addison – Wesley Publishing Company, Inc.(1970).